Maxwell's equations

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Overview

Maxwell's equations

Maxwell's equations are a set of partial differential equations that, together with the Lorentz force law, form the foundation of classical electrodynamics, classical optics, and electric circuits. These fields in turn underlie modern electrical and communications technologies.

Maxwell's equations have two major variants. The "microscopic" set of Maxwell's equations uses total charge and total current including the difficult-to-calculate atomic level charges and currents in materials. The "macroscopic" set of Maxwell's equations defines two new auxiliary fields that can sidestep having to know these 'atomic' sized charges and currents.

Maxwell's equations are named after the Scottish physicist and mathematician James Clerk Maxwell, since in an early form they are all found in a four-part paper, "On Physical Lines of Force," which he published between 1861 and 1862. The mathematical form of the Lorentz force law also appeared in this paper.

It is often useful to write Maxwell's equations in other forms; these representations are still formally termed "Maxwell's equations". A relativistic formulation in terms of covariant field tensors is used in special relativity, while, in quantum mechanics, a version based on the electric and magnetic potentials is preferred.

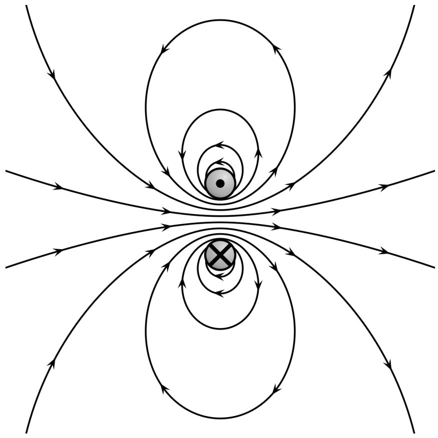
Conceptual description

Conceptually, Maxwell's equations describe how electric charges and electric currents act as sources for the electric and magnetic fields. Further, it describes how a time varying electric field generates a time varying magnetic field and vice versa. (See below for a mathematical description of these laws.) Of the four equations, two of them, Gauss's law and Gauss's law for magnetism, describe how the fields emanate from charges. (For the magnetic field there is no magnetic charge and therefore magnetic fields lines neither begin nor end anywhere.) The other two equations describe how the fields 'circulate' around their respective sources; the magnetic field 'circulates' around electric currents and time varying electric field in Ampère's law with Maxwell's correction, while the electric field 'circulates' around time varying magnetic fields in Faraday's law.

Gauss's law

Gauss's law describes the relationship between an electric field and the electric charges that cause it: The electric field points away from positive charges and towards negative charges. In the field line description, electric field lines begin only at positive electric charges and end only at negative electric charges. 'Counting' the number of field lines in a closed surface, therefore, yields the total charge enclosed by that surface. More technically, it relates the electric flux through any hypothetical closed "Gaussian surface" to the enclosed electric charge.

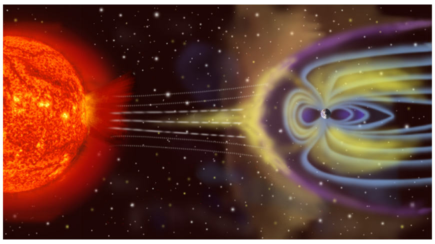
Gauss's law for magnetism



Gauss's law for magnetism: magnetic field lines never begin nor end but form loops or extend to infinity as shown here with the magnetic field due to a ring of current.

Gauss's law for magnetism states that there are no "magnetic charges" (also called magnetic monopoles), analogous to electric charges.[1] Instead, the magnetic field due to materials is generated by a configuration called a dipole. Magnetic dipoles are best represented as loops of current but resemble positive and negative 'magnetic charges', inseparably bound together, having no net 'magnetic charge'. In terms of field lines, this equation states that magnetic field lines neither begin nor end but make loops or extend to infinity and back. In other words, any magnetic field line that enters a given volume must somewhere exit that volume. Equivalent technical statements are that the sum total magnetic flux through any Gaussian surface is zero, or that the magnetic field is a solenoidal vector field.

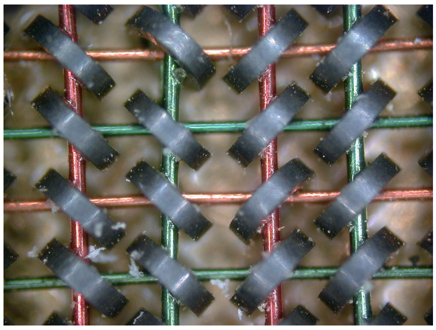
Faraday's law



In a geomagnetic storm, a surge in the flux of charged particles temporarily alters Earth's magnetic field, which induces electric fields in Earth's atmosphere, thus causing surges in our electrical power grids.

Faraday's law describes how a time varying magnetic field creates ("induces") an electric field.[1] This aspect of electromagnetic induction is the operating principle behind many electric generators: for example a rotating bar magnet creates a changing magnetic field, which in turn generates an electric field in a nearby wire. (Note: there are two closely related equations which are called Faraday's law. The form used in Maxwell's equations is always valid but more restrictive than that originally formulated by Michael Faraday.)

Ampère's law with Maxwell's correction



An Wang's magnetic core memory (1954) is an application of Ampère's law. Each core stores one bit of data.

Ampère's law with Maxwell's correction states that magnetic fields can be generated in two ways: by electrical current (this was the original "Ampère's law") and by changing electric fields (this was "Maxwell's correction").

Maxwell's correction to Ampère's law is particularly important: it shows that not only a changing magnetic field induces an electric field, but also a changing electric field induces a magnetic field.[1] [2] Therefore, these equations allow self-sustaining "electromagnetic waves" to travel through empty space (see electromagnetic wave equation).

The speed calculated for electromagnetic waves, which could be predicted from experiments on charges and currents,[3] exactly matches the speed of light; indeed, light *is* one form of electromagnetic radiation (as are X-rays, radio waves, and others). Maxwell understood the connection between electromagnetic waves and light in 1861, thereby unifying the theories of electromagnetism and optics.

Units and summary of equations

Maxwell's equations vary with the unit system used. Though the general form remains the same, various definitions get changed and different constants appear at different places. (This may seem strange at first, but this is because some unit systems, e.g. variants of cgs, define their units in such a way that certain physical constants are fixed, dimensionless constants, e.g. 1, so these constants disappear from the equations.) The equations in this section are given in SI units. Other units commonly used are Gaussian units (based on the cgs system[4]), Lorentz–Heaviside units (used mainly in particle physics) and Planck units (used in theoretical physics). See below for CGS-Gaussian units.

For a description of the difference between the microscopic and macroscopic variants of Maxwell's equations see the relevant sections below.

In the equations given below, symbols in **bold** represent vector quantities, and symbols in *italics* represent scalar quantities. The definitions of terms used in the two tables of equations are given in another table immediately following.

Table of 'microscopic' equations

Formulation in terms of *total* charge and current[5]

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q(V)}{\varepsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_S(\mathbf{E})}{\partial t}$

Here, in agreement with the common definition, it is assumed that one works in a system where the integration regions are constant. Thus, for example, $\frac{\partial \Phi_S(\mathbf{B})}{\partial t} := \iint_S \frac{\partial(\mathbf{B})}{\partial t} \cdot \mathrm{d}\mathbf{S}$ .However, exactly this equation, and *not* $\frac{\partial \Phi_S(\mathbf{B})}{\partial t} := \frac{\mathrm{d}}{\mathrm{dt}} \iint_{S(t)} \mathbf{B}(t) \cdot \mathrm{d}\mathbf{S}$,would also be true, if S were to depend on time as well.

Table of 'macroscopic' equations

Formulation in terms of *free* charge and current

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_f$	$\oiint_{\partial V} \mathbf{D} \cdot \mathrm{d}\mathbf{A} = Q_f(V)$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_{\partial S} \mathbf{H} \cdot \mathrm{d}\mathbf{l} = I_{f,S} + \frac{\partial \Phi_S(\mathbf{D})}{\partial t}$

Table of terms used in Maxwell's equations

The following table provides the meaning of each symbol and the SI unit of measure:

Definitions and units

Symbol	Meaning (first term is the most common)	SI Unit of Measure
$\mathbf{E}$	electric field also called the electric field intensity	volt per meter or, equivalently, newton per coulomb
$\mathbf{B}$	magnetic field also called the magnetic induction also called the magnetic field density also called the magnetic flux density	tesla, or equivalently, weber per square meter, volt-second per square meter
$\mathbf{D}$	electric displacement field also called the electric induction also called the electric flux density	coulombs per square meter or equivalently, newton per volt-meter
$\mathbf{H}$	magnetizing field also called auxiliary magnetic field also called magnetic field intensity also called magnetic field	ampere per meter
$\nabla \cdot$	the divergence operator	per meter (factor contributed by applying either operator)
$\nabla \times$	the curl operator	
$\frac{\partial}{\partial t}$	partial derivative with respect to time	per second (factor contributed by applying the operator)
S and ∂S	S is any surface, and ∂S is its boundary curve. The surface is fixed (unchanging in time).	
V and ∂V	V is any three-dimensional volume, and ∂V is its boundary surface. The volume is fixed (unchanging in time).	
$\mathrm{d}\mathbf{A}$	differential vector element of surface area *A*, with infinitesimally small magnitude and direction normal to surface *S*	square meters
$\mathrm{d}\mathbf{l}$	differential vector element of *path length* tangential to the path/curve	meters
ε_0	permittivity of free space, also called the electric constant, a universal constant	farads per meter
μ_0	permeability of free space, also called the magnetic constant, a universal constant	henries per meter, or newtons per ampere squared
ρ_f	free charge density (not including bound charge)	coulombs per cubic meter

ρ	total charge density (including both free and bound charge)	coulombs per cubic meter
$\mathbf{J}_f$	free current density (not including bound current)	amperes per square meter
$\mathbf{J}$	total current density (including both free and bound current)	amperes per square meter
$Q_f(V)$	net free electric charge within the three-dimensional volume *V* (not including bound charge)	coulombs
$Q(V)$	net electric charge within the three-dimensional volume *V* (including both free and bound charge)	coulombs
$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l}$	line integral of the electric field along the boundary ∂S of a surface S (∂S is always a closed curve).	joules per coulomb
$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l}$	line integral of the magnetic field over the closed boundary ∂S of the surface S	tesla-meters
$\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A}$	the electric flux (surface integral of the electric field) through the (closed) surface ∂V (the boundary of the volume *V*)	joule-meter per coulomb
$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A}$	the magnetic flux (surface integral of the magnetic B-field) through the (closed) surface ∂V (the boundary of the volume *V*)	tesla meters-squared or webers
$\iint_S \mathbf{B} \cdot \mathrm{d}\mathbf{A} = \Phi_S(\mathbf{B})$	magnetic flux through any surface S, not necessarily closed	webers or equivalently, volt-seconds
$\iint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \Phi_S(\mathbf{E})$	electric flux through any surface S, not necessarily closed	joule-meters per coulomb
$\iint_S \mathbf{D} \cdot \mathrm{d}\mathbf{A} = \Phi_S(\mathbf{D})$	flux of electric displacement field through any surface S, not necessarily closed	coulombs
$\iint_S \mathbf{J}_f \cdot \mathrm{d}\mathbf{A} = I_{f,s}$	net free electrical current passing through the surface S (not including bound current)	amperes
$\iint_S \mathbf{J} \cdot \mathrm{d}\mathbf{A} = I_S$	net electrical current passing through the surface S (including both free and bound current)	amperes

Proof that the two general formulations are equivalent

The two alternate general formulations of Maxwell's equations given above are mathematically equivalent and related by the following relations:

$$\rho_b = -\nabla \cdot \mathbf{P},$$

$$\mathbf{J}_b = \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t},$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P},$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}),$$

$$\rho = \rho_b + \rho_f,$$

$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f,$$

where **P** and **M** are polarization and magnetization, and ϱ_b and $\mathbf{J}_b$ are bound charge and current, respectively. Substituting these equations into the 'macroscopic' Maxwell's equations gives identically the microscopic equations.

Relationship between differential and integral forms

The differential and integral forms of the equations are mathematically equivalent, by the divergence theorem in the case of Gauss's law and Gauss's law for magnetism, and by the Kelvin–Stokes theorem in the case of Faraday's law and Ampère's law. Both the differential and integral forms are useful. The integral forms can often be used to simply and directly calculate fields from symmetric distributions of charges and currents. On the other hand, the differential forms are a more natural starting point for calculating the fields in more complicated (less symmetric) situations, for example using finite element analysis.[6]

Maxwell's 'microscopic' equations

The *microscopic* variant of Maxwell's equation expresses the electric **E** field and the magnetic **B** field in terms of the *total charge* and total *current* present including the charges and currents at the atomic level. It is sometimes called the general form of Maxwell's equations or "Maxwell's equations in a vacuum". Both variants of Maxwell's equations are equally general, though, as they are mathematically equivalent. The microscopic equations are most useful in waveguides for example, when there are no dielectric or magnetic materials nearby.

Formulation in terms of *total* charge and current[7]

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$	$\oint\!\!\oint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \dfrac{Q(V)}{\varepsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint\!\!\oint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\dfrac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \dfrac{\partial \Phi_S(\mathbf{E})}{\partial t}$

With neither charges nor currents

Further information: Electromagnetic wave equation and Sinusoidal plane-wave solutions of the electromagnetic wave equation

In a region with no charges ($\varrho = 0$) and no currents ($\mathbf{J} = 0$), such as in a vacuum, Maxwell's equations reduce to:

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

These equations lead directly to **E** and **B** satisfying the wave equation for which the solutions are linear combinations of plane waves traveling at the speed of light,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

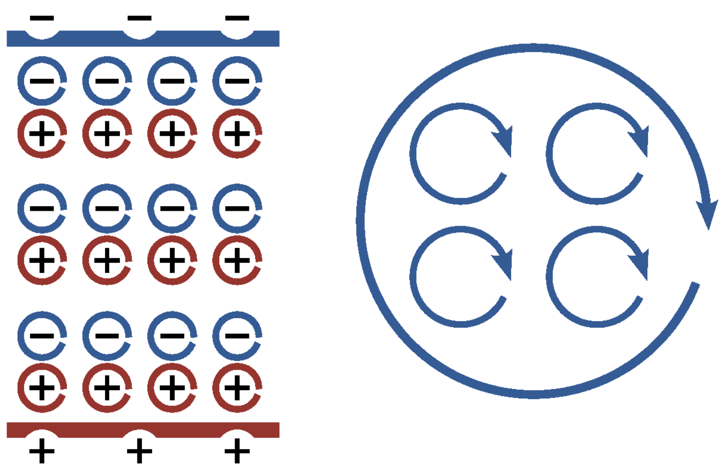
In addition, **E** and **B** are mutually perpendicular to each other and the direction of motion and are in phase with each other. A sinusoidal plane wave is one special solution of these equations.

In fact, Maxwell's equations explain how these waves can physically propagate through space. The changing magnetic field creates a changing electric field through Faraday's law. In turn, that electric field creates a changing magnetic field through Maxwell's correction to Ampère's law. This perpetual cycle allows these waves, now known as electromagnetic radiation, to move through space at velocity *c*.

Maxwell's 'macroscopic' equations

Unlike the 'microscopic' equations, "Maxwell's macroscopic equations", also known as **Maxwell's equations in matter**, factor out the bound charge and current to obtain equations that depend only on the free charges and currents. These equations are more similar to those that Maxwell himself introduced. The cost of this factorization is that additional fields need to be defined: the displacement field **D** which is defined in terms of the electric field **E** and the polarization **P** of the material, and the magnetic-**H** field, which is defined in terms of the magnetic-**B** field and the magnetization **M** of the material.

Bound charge and current



Left: A schematic view of how an assembly of microscopic dipoles produces opposite surface charges as shown at top and bottom. *Right:* How an assembly of microscopic current loops add together to produce a macroscopically circulating current loop. Inside the boundaries, the individual contributions tend to cancel, but at the boundaries no cancellation occurs.

When an electric field is applied to a dielectric material its molecules respond by forming microscopic electric dipoles—their atomic nuclei move a tiny distance in the direction of the field, while their electrons move a tiny distance in the opposite direction. This produces a *macroscopic bound charge* in the material even though all of the charges involved are bound to individual molecules. For example, if every molecule responds the same, similar to that shown in the figure, these tiny movements of charge combine to produce a layer of positive bound charge on one side of the material and a layer of negative charge on the other side. The bound charge is most conveniently described in terms of a polarization, **P**, in the material. If **P** is uniform, a macroscopic separation of charge is produced only at the surfaces where **P** enter and leave the material. For non-uniform **P**, a charge is also produced in the bulk.[8]

Somewhat similarly, in all materials the constituent atoms exhibit magnetic moments that are intrinsically linked to the angular momentum of the atoms' components, most notably their electrons. The connection to angular momentum suggests the picture of an assembly of microscopic current loops. Outside the material, an assembly of such microscopic current loops is not different from a macroscopic current circulating around the material's surface, despite the fact that no individual magnetic moment is traveling a large distance. These *bound currents* can be described using the magnetization **M**.[9]

The very complicated and granular bound charges and bound currents, therefore can be represented on the macroscopic scale in terms of **P** and **M** which average these charges and currents on a sufficiently large scale so as not to see the granularity of individual atoms, but also sufficiently small that they vary with location in the material. As such, the *Maxwell's macroscopic equations* ignores many details on a fine scale that may be unimportant to understanding matters on a grosser scale by calculating fields that are averaged over some suitably sized volume.

Equations

Formulation in terms of *free* charge and current

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_f$	$\oiint_{\partial V} \mathbf{D} \cdot \mathrm{d}\mathbf{A} = Q_f(V)$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_{\partial S} \mathbf{H} \cdot \mathrm{d}\mathbf{l} = I_{f,S} + \frac{\partial \Phi_S(\mathbf{D})}{\partial t}$

Constitutive relations

In order to apply 'Maxwell's macroscopic equations', it is necessary to specify the relations between displacement field **D** and **E**, and the magnetic H-field **H** and **B**. These equations specify the response of bound charge and current to the applied fields and are called constitutive relations.

Determining the constitutive relationship between the auxiliary fields **D** and **H** and the **E** and **B** fields starts with the definition of the auxiliary fields themselves:

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t)$$
$$\mathbf{H}(\mathbf{r},t) = \frac{1}{\mu_0}\mathbf{B}(\mathbf{r},t) - \mathbf{M}(\mathbf{r},t),$$

where **P** is the polarization field and **M** is the magnetization field which are defined in terms of microscopic bound charges and bound current respectively. Before getting to how to calculate **M** and **P** it is useful to examine some special cases, though.

Without magnetic or dielectric materials

In the absence of magnetic or dielectric materials, the constitutive relations are simple:

$$\mathbf{D} = \varepsilon_0 \mathbf{E}, \quad \mathbf{H} = \mathbf{B}/\mu_0$$

where ε_0 and μ_0 are two universal constants, called the permittivity of free space and permeability of free space, respectively. Substituting these back into Maxwell's macroscopic equations lead directly to Maxwell's microscopic equations, except that the currents and charges are replaced with free currents and free charges. This is expected since there are no bound charges nor currents.

Isotropic Linear materials

In an (isotropic[10]) linear material, where **P** is proportional to **E** and **M** is proportional to **B** the constitutive relations are also straightforward. In terms of the polarization **P** and the magnetization **M** they are:

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}, \quad \mathbf{M} = \chi_m \mathbf{H},$$

where χ_e and χ_m are the electric and magnetic susceptibilities of a given material respectively. In terms of **D** and **H** the constitutive relations are:

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{H} = \mathbf{B}/\mu,$$

where ε and μ are constants (which depend on the material), called the permittivity and permeability, respectively, of the material. These are related to the susceptibilities by:

$$\varepsilon = \varepsilon_0(1+\chi_e) \quad \mu = \mu_0(1+\chi_m)$$

Substituting in the constitutive relations above into Maxwell's equations in linear, dispersionless, time-invariant materials (differential form only) are:

$$\nabla \cdot (\varepsilon \mathbf{E}) = \rho_f$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times (\mathbf{B}/\mu) = \mathbf{J}_f + \varepsilon \frac{\partial \mathbf{E}}{\partial t}.$$

These are formally identical to the *general* formulation in terms of **E** and **B** (given above), except that the permittivity of free space was replaced with the permittivity of the material, the permeability of free space was replaced with the permeability of the material, and only free charges and currents are included (instead of all charges and currents). Unless that material is homogeneous in space, ε and μ cannot be factored out of the derivative expressions on the left sides.

General case

For real-world materials, the constitutive relations are not linear, except approximately. Calculating the constitutive relations from first principles involves determining how **P** and **M** are created from a given **E** and **B**.[11] These relations may be empirical (based directly upon measurements), or theoretical (based upon statistical mechanics, transport theory or other tools of condensed matter physics). The detail employed may be macroscopic or microscopic, depending upon the level necessary to the problem under scrutiny.

In general, though the constitutive relations can usually still be written:

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{H} = \mathbf{B}/\mu$$

but ε and μ are not, in general, simple constants, but rather functions. Examples are:

- *Dispersion and absorption* where ε and μ are functions of frequency. (Causality does not permit materials to be nondispersive; see, for example, Kramers–Kronig relations). Neither do the fields need to be in phase which leads to ε and μ being complex. This also leads to absorption.
- Bi-(an)isotropy where **H** and **D** depend on both **B** and **E**:[12]

$$D = \varepsilon E + \xi H \quad B = \mu H + \zeta E.$$

- *Nonlinearity* where ε and μ are functions of **E** and **B**.
- *Anisotropy* (such as *birefringence* or *dichroism*) which occurs when ε and μ are second-rank tensors,

$$D_i = \sum_j \varepsilon_{ij} E_j \quad B_i = \sum_j \mu_{ij} H_j.$$

- Dependence of **P** and **M** on **E** and **B** at other locations and times. This could be due to *spatial inhomogeneity*; for example in a domained structure, heterostructure or a liquid crystal, or most commonly in the situation where there are simply multiple materials occupying different regions of space). Or it could be due to a time varying medium or due to hysteresis. In such cases **P** and **M** can be calculated as:[13] [14]

$$\mathbf{P}(\mathbf{r}, t) = \varepsilon_0 \int \mathrm{d}^3\mathbf{r}'\mathrm{d}t' \; \hat{\chi}_{\text{elec}}(\mathbf{r}, \mathbf{r}', t, t'; \mathbf{E})\, \mathbf{E}(\mathbf{r}', t')$$
$$\mathbf{M}(\mathbf{r}, t) = \frac{1}{\mu_0} \int \mathrm{d}^3\mathbf{r}'\mathrm{d}t' \; \hat{\chi}_{\text{magn}}(\mathbf{r}, \mathbf{r}', t, t'; \mathbf{B})\, \mathbf{B}(\mathbf{r}', t'),$$

  in which the permittivity and permeability functions are replaced by integrals over the more general electric and magnetic susceptibilities.[15]

In practice, some materials properties have a negligible impact in particular circumstances, permitting neglect of small effects. For example: optical nonlinearities can be neglected for low field strengths; material dispersion is unimportant when frequency is limited to a narrow bandwidth; material absorption can be neglected for wavelengths for which a material is transparent; and metals with finite conductivity often are approximated at microwave or longer wavelengths as perfect metals with infinite conductivity (forming hard barriers with zero skin depth of field penetration).

It may be noted that man-made materials can be designed to have customized permittivity and permeability, such as metamaterials and photonic crystals.

Calculation of constitutive relations

In general, the constitutive equations are theoretically determined by calculating how a molecule responds to the local fields through the Lorentz force. Other forces may need to be modeled as well such as lattice vibrations in crystals or bond forces. Including all of the forces leads to changes in the molecule which are used to calculate **P** and **M** as a function of the local fields.

The local fields differ from the applied fields due to the fields produced by the polarization and magnetization of nearby material; an effect which also needs to be modeled. Further, real materials are not continuous media; the local fields of real materials vary wildly on the atomic scale. The fields need to be averaged over a suitable volume to form a continuum approximation.

These continuum approximations often require some type of quantum mechanical analysis such as quantum field theory as applied to condensed matter physics. See, for example, density functional theory, Green–Kubo relations and Green's function. Various approximate transport equations have evolved, for example, the Boltzmann equation or the Fokker–Planck equation or the Navier–Stokes equations. Some examples where these equations are applied are magnetohydrodynamics, fluid dynamics, electrohydrodynamics, superconductivity, plasma modeling. An entire physical apparatus for dealing with these matters has developed. A different set of *homogenization methods* (evolving from a tradition in treating materials such as conglomerates and laminates) are based upon approximation of an inhomogeneous material by a homogeneous *effective medium*[16] [17] (valid for excitations with wavelengths much larger than the scale of the inhomogeneity).[18] [19] [20] [21]

The theoretical modeling of the continuum-approximation properties of many real materials often rely upon measurement as well,[22] for example, ellipsometry measurements.

History

Relation between electricity, magnetism, and the speed of light

The relation between electricity, magnetism, and the speed of light can be summarized by the modern equation:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \ .$$

The left-hand side is the speed of light, and the right-hand side is a quantity related to the equations governing electricity and magnetism. Although the right-hand side has units of velocity, it can be inferred from measurements of electric and magnetic forces, which involve no physical velocities. Therefore, establishing this relationship provided convincing evidence that light is an electromagnetic phenomenon.

The discovery of this relationship started in 1855, when Wilhelm Eduard Weber and Rudolf Kohlrausch determined that there was a quantity related to electricity and magnetism, "the ratio of the absolute electromagnetic unit of charge to the absolute electrostatic unit of charge" (in modern language, the value $1/\sqrt{\mu_0\varepsilon_0}$), and determined that it should have units of velocity. They then measured this ratio by an experiment which involved charging and discharging a Leyden jar and measuring the magnetic force from the discharge current, and found a value 3.107×10^8 m/s,[23] remarkably close to the speed of light, which had recently been measured at 3.14×10^8 m/s by Hippolyte Fizeau in 1848 and at 2.98×10^8 m/s by Léon Foucault in 1850.[23] However, Weber and Kohlrausch did not make the connection to the speed of light.[23] Towards the end of 1861 while working on part III of his paper *On*

Physical Lines of Force, Maxwell travelled from Scotland to London and looked up Weber and Kohlrausch's results. He converted them into a format which was compatible with his own writings, and in doing so he established the connection to the speed of light and concluded that light is a form of electromagnetic radiation.[24]

The term *Maxwell's equations*

The four modern Maxwell's equations can be found individually throughout his 1861 paper, derived theoretically using a molecular vortex model of Michael Faraday's "lines of force" and in conjunction with the experimental result of Weber and Kohlrausch. But it wasn't until 1884 that Oliver Heaviside,[25] concurrently with similar work by Willard Gibbs and Heinrich Hertz,[26] grouped the four together into a distinct set. This group of four equations was known variously as the Hertz-Heaviside equations and the Maxwell-Hertz equations,[25] and are sometimes still known as the Maxwell–Heaviside equations.[27]

Maxwell's contribution to science in producing these equations lies in the correction he made to Ampère's circuital law in his 1861 paper *On Physical Lines of Force*. He added the displacement current term to Ampère's circuital law and this enabled him to derive the electromagnetic wave equation in his later 1865 paper *A Dynamical Theory of the Electromagnetic Field* and demonstrate the fact that light is an electromagnetic wave. This fact was then later confirmed experimentally by Heinrich Hertz in 1887. The physicist Richard Feynman predicted that, "The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade."[28]

The concept of fields was introduced by, among others, Faraday. Albert Einstein wrote:

> The precise formulation of the time-space laws was the work of Maxwell. Imagine his feelings when the differential equations he had formulated proved to him that electromagnetic fields spread in the form of polarised waves, and at the speed of light! To few men in the world has such an experience been vouchsafed ... it took physicists some decades to grasp the full significance of Maxwell's discovery, so bold was the leap that his genius forced upon the conceptions of his fellow-workers
>
> —(*Science*, May 24, 1940)

Heaviside worked to eliminate the potentials (electric potential and magnetic potential) that Maxwell had used as the central concepts in his equations;[25] this effort was somewhat controversial,[29] though it was understood by 1884 that the potentials must propagate at the speed of light like the fields, unlike the concept of instantaneous action-at-a-distance like the then conception of gravitational potential.[26] Modern analysis of, for example, radio antennas, makes full use of Maxwell's vector and scalar potentials to separate the variables, a common technique used in formulating the solutions of differential equations. However, the potentials can be introduced by algebraic manipulation of the four fundamental equations.

On Physical Lines of Force

The four modern day Maxwell's equations appeared throughout Maxwell's 1861 paper *On Physical Lines of Force*:

1. Equation (56) in Maxwell's 1861 paper is $\nabla \cdot \mathbf{B} = 0$.
2. Equation (112) is Ampère's circuital law with Maxwell's displacement current added. It is the addition of displacement current that is the most significant aspect of Maxwell's work in electromagnetism, as it enabled him to later derive the electromagnetic wave equation in his 1865 paper A Dynamical Theory of the Electromagnetic Field, and hence show that light is an electromagnetic wave. It is therefore this aspect of Maxwell's work which gives the equations their full significance. (Interestingly, Kirchhoff derived the telegrapher's equations in 1857 without using displacement current. But he did use Poisson's equation and the equation of continuity which are the mathematical ingredients of the displacement current. Nevertheless, Kirchhoff believed his equations to be applicable only inside an electric wire and so he is not credited with having discovered that light is an electromagnetic wave).
3. Equation (115) is Gauss's law.

4. Equation (54) is an equation that Oliver Heaviside referred to as 'Faraday's law'. This equation caters for the time varying aspect of electromagnetic induction, but not for the motionally induced aspect, whereas Faraday's original flux law caters for both aspects.[30] [31] Maxwell deals with the motionally dependent aspect of electromagnetic induction, $\mathbf{v} \times \mathbf{B}$, at equation (77). Equation (77) which is the same as equation (D) in the original eight Maxwell's equations listed below, corresponds to all intents and purposes to the modern day force law $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ which sits adjacent to Maxwell's equations and bears the name Lorentz force, even though Maxwell derived it when Lorentz was still a young boy.

The difference between the **B** and the **H** vectors can be traced back to Maxwell's 1855 paper entitled *On Faraday's Lines of Force* which was read to the Cambridge Philosophical Society. The paper presented a simplified model of Faraday's work, and how the two phenomena were related. He reduced all of the current knowledge into a linked set of differential equations.

It is later clarified in his concept of a sea of molecular vortices that appears in his 1861 paper *On Physical Lines of Force*. Within that context, **H** represented pure vorticity (spin), whereas **B** was a weighted vorticity that was weighted for the density of the vortex sea. Maxwell considered magnetic permeability μ to be a measure of the density of the vortex sea. Hence the relationship,

1. **Magnetic induction current** causes a magnetic current density

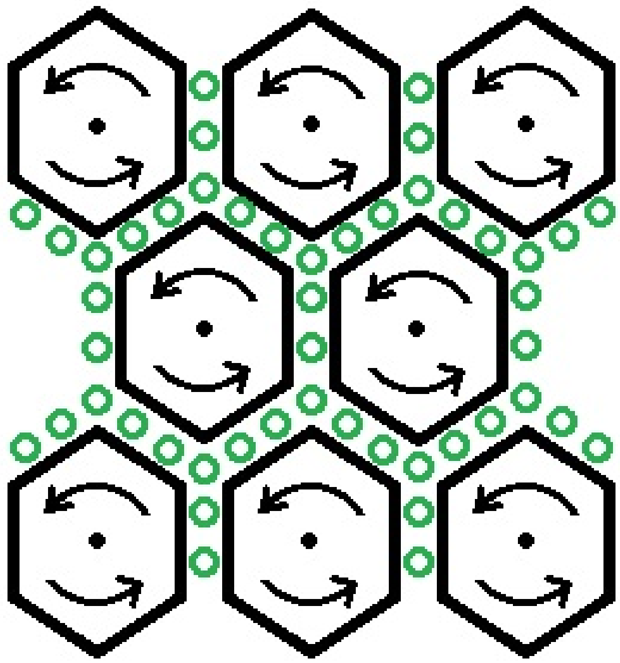


Figure of Maxwell's molecular vortex model. For a uniform magnetic field, the field lines point outward from the display screen, as can be observed from the black dots in the middle of the hexagons. The vortex of each hexagonal molecule rotates counter-clockwise. The small green circles are clockwise rotating particles sandwiching between the molecular vortices.

$$\mathbf{B} = \mu \mathbf{H}$$

was essentially a rotational analogy to the linear electric current relationship,

1. **Electric convection current**

$$\mathbf{J} = \rho \mathbf{v}$$

where ϱ is electric charge density. **B** was seen as a kind of magnetic current of vortices aligned in their axial planes, with **H** being the circumferential velocity of the vortices. With μ representing vortex density, it follows that the product of μ with vorticity **H** leads to the magnetic field denoted as **B**.

The electric current equation can be viewed as a convective current of electric charge that involves linear motion. By analogy, the magnetic equation is an inductive current involving spin. There is no linear motion in the inductive current along the direction of the **B** vector. The magnetic inductive current represents lines of force. In particular, it represents lines of inverse square law force.

The extension of the above considerations confirms that where **B** is to **H**, and where **J** is to ϱ, then it necessarily follows from Gauss's law and from the equation of continuity of charge that **E** is to **D**. i.e. **B** parallels with **E**, whereas **H** parallels with **D**.

A Dynamical Theory of the Electromagnetic Field

In 1864 Maxwell published *A Dynamical Theory of the Electromagnetic Field* in which he showed that light was an electromagnetic phenomenon. Confusion over the term "Maxwell's equations" sometimes arises because it has been used for a set of eight equations that appeared in Part III of Maxwell's 1864 paper A Dynamical Theory of the Electromagnetic Field, entitled "General Equations of the Electromagnetic Field,"[32] and this confusion is compounded by the writing of six of those eight equations as three separate equations (one for each of the Cartesian axes), resulting in twenty equations and twenty unknowns. (As noted above, this terminology is not common: Modern references to the term "Maxwell's equations" refer to the Heaviside restatements.)

The eight original Maxwell's equations can be written in modern vector notation as follows:

(A) The law of total currents

$$\mathbf{J}_{tot} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

(B) The equation of magnetic force

$$\mu \mathbf{H} = \nabla \times \mathbf{A}$$

(C) Ampère's circuital law

$$\nabla \times \mathbf{H} = \mathbf{J}_{\text{tot}}$$

(D) Electromotive force created by convection, induction, and by static electricity. (This is in effect the Lorentz force)

$$\mathbf{E} = \mu \mathbf{v} \times \mathbf{H} - \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi$$

(E) The electric elasticity equation

$$\mathbf{E} = \frac{1}{\varepsilon} \mathbf{D}$$

(F) Ohm's law

$$\mathbf{E} = \frac{1}{\sigma} \mathbf{J}$$

(G) Gauss's law

$$\nabla \cdot \mathbf{D} = \rho$$

(H) Equation of continuity

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

or

$$\nabla \cdot \mathbf{J}_{\text{tot}} = 0$$

Notation

H is the magnetizing field, which Maxwell called the *magnetic intensity*.

J is the current density (with$\mathbf{J}_{\text{tot}}$ being the total current including displacement current).[33]

D is the displacement field (called the *electric displacement* by Maxwell).

ϱ is the free charge density (called the *quantity of free electricity* by Maxwell).

A is the magnetic potential (called the *angular impulse* by Maxwell).

E is called the *electromotive force* by Maxwell. The term electromotive force is nowadays used for voltage, but it is clear from the context that Maxwell's meaning corresponded more to the modern term electric field.

φ is the electric potential (which Maxwell also called *electric potential*).

σ is the electrical conductivity (Maxwell called the inverse of conductivity the *specific resistance*, what is now called the resistivity).

It is interesting to note the $\mu\mathbf{v} \times \mathbf{H}$ term that appears in equation D. Equation D is therefore effectively the Lorentz force, similarly to equation (77) of his 1861 paper (see above).

When Maxwell derives the electromagnetic wave equation in his 1865 paper, he uses equation D to cater for electromagnetic induction rather than Faraday's law of induction which is used in modern textbooks. (Faraday's law itself does not appear among his equations.) However, Maxwell drops the $\mu\mathbf{v} \times \mathbf{H}$ term from equation D when he is deriving the electromagnetic wave equation, as he considers the situation only from the rest frame.

A Treatise on Electricity and Magnetism

In *A Treatise on Electricity and Magnetism*, an 1873 treatise on electromagnetism written by James Clerk Maxwell, eleven general equations of the electromagnetic field are listed and these include the eight that are listed in the 1865 paper.[34]

Maxwell's equations and relativity

Maxwell's original equations are based on the idea that light travels through a sea of molecular vortices known as the 'luminiferous aether', and that the speed of light has to be respective to the reference frame of this aether. Measurements designed to measure the speed of the Earth through the aether conflicted, though.[35]

A more theoretical approach was suggested by Hendrik Lorentz along with George FitzGerald and Joseph Larmor. Both Larmor (1897) and Lorentz (1899, 1904) derived the Lorentz transformation (so named by Henri Poincaré) as one under which Maxwell's equations were invariant. Poincaré (1900) analyzed the coordination of moving clocks by exchanging light signals. He also established mathematically the group property of the Lorentz transformation (Poincaré 1905).

Einstein dismissed the aether as unnecessary and concluded that Maxwell's equations predict the existence of a fixed speed of light, independent of the speed of the observer, and as such he used Maxwell's equations as the starting point for his special theory of relativity. In doing so, he established the Lorentz transformation as being valid for all matter and not just Maxwell's equations. Maxwell's equations played a key role in Einstein's famous paper on special relativity; for example, in the opening paragraph of the paper, he motivated his theory by noting that a description of a conductor moving with respect to a magnet must generate a consistent set of fields irrespective of whether the force is calculated in the rest frame of the magnet or that of the conductor.[36]

General relativity has also had a close relationship with Maxwell's equations. For example, Theodor Kaluza and Oskar Klein showed in the 1920s that Maxwell's equations can be derived by extending general relativity into five dimensions. This strategy of using higher dimensions to unify different forces remains an active area of research in particle physics.

Modified to include magnetic monopoles

Maxwell's equations provide for an electric charge, but posit no magnetic charge. Magnetic charge has never been seen[37] and may not exist. Nevertheless, Maxwell's equations including magnetic charge (and magnetic current) is of some theoretical interest.[38]

For one reason, Maxwell's equations can be made fully symmetric under interchange of electric and magnetic field by allowing for the possibility of magnetic charges with magnetic charge density ϱ_{m} and currents with magnetic current density $\mathbf{J}_{\mathrm{m}}$.[39] The extended Maxwell's equations (in cgs-Gaussian units) are:

Name	Without magnetic monopoles	With magnetic monopoles (hypothetical)
Gauss's law:	$\nabla \cdot \mathbf{E} = 4\pi\rho_{\mathrm{e}}$	$\nabla \cdot \mathbf{E} = 4\pi\rho_{\mathrm{e}}$
Gauss's law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 4\pi\rho_{\mathrm{m}}$
Maxwell–Faraday equation (Faraday's law of induction):	$-\nabla \times \mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$-\nabla \times \mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_{\mathrm{m}}$
Ampère's law (with Maxwell's extension):	$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_{\mathrm{e}}$	$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_{\mathrm{e}}$

If magnetic charges do not exist, or if they exist but not in the region studied, then the new variables are zero, and the symmetric equations reduce to the conventional equations of electromagnetism such as $\nabla \cdot \mathbf{B} = 0$. Further, if every particle has the same ratio of electric to magnetic charge, then an E and a B field can be defined that obeys the normal Maxwell's equation (having no magnetic charges or currents) with its own charge and current densities.[40]

Solving Maxwell's equations

Maxwell's equations are partial differential equations that relate the electric and magnetic fields to each other and to the electric charges and currents. Often, the charges and currents are themselves dependent on the electric and magnetic fields via the Lorentz force equation and the constitutive relations. These all form a set of coupled partial differential equations, which are often very difficult to solve. In fact, the solutions of these equations encompass all the diverse phenomena in the entire field of classical electromagnetism. A thorough discussion is far beyond the scope of the article, but some general notes follow:

- Like any differential equation, boundary conditions[41] [42] [43] and initial conditions[44] are necessary for a unique solution. For example, even with no charges and no currents anywhere in spacetime, many solutions to Maxwell's equations are possible, not just the obvious solution **E**=**B**=0. Another solution is **E**=constant, **B**=constant, while yet other solutions have electromagnetic waves filling spacetime. In some cases, Maxwell's equations are solved through infinite space, and boundary conditions are given as asymptotic limits at infinity.[45] In other cases, Maxwell's equations are solved in just a finite region of space, with appropriate boundary conditions on that region: For example, the boundary could be a artificial absorbing boundary representing the rest of the universe,[46] [47] or periodic boundary conditions, or (as with a waveguide or cavity resonator) the boundary conditions may describe the walls that isolate a small region from the outside world.[48]
- Jefimenko's equations (or the closely related Liénard–Wiechert potentials) are the explicit solution to Maxwell's equations for the electric and magnetic fields created by any given distribution of charges and currents. It assumes specific initial conditions to obtain the so-called "retarded solution", where the only fields present are the ones created by the charges. Jefimenko's equations are not so helpful in situations when the charges and currents are themselves affected by the fields they create.
- Numerical methods for differential equations can be used to approximately solve Maxwell's equations when an exact solution is impossible. These methods usually require a computer, and include the finite element method and finite-difference time-domain method.[41] [43] [49] [50] [51] For more details, see Computational electromagnetics.

Gaussian units

Gaussian units is a popular electromagnetism variant of the centimetre gram second system of units (cgs). In gaussian units, Maxwell's equations are:[52]

$$\nabla \cdot \mathbf{D} = 4\pi\rho_{\mathrm{f}}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J}_{\mathrm{f}}$$

where c is the speed of light in a vacuum. The microscopic equations are:

$$\nabla \cdot \mathbf{E} = 4\pi\rho_{\mathrm{tot}}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}_{\mathrm{tot}}.$$

The relation between electric displacement field, electric field and polarization density is:

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}.$$

And likewise the relation between magnetic induction, magnetic field and total magnetization is:

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}.$$

In the linear approximation, the electric susceptibility and magnetic susceptibility are defined so that:

$$\mathbf{P} = \chi_{\mathrm{e}}\mathbf{E}, \quad \mathbf{M} = \chi_{\mathrm{m}}\mathbf{H}.$$

(Note: although the susceptibilities are dimensionless numbers in both cgs and SI, they differ in value by a factor of 4π.) The permittivity and permeability are:

$$\varepsilon = 1 + 4\pi\chi_{\mathrm{e}}, \quad \mu = 1 + 4\pi\chi_{\mathrm{m}},$$

so that

$$\mathbf{D} = \varepsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}.$$

In vacuum, $\varepsilon = \mu = 1$, therefore **D** = **E**, and **B** = **H**.

The force exerted upon a charged particle by the electric field and magnetic field is given by the Lorentz force equation:

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right),$$

where q is the charge on the particle and **v** is the particle velocity. This is slightly different from the SI-unit expression above. For example, the magnetic field **B** has the same units as the electric field **E**.

Some equations in the article are given in Gaussian units but not SI or vice-versa. Fortunately, there are general rules to convert from one to the other; see the article Gaussian units for details.

Alternative formulations of Maxwell's equations

Special relativity motivated a compact mathematical formulation of Maxwell's equations, in terms of covariant tensors. Quantum mechanics also motivated other formulations.

For example, consider a conductor moving in the field of a magnet.[53] In the frame of the magnet, that conductor experiences a *magnetic* force. But in the frame of a conductor moving relative to the magnet, the conductor experiences a force due to an *electric* field. The following formulation shows how Maxwell's equations take the same form in any inertial coordinate system.

In terms of a minimum action principle

For the field formulation of Maxwell's equations in terms of a principle of extremal action, see the article on the electromagnetic tensor.

Covariant formulation of Maxwell's equations

In special relativity, in order to more clearly express the fact that Maxwell's ('microscopic') equations take the same form in any inertial coordinate system, Maxwell's equations are written in terms of four-vectors and tensors in the "manifestly covariant" form. The purely spatial components of the following are in SI units.

One ingredient in this formulation is the electromagnetic tensor, a rank-2 covariant antisymmetric tensor combining the electric and magnetic fields:

$$F_{\alpha\beta} = \begin{pmatrix} 0 & \frac{-E_x}{c} & \frac{-E_y}{c} & \frac{-E_z}{c} \\ \frac{E_x}{c} & 0 & B_z & -B_y \\ \frac{E_y}{c} & -B_z & 0 & B_x \\ \frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

and the result of raising its indices

$$F^{\mu\nu} \stackrel{\text{def}}{=} \eta^{\mu\alpha} F_{\alpha\beta} \eta^{\beta\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ \frac{-E_x}{c} & 0 & B_z & -B_y \\ \frac{-E_y}{c} & -B_z & 0 & B_x \\ \frac{-E_z}{c} & B_y & -B_x & 0 \end{pmatrix}.$$

The other ingredient is the four-current:

$$J^\alpha = (c\rho, \vec{J})$$

where ρ is the charge density and **J** is the current density.

With these ingredients, Maxwell's equations can be written:

$$\mu_0 J^\beta = \frac{\partial F^{\beta\alpha}}{\partial x^\alpha} \stackrel{\text{def}}{=} \partial_\alpha F^{\beta\alpha} \stackrel{\text{def}}{=} F^{\beta\alpha}{}_{,\alpha}$$

and

$$0 = \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} \stackrel{\text{def}}{=} F_{\alpha\beta,\gamma} + F_{\gamma\alpha,\beta} + F_{\beta\gamma,\alpha}.$$

The first tensor equation is an expression of the two inhomogeneous Maxwell's equations, Gauss's law and Ampère's law with Maxwell's correction. The second equation is an expression of the two homogeneous equations, Faraday's law of induction and Gauss's law for magnetism. The second equation is equivalent to

$$0 = \epsilon^{\delta\alpha\beta\gamma} F_{\beta\gamma,\alpha}$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is the contravariant version of the Levi-Civita symbol, and

$$\frac{\partial}{\partial x^\alpha} \stackrel{\text{def}}{=} \partial_\alpha \stackrel{\text{def}}{=} {}_{,\alpha} \stackrel{\text{def}}{=} \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right)$$

is the 4-gradient. In the tensor equations above, repeated indices are summed over according to Einstein summation convention. We have displayed the results in several common notations. Upper and lower components of a vector, v^α and v^α respectively, are interchanged with the fundamental tensor g, e.g., $g = \eta = \mathrm{diag}(-1, +1, +1, +1)$.

Alternative covariant presentations of Maxwell's equations also exist, for example in terms of the four-potential; see Covariant formulation of classical electromagnetism for details.

Potential formulation

In advanced classical mechanics and in quantum mechanics (where it is necessary) it is sometimes useful to express Maxwell's equations in a 'potential formulation' involving the electric potential (also called scalar potential), φ, and the magnetic potential, **A**, (also called vector potential). These are defined such that:

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t},$$
$$\mathbf{B} = \nabla \times \mathbf{A}.$$

With these definitions, the two homogeneous Maxwell's equations (Faraday's Law and Gauss's law for magnetism) are automatically satisfied and the other two (inhomogeneous) equations give the following equations (for "Maxwell's microscopic equations"):

$$\nabla^2\varphi + \frac{\partial}{\partial t}\left(\nabla \cdot \mathbf{A}\right) = -\frac{\rho}{\varepsilon_0}$$
$$\left(\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2}\right) - \nabla\left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t}\right) = -\mu_0 \mathbf{J}.$$

These equations, taken together, are as powerful and complete as Maxwell's equations. Moreover, if we work only with the potentials and ignore the fields, the problem has been reduced somewhat, as the electric and magnetic fields each have three components which need to be solved for (six components altogether), while the electric and magnetic potentials have only four components altogether.

Many different choices of **A** and φ are consistent with a given **E** and **B**, making these choices physically equivalent – a flexibility known as gauge freedom. Suitable choice of **A** and φ can simplify these equations, or can adapt them to suit a particular situation.

Four-potential

In the Lorenz gauge, the two equations that represent the potentials can be reduced to one manifestly Lorentz invariant equation, using four-vectors: the four-current defined by

$$j^\mu = (\rho c, \mathbf{j})$$

formed from the current density **j** and charge density ρ, and the electromagnetic four-potential defined by

$$A^\mu = (\varphi, \mathbf{A}c)$$

formed from the vector potential **A** and the scalar potential φ. The resulting single equation, due to Arnold Sommerfeld, a generalization of an equation due to Bernhard Riemann and known as the Riemann–Sommerfeld equation[54] or the covariant form of the Maxwell equations,[55] is:

$$\Box A^\mu = \mu_0 j^\mu,$$

where $\Box = \partial^2 = \partial_\alpha \partial^\alpha$ is the d'Alembertian operator, or four-Laplacian, $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)$, sometimes written $\Box^2$, or $\Box \cdot \Box$, where $\Box$ is the four-gradient.

Differential geometric formulations

In free space, where $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ are constant everywhere, Maxwell's equations simplify considerably once the language of differential geometry and differential forms is used. In what follows, cgs-Gaussian units, not SI units are used. (To convert to SI, see here.) The electric and magnetic fields are now jointly described by a 2-form **F** in a 4-dimensional spacetime manifold. Maxwell's equations then reduce to the Bianchi identity

$$\mathrm{d}\mathbf{F} = 0$$

where d denotes the exterior derivative — a natural coordinate and metric independent differential operator acting on forms — and the source equation

$$\mathrm{d} * \mathbf{F} = \mathbf{J}$$

where the (dual) Hodge star operator * is a linear transformation from the space of 2-forms to the space of (4-2)-forms defined by the metric in Minkowski space (in four dimensions even by any metric conformal to this metric), and the fields are in natural units where $1/4\pi\varepsilon_0 = 1$. Here, the 3-form **J** is called the *electric current form* or *current 3-form* satisfying the continuity equation

$$\mathrm{d}\mathbf{J} = 0.$$

The current 3-form can be integrated over a 3-dimensional space-time region. The physical interpretation of this integral is the charge in that region if it is spacelike, or the amount of charge that flows through a surface in a certain amount of time if that region is a spacelike surface cross a timelike interval. As the exterior derivative is defined on any manifold, the differential form version of the Bianchi identity makes sense for any 4-dimensional manifold, whereas the source equation is defined if the manifold is oriented and has a Lorentz metric. In particular the differential form version of the Maxwell equations are a convenient and intuitive formulation of the Maxwell equations in general relativity.

In a linear, macroscopic theory, the influence of matter on the electromagnetic field is described through more general linear transformation in the space of 2-forms. We call

$$C : \Lambda^2 \ni \mathbf{F} \mapsto \mathbf{G} \in \Lambda^{(4-2)}$$

the constitutive transformation. The role of this transformation is comparable to the Hodge duality transformation. The Maxwell equations in the presence of matter then become:

$$\mathrm{d}\mathbf{F} = 0$$

$$\mathrm{d}\mathbf{G} = \mathbf{J}$$

where the current 3-form **J** still satisfies the continuity equation d**J** = 0.

When the fields are expressed as linear combinations (of exterior products) of basis forms θ^p,

$$\mathbf{F} = \frac{1}{2} F_{pq} \theta^p \wedge \theta^q.$$

the constitutive relation takes the form

$$G_{pq} = C_{pq}^{mn} F_{mn}$$

where the field coefficient functions are antisymmetric in the indices and the constitutive coefficients are antisymmetric in the corresponding pairs. In particular, the Hodge duality transformation leading to the vacuum equations discussed above are obtained by taking

$$C_{pq}^{mn} = \frac{1}{2} g^{ma} g^{nb} \epsilon_{abpq} \sqrt{-g}$$

which up to scaling is the only invariant tensor of this type that can be defined with the metric.

In this formulation, electromagnetism generalises immediately to any 4-dimensional oriented manifold or with small adaptations any manifold, requiring not even a metric. Thus the expression of Maxwell's equations in terms of differential forms leads to a further notational and conceptual simplification. Whereas Maxwell's Equations could be written as two tensor equations instead of eight scalar equations, from which the propagation of electromagnetic disturbances and the continuity equation could be derived with a little effort, using differential forms leads to an even simpler derivation of these results.

Conceptual insight from this formulation

On the conceptual side, from the point of view of physics, this shows that the second and third Maxwell equations should be grouped together, be called the homogeneous ones, and be seen as geometric *identities* expressing nothing else than: the *field* **F** derives from a more "fundamental" *potential* **A**. While the first and last one should be seen as the dynamical *equations of motion*, obtained via the Lagrangian principle of least action, from the "interaction term" **A J** (introduced through gauge covariant derivatives), coupling the field to matter.

Often, the time derivative in the third law motivates calling this equation "dynamical", which is somewhat misleading; in the sense of the preceding analysis, this is rather an artifact of breaking relativistic covariance by choosing a preferred time direction. To have physical degrees of freedom propagated by these field equations, one must include a kinetic term **F *F** for **A**; and take into account the non-physical degrees of freedom which can be removed by gauge transformation **A** → **A'** = **A** − dα. See also gauge fixing and Faddeev–Popov ghosts.

Geometric algebra (GA) formulation

In geometric algebra, Maxwell's equations are reduced to a single equation,

$$\left(\frac{1}{c}\partial_t + \boldsymbol{\nabla}\right) F = \mu_0 c J,$$ [56]

where F and J are multivectors

$$F = \mathbf{E} + Ic\mathbf{B}$$

and

$$J = c\rho + \mathbf{J}.$$

with the unit pseudoscalar $I^2 = -1$

The GA spatial gradient operator $\boldsymbol{\nabla}$ acts on a vector field, such that

$$\boldsymbol{\nabla}\mathbf{F} = \boldsymbol{\nabla} \cdot \mathbf{F} + I\boldsymbol{\nabla} \times \mathbf{F},$$

In spacetime algebra using the same geometric product the equation is simply

$$\nabla F = \mu_0 c J,$$

the spacetime derivative of the electromagnetic field is its source. Here the (non-bold) spacetime gradient

$$\nabla = \gamma^\mu \partial_\mu$$

is a four vector, as is the current density

$$J = \gamma_\mu J^\mu = \gamma_0 c\rho + J^k \gamma_k = (c\rho + \mathbf{J})\gamma_0.$$

For a demonstration that the equations given reproduce Maxwell's equations see the main article.

Classical electrodynamics as the curvature of a line bundle

An elegant and intuitive way to formulate Maxwell's equations is to use complex line bundles or principal bundles with fibre U(1). The connection ∇ on the line bundle has a curvature $\mathbf{F} = \nabla^2$ which is a two-form that automatically satisfies $\mathrm{d}\mathbf{F} = 0$ and can be interpreted as a field-strength. If the line bundle is trivial with flat reference connection *d* we can write $\nabla = \mathrm{d} + \mathbf{A}$ and **F** = d**A** with **A** the 1-form composed of the electric potential and the magnetic vector potential.

In quantum mechanics, the connection itself is used to define the dynamics of the system. This formulation allows a natural description of the Aharonov–Bohm effect. In this experiment, a static magnetic field runs through a long magnetic wire (e.g., an iron wire magnetized longitudinally). Outside of this wire the magnetic induction is zero, in contrast to the vector potential, which essentially depends on the magnetic flux through the cross-section of the wire and does not vanish outside. Since there is no electric field either, the Maxwell tensor $\mathbf{F} = \mathbf{0}$ throughout the space-time region outside the tube, during the experiment. This means by definition that the connection ∇ is flat there.

However, as mentioned, the connection depends on the magnetic field through the tube since the holonomy along a non-contractible curve encircling the tube is the magnetic flux through the tube in the proper units. This can be detected quantum-mechanically with a double-slit electron diffraction experiment on an electron wave traveling around the tube. The holonomy corresponds to an extra phase shift, which leads to a shift in the diffraction pattern.[57] [58]

Curved spacetime

Traditional formulation

Matter and energy generate curvature of spacetime. This is the subject of general relativity. Curvature of spacetime affects electrodynamics. An electromagnetic field having energy and momentum also generates curvature in spacetime. Maxwell's equations in curved spacetime can be obtained by replacing the derivatives in the equations in flat spacetime with covariant derivatives. (Whether this is the appropriate generalization requires separate investigation.) The sourced and source-free equations become (cgs-Gaussian units):

$$\frac{4\pi}{c}j^{\beta} = \partial_\alpha F^{\alpha\beta} + {\Gamma^\alpha}_{\mu\alpha}F^{\mu\beta} + {\Gamma^\beta}_{\mu\alpha}F^{\alpha\mu} \stackrel{\text{def}}{=} \nabla_\alpha F^{\alpha\beta} \stackrel{\text{def}}{=} {F^{\alpha\beta}}_{;\alpha}$$

and

$$0 = \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = \nabla_\gamma F_{\alpha\beta} + \nabla_\beta F_{\gamma\alpha} + \nabla_\alpha F_{\beta\gamma}.$$

Here,

$${\Gamma^\alpha}_{\mu\beta}$$

is a Christoffel symbol that characterizes the curvature of spacetime and ∇_α is the covariant derivative.

Formulation in terms of differential forms

The formulation of the Maxwell equations in terms of differential forms can be used without change in general relativity. The equivalence of the more traditional general relativistic formulation using the covariant derivative with the differential form formulation can be seen as follows. Choose local coordinates x^α which gives a basis of 1-forms dx^α in every point of the open set where the coordinates are defined. Using this basis and cgs-Gaussian units we define

- The antisymmetric infinitesimal field tensor $F_{\alpha\beta}$, corresponding to the field 2-form **F**

$$\mathbf{F} := \frac{1}{2}F_{\alpha\beta}\,\mathrm{d}\,x^\alpha \wedge \mathrm{d}\,x^\beta.$$

- The current-vector infinitesimal 3-form **J**

$$\mathbf{J} := \frac{4\pi}{c}j^\alpha\sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}\mathrm{d}\,x^\beta \wedge \mathrm{d}\,x^\gamma \wedge \mathrm{d}\,x^\delta.$$

Here g is as usual the determinant of the metric tensor $g_{\alpha\beta}$. A small computation that uses the symmetry of the Christoffel symbols (i.e., the torsion-freeness of the Levi-Civita connection) and the covariant constantness of the Hodge star operator then shows that in this coordinate neighborhood we have:

- the Bianchi identity

$$\mathrm{d}\mathbf{F} = 2(\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma})\mathrm{d}\,x^\alpha \wedge \mathrm{d}\,x^\beta \wedge \mathrm{d}\,x^\gamma = 0,$$

- the source equation

$$\mathrm{d} * \mathbf{F} = F^{\alpha\beta}{}_{;\alpha}\sqrt{-g}\,\epsilon_{\beta\gamma\delta\eta}\mathrm{d}\,x^\gamma \wedge \mathrm{d}\,x^\delta \wedge \mathrm{d}\,x^\eta = \mathbf{J},$$

- the continuity equation

$$\mathrm{d}\mathbf{J} = \frac{4\pi}{c}j^\alpha{}_{;\alpha}\sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}\mathrm{d}\,x^\alpha \wedge \mathrm{d}\,x^\beta \wedge \mathrm{d}\,x^\gamma \wedge \mathrm{d}\,x^\delta = 0.$$

Notes

[1] J.D. Jackson, "Maxwell's Equations" video glossary entry (http://videoglossary.lbl.gov/2009/maxwells-equations/)

[2] *Principles of physics: a calculus-based text* (http://books.google.com/books?id=1DZz341Pp50C&pg=PA809), by R.A. Serway, J.W. Jewett, page 809.

[3] The quantity we would now call $1/\sqrt{\mu_0\varepsilon_0}$, with units of velocity, was directly measured before Maxwell's equations, in an 1855 experiment by Wilhelm Eduard Weber and Rudolf Kohlrausch. They charged a leyden jar (a kind of capacitor), and measured the electrostatic force associated with the potential; then, they discharged it while measuring the magnetic force from the current in the discharge-wire. Their result was 3.107×10^8 m/s, remarkably close to the speed of light. See The story of electrical and magnetic measurements: from 500 B.C. to the 1940s, by Joseph F. Keithley, p115 (http://books.google.com/books?id=uwgNAtqSHuQC&pg=PA115)

[4] David J Griffiths (1999). *Introduction to electrodynamics* (http://worldcat.org/isbn/013805326X) (Third ed.). Prentice Hall. pp. 559–562. ISBN 013805326X. .

[5] In some books *e.g., inU. Krey and A. Owen's *Basic Theoretical Physics* (Springer 2007)), the term *effective charge* is used instead of *total charge*, while *free charge* is simply called *charge*.

[6] Šolín, Pavel (2006). *Partial differential equations and the finite element method* (http://books.google.com/books?id=-hIG3NZrnd8C&pg=PA273). John Wiley and Sons. p. 273. ISBN 0471720704. .

[7] In some books *e.g., in), the term *effective charge* is used instead of *total charge*, while *free charge* is simply called *charge*.

[8] See David J. Griffiths (1999). *Introduction to Electrodynamics* (third ed.). Prentice Hall. for a good description of how **P** relates to the bound charge.

[9] See David J. Griffiths (1999). *Introduction to Electrodynamics* (third ed.). Prentice Hall. for a good description of how **M** relates to the bound current.

[10] The generalization to non-isotropic materials is straight forward; simply replace the constants with tensor quantities.

[11] The *free* charges and currents respond to the fields through the Lorentz force law and this response is calculated at a fundamental level using mechanics. The response of *bound* charges and currents is dealt with using grosser methods subsumed under the notions of magnetization and polarization. Depending upon the problem, one may choose to have *no* free charges whatsoever.

[12] In general materials are bianisotropic. TG Mackay and A Lakhtakia (2010). *Electromagnetic Anisotropy and Bianisotropy: A Field Guide* (http://www.worldscibooks.com/physics/7515.html). World Scientific. .

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[14] Jackson, John David (1999). *Classical Electrodynamics* (3rd ed.). New York: Wiley. ISBN 0-471-30932-X.

[15] Note that the 'magnetic susceptibility' term used here is in terms of **B** and is different from the standard definition in terms of **H**.

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[23] The story of electrical and magnetic measurements: from 500 B.C. to the 1940s, by Joseph F. Keithley, p115 (http://books.google.com/books?id=uwgNAtqSHuQC&pg=PA115)

[24] "The Dictionary of Scientific Biography", by Charles Coulston Gillispie

[25] but are now universally known as *Maxwell's equations*. However, in 1940 Einstein referred to the equations as *Maxwell's equations* in "The Fundamentals of Theoretical Physics" published in the Washington periodical *Science*, May 24, 1940.

Paul J. Nahin (2002-10-09). *Oliver Heaviside: the life, work, and times of an electrical genius of the Victorian age* (http://books.google.com/?id=e9wEntQmA0IC&pg=PA111&dq=nahin+hertz-heaviside+maxwell-hertz). JHU Press. pp. 108–112. ISBN 9780801869099. .

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[32] page 480. (http://upload.wikimedia.org/wikipedia/commons/1/19/A_Dynamical_Theory_of_the_Electromagnetic_Field.pdf)

[33] Here it is noted that a quite different quantity, the *magnetic polarization*, $\mu_0\mathbf{M}$ by decision of an international IUPAP commission has been given the same name **J**. So for the electric current density, a name with small letters, **j** would be better. But even then the mathematicians would still use the large-letter-name **J** for the corresponding current-twoform (see below).

[34] http://www.mathematik.tu-darmstadt.de/~bruhn/Original-MAXWELL.htm

[35] Experiments like the Michelson-Morley experiment in 1887 showed that the 'aether' moved at the same speed as Earth. While other experiments, such as measurements of the aberration of light from stars, showed that the ether is moving relative to earth.

[36] "On the Electrodynamics of Moving Bodies" (http://www.fourmilab.ch/etexts/einstein/specrel/www/). Fourmilab.ch. . Retrieved 2008-10-19.

[37] Recently, scientists have described behavior in a crystalline state of matter known as spin-ice which have macroscopic behavior like magnetic monopoles. (See http://www.sciencemag.org/cgi/content/abstract/1178868 and http://www.nature.com/nature/journal/v461/n7266/full/nature08500.html .) The divergence of B is still zero for this system, though.

[38] J.D. Jackson. "6.12". *Classical Electrodynamics* (3rd ed.). ISBN 047143132x.

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Further reading

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External links

- Mathematical aspects of Maxwell's equation are discussed on the Dispersive PDE Wiki (http://tosio.math.toronto.edu/wiki/index.php/Main_Page).

Modern treatments

- Electromagnetism (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html), B. Crowell, Fullerton College
- Lecture series: Relativity and electromagnetism (http://farside.ph.utexas.edu/~rfitzp/teaching/jk1/lectures/node6.html), R. Fitzpatrick, University of Texas at Austin
- *Electromagnetic waves from Maxwell's equations* (http://www.physnet.org/modules/pdf_modules/m210.pdf) on Project PHYSNET (http://www.physnet.org).
- MIT Video Lecture Series (36 x 50 minute lectures) (in .mp4 format) - Electricity and Magnetism (http://ocw.mit.edu/OcwWeb/Physics/8-02Electricity-and-MagnetismSpring2002/VideoAndCaptions/index.htm) Taught by Professor Walter Lewin.

Historical

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- On Faraday's Lines of Force - 1855/56 (http://blazelabs.com/On Faraday's Lines of Force.pdf) Maxwell's first paper (Part 1 & 2) - Compiled by Blaze Labs Research (PDF)
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- A Dynamical Theory Of The Electromagnetic Field - 1865 (http://books.google.com/books?id=5HE_cmxXt2MC&vid=02IWHrbcLC9ECI_wQx&dq=Proceedings+of+the+Royal+Society+Of+London+Vol+XIII&ie=UTF-8&jtp=531) Maxwell's 1865 paper describing his 20 Equations in 20 Unknowns - Predecessor to the 1873 Treatise

Other

- Feynman's derivation of Maxwell equations and extra dimensions (http://uk.arxiv.org/abs/hep-ph/0106235)
- *Nature Milestones: Photons – Milestone 2 (1861) Maxwell's equations* (http://www.nature.com/milestones/milephotons/full/milephotons02.html)

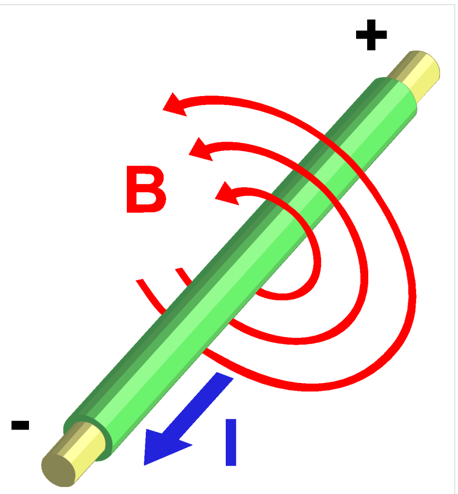
Equations

Ampère's circuital law

In classical electromagnetism, **Ampère's circuital law**, discovered by André-Marie Ampère in 1826,[1] relates the integrated magnetic field around a closed loop to the electric current passing through the loop. James Clerk Maxwell derived it again using hydrodynamics in his 1861 paper *On Physical Lines of Force* and it is now one of the Maxwell equations, which form the basis of classical electromagnetism.

Original Ampère's circuital law

It relates magnetic fields to electric currents that produce them. Using Ampere's law, one can determine the magnetic field associated with a given current or current associated with a given magnetic field, providing there is no time changing electric field present. In its historically original form, Ampère's Circuital Law relates the magnetic field to its electric current source. The law can be written in two forms, the "integral form" and the "differential form". The forms are equivalent, and related by the Kelvin–Stokes theorem. It can also be written in terms of either the **B** or **H** magnetic fields. Again, the two forms are equivalent (see the "proof" section below).



An electric current produces a magnetic field.

Ampère's circuital law is now known to be a correct law of physics in a magnetostatic situation: The system is static except possibly for continuous steady currents within closed loops. In all other cases the law is incorrect unless Maxwell's correction is included (see below).

Integral form

In SI units (the version in cgs units is in a later section), the "integral form" of the original Ampère's circuital law is:[2] [3]

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_S \mathbf{J} \cdot \mathrm{d}\mathbf{S}$$

$$\oint_C \mathbf{H} \cdot \mathrm{d}\boldsymbol{\ell} = \iint_S \mathbf{J}_\mathrm{f} \cdot \mathrm{d}\mathbf{S}$$

or equivalently,

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I_\mathrm{enc}$$

$$\oint_C \mathbf{H} \cdot \mathrm{d}\boldsymbol{\ell} = I_\mathrm{f,enc}$$

where

$\oint_C$ is the closed line integral around the closed curve C;

B is the magnetic B-field in teslas;

H is the magnetic H-field in ampere per metre;

$\cdot$ is the vector dot product;

$d\boldsymbol{\ell}$ is an infinitesimal element (a differential) of the curve *C* (i.e. a vector with magnitude equal to the length of the infinitesimal line element, and direction given by the tangent to the curve *C*, see below);

$\iint_S$ denotes an integral over the surface *S* enclosed by the curve *C* (see below; the double integral sign is meant simply to denote that the integral is two-dimensional in nature);

μ_0 is the magnetic constant;

$\mathbf{J}_\mathrm{f}$ is the free current density through the surface *S* enclosed by the curve *C* (see below);

$\mathbf{J}$ is the total current density through the surface *S* enclosed by the curve *C*, including both free and bound current (see below);

$d\mathbf{S}$ is the vector area of an infinitesimal element of surface *S* (that is, a vector with magnitude equal to the area of the infinitesimal surface element, and direction normal to surface *S*. The direction of the normal must correspond with the orientation of *C* by the right hand rule, see below for further discussion);

$I_\mathrm{f,enc}$ is the net free current that penetrates through the surface *S* (see below);

I_enc is the total net current that penetrates through the surface *S*, including both free and bound current (see below).

There are a number of ambiguities in the above definitions that warrant elaboration.

First, three of these terms are associated with sign ambiguities: the line integral $\oint_C$ could go around the loop in either direction (clockwise or counterclockwise); the vector area $d\mathbf{S}$ could point in either of the two directions normal to the surface; and I_enc is the net current passing through the surface *S*, meaning the current passing through in one direction, minus the current in the other direction—but either direction could be chosen as positive. These ambiguities are resolved by the right-hand rule: With the palm of the right-hand toward the area of integration, and the index-finger pointing along the direction of line-integration, the outstretched thumb points in the direction that must be chosen for the vector area $d\mathbf{S}$. Also the current passing in the same direction as $d\mathbf{S}$ must be counted as positive. The right hand grip rule can also be used to determine the signs.

Second, there are infinitely many possible surfaces *S* that have the curve *C* as their border. (Imagine a soap film on a wire loop, which can be deformed by blowing gently at it.) Which of those surfaces is to be chosen? If the loop does not lie in a single plane, for example, there is no one obvious choice. The answer is that it does not matter; it can be proven that any surface with boundary *C* can be chosen.

Differential form

By the Kelvin–Stokes theorem, this equation can also be written in a "differential form". Again, this equation only applies in the case where the electric field is constant in time; see below for the more general form. In SI units, the equation states:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \times \mathbf{H} = \mathbf{J_f}$$

where

$\nabla \times$ is the curl operator.

Note on free current versus bound current

The electric current that arises in the simplest textbook situations would be classified as "free current"—for example, the current that passes through a wire or battery. In contrast, "bound current" arises in the context of bulk materials that can be magnetized and/or polarized. (All materials can to some extent.)

When a material is magnetized (for example, by placing it in an external magnetic field), the electrons remain bound to their respective atoms, but behave as if they were orbiting the nucleus in a particular direction, creating a microscopic current. When the currents from all these atoms are put together, they create the same effect as a macroscopic current, circulating perpetually around the magnetized object. This magnetization current $\boldsymbol{J_M}$ is one contribution to "bound current".

The other source of bound current is bound charge. When an electric field is applied, the positive and negative bound charges can separate over atomic distances in polarizable materials, and when the bound charges move, the polarization changes, creating another contribution to the "bound current", the polarization current $\boldsymbol{J_P}$.

The total current density $\boldsymbol{J}$ due to free and bound charges is then:

$$\mathbf{J} = \mathbf{J_f} + \mathbf{J_M} + \mathbf{J_P} \ ,$$

with $\boldsymbol{J_f}$ the "free" or "conduction" current density.

All current is fundamentally the same, microscopically. Nevertheless, there are often practical reasons for wanting to treat bound current differently from free current. For example, the bound current usually originates over atomic dimensions, and one may wish to take advantage of a simpler theory intended for larger dimensions. The result is that the more microscopic Ampère's law, expressed in terms of **B** and the microscopic current (which includes free, magnetization and polarization currents), is sometimes put into the equivalent form below in terms of **H** and the free current only. For a detailed definition of free current and bound current, and the proof that the two formulations are equivalent, see the "proof" section below.

Shortcomings of the original formulation of Ampère's circuital law

There are two important issues regarding Ampère's law that require closer scrutiny. First, there is an issue regarding the continuity equation for electrical charge. There is a theorem in vector calculus that states the divergence of a curl must always be zero. Hence $\nabla\cdot(\nabla\times\mathbf{B}) = 0$ and so the original Ampère's law implies that $\nabla\cdot\mathbf{J} = 0$. But in general $\nabla\cdot\mathbf{J} = -\partial\varrho/\partial t$, which is non-zero for a time-varying charge density. An example occurs in a capacitor circuit where time-varying charge densities exist on the plates.[4] [5] [6] [7] [8]

Second, there is an issue regarding the propagation of electromagnetic waves. For example, in free space, where $\mathbf{J} = 0$, Ampère's law implies that $\nabla\times\mathbf{B} = 0$, but instead $\nabla\times\mathbf{B} = -(1/c^2)\ \partial\mathbf{E}/\partial t$.

To treat these situations, the contribution of displacement current must be added to the current term in Ampère's law.

James Clerk Maxwell conceived of displacement current as a polarization current in the dielectric vortex sea, which he used to model the magnetic field hydrodynamically and mechanically.[9] He added this displacement current to Ampère's circuital law at equation (112) in his 1861 paper *On Physical Lines of Force* .[10]

Displacement current

In free space, the displacement current is related to the time rate of change of electric field.

In a dielectric the above contribution to displacement current is present too, but a major contribution to the displacement current is related to the polarization of the individual molecules of the dielectric material. Even though charges cannot flow freely in a dielectric, the charges in molecules can move a little under the influence of an electric field. The positive and negative charges in molecules separate under the applied field, causing an increase in the state of polarization, expressed as the polarization density $\boldsymbol{P}$. A changing state of polarization is equivalent to a current.

Both contributions to the displacement current are combined by defining the displacement current as:[4]

$$\mathbf{J_D} = \frac{\partial}{\partial t}\mathbf{D}(\boldsymbol{r},\ t)\ ,$$

where the electric displacement field is defined as:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r \mathbf{E}\ ,$$

where ε_0 is the electric constant, ε_r the relative static permittivity, and ***P*** is the polarization density. Substituting this form for ***D*** in the expression for displacement current, it has two components:

$$\mathbf{J}_\mathrm{D} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t}.$$

The first term on the right hand side is present everywhere, even in a vacuum. It doesn't involve any actual movement of charge, but it nevertheless has an associated magnetic field, as if it were an actual current. Some authors apply the name *displacement current* to only this contribution.[11]

The second term on the right hand side is the displacement current as originally conceived by Maxwell, associated with the polarization of the individual molecules of the dielectric material.

Maxwell's original explanation for displacement current focused upon the situation that occurs in dielectric media. In the modern post-aether era, the concept has been extended to apply to situations with no material media present, for example, to the vacuum between the plates of a charging vacuum capacitor. The displacement current is justified today because it serves several requirements of an electromagnetic theory: correct prediction of magnetic fields in regions where no free current flows; prediction of wave propagation of electromagnetic fields; and conservation of electric charge in cases where charge density is time-varying. For greater discussion see Displacement current.

Extending the original law: the Maxwell–Ampère equation

Next Ampère's equation is extended by including the polarization current, thereby remedying the limited applicability of the original Ampère's circuital law.

Treating free charges separately from bound charges, Ampère's equation including Maxwell's correction in terms of the **H**-field is (the **H**-field is used because it includes the magnetization currents, so $\mathbf{J}_\mathrm{M}$ does not appear explicitly, see H-field and also Note):[12]

$$\oint_C \mathbf{H} \cdot \mathrm{d}\boldsymbol{\ell} = \iint_S \left(\mathbf{J}_\mathrm{f} + \frac{\partial}{\partial t}\mathbf{D} \right) \cdot \mathrm{d}\mathbf{A}$$

(integral form), where **H** is the magnetic H field (also called "auxiliary magnetic field", "magnetic field intensity", or just "magnetic field", **D** is the electric displacement field, and $\mathbf{J}_\mathrm{f}$ is the enclosed conduction current or free current density. In differential form,

$$\nabla \times \mathbf{H} = \mathbf{J}_\mathrm{f} + \frac{\partial}{\partial t}\mathbf{D}\ .$$

On the other hand, treating all charges on the same footing (disregarding whether they are bound or free charges), the generalized Ampère's equation (also called the Maxwell–Ampère equation) is (see the "proof" section below):

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \iint_S \left(\mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\mathbf{E} \right) \cdot \mathrm{d}\mathbf{A}$$

in integral form. In differential form,

$$\nabla \times \mathbf{B} = \left(\mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\mathbf{E} \right)\ .$$

In both forms **J** includes magnetization current density[13] as well as conduction and polarization current densities. That is, the current density on the right side of the Ampère–Maxwell equation is:

$$\mathbf{J_f} + \mathbf{J_D} + \mathbf{J_M} = \mathbf{J_f} + \mathbf{J_P} + \mathbf{J_M} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \ ,$$

where current density $\mathbf{J}_\mathrm{D}$ is the *displacement current*, and **J** is the current density contribution actually due to movement of charges, both free and bound. Because $\nabla \cdot \mathbf{D} = \varrho$, the charge continuity issue with Ampère's original formulation is no longer a problem.[14] Because of the term in $\varepsilon_0 \partial E / \partial t$, wave propagation in free space now is possible.

With the addition of the displacement current, Maxwell was able to hypothesize (correctly) that light was a form of electromagnetic wave. See electromagnetic wave equation for a discussion of this important discovery.

Proof of equivalence

Proof that the formulations of Ampère's law in terms of free current are equivalent to the formulations involving total current.

In this proof, we will show that the equation

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

is equivalent to the equation

$$\nabla \times \mathbf{B}/\mu_0 = \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Note that we're only dealing with the differential forms, not the integral forms, but that is sufficient since the differential and integral forms are equivalent in each case, by the Kelvin–Stokes theorem.

We introduce the polarization density **P**, which has the following relation to **E** and **D**:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

Next, we introduce the magnetization density **M**, which has the following relation to **B** and **H**:

$$\mathbf{B}/\mu_0 = \mathbf{H} + \mathbf{M}$$

and the following relation to the bound current:

$$\begin{aligned} \mathbf{J}_\mathrm{bound} &= \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t} \ , \\ &= \mathbf{J}_\mathrm{M} + \mathbf{J}_\mathrm{P} \ , \end{aligned}$$

where

$$\mathbf{J}_\mathrm{M} = \nabla \times \mathbf{M} \ ,$$

is called the magnetization current density, and

$$\mathbf{J}_\mathrm{P} = \frac{\partial \mathbf{P}}{\partial t} \ ,$$

is the polarization current density. Taking the equation for ***B***:

$$\begin{aligned} \nabla \times \mathbf{B}/\mu_0 &= \nabla \times (\mathbf{H} + \mathbf{M}) \\ &= \nabla \times \mathbf{H} + \mathbf{J_M} \\ &= \mathbf{J_f} + \mathbf{J_P} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J_M} \end{aligned}$$

Consequently, referring to the definition of the bound current:

$$\begin{aligned} \nabla \times \mathbf{B}/\mu_0 &= \mathbf{J_f} + \mathbf{J}_\mathrm{bound} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\ &= \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \ , \end{aligned}$$

as was to be shown.

Ampère's law in cgs units

In cgs units, the integral form of the equation, including Maxwell's correction, reads

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \frac{1}{c} \iint_S \left(4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right) \cdot \mathrm{d}\mathbf{S}$$

where c is the speed of light.

The differential form of the equation (again, including Maxwell's correction) is

$$\nabla \times \mathbf{B} = \frac{1}{c} \left(4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right).$$

Notes

[1] Richard Fitzpatrick (2007). "Ampère's Circuital Law" (http://farside.ph.utexas.edu/teaching/316/lectures/node75.html). .

[2] Heinz E Knoepfel (2000). *Magnetic Fields: A comprehensive theoretical treatise for practical use* (http://books.google.com/?id=1n7NPasgh0sC&pg=PA210&dq=isbn=0471322059#PPA4,M1). Wiley. p. 4. ISBN 0471322059. .

[3] George E. Owen (2003). *Electromagnetic Theory* (http://books.google.com/?id=VLm_dqhZUOYC&pg=PA213&dq="Ampere's+circuital+law") (Reprint of 1963 ed.). Courier-Dover Publications. p. 213. ISBN 0486428303. .

[4] John David Jackson (1999). *Classical Electrodynamics* (3rd ed.). Wiley. p. 238. ISBN 047130932X.

[5] David J Griffiths (1999). *Introduction to Electrodynamics* (http://books.google.com/?id=w0YgJgAACAAJ&dq=isbn=013805326X) (3rd ed.). Pearson/Addison-Wesley. pp. 322–323. ISBN 013805326X. .

[6] George E. Owen (2003). op. cit. (http://books.google.com/?id=VLm_dqhZUOYC&pg=PA213&dq="Ampere's+circuital+law"). Mineola, N.Y.: Dover Publications. p. 285. ISBN 0486428303. .

[7] J. Billingham, A. C. King (2006). *Wave Motion* (http://books.google.com/?id=bNePaHM20LQC&pg=PA179&dq=displacement+"ampere's+law"). Cambridge University Press. p. 179. ISBN 0521634504. .

[8] J.C. Slater and N.H. Frank (1969). *Electromagnetism* (http://books.google.com/?id=GYsphnFwUuUC&pg=PA83&dq=displacement+"ampere's+law") (Reprint of 1947 ed.). Courier Dover Publications. p. 83. ISBN 0486622630. .

[9] Daniel M. Siegel (2003). *Innovation in Maxwell's Electromagnetic Theory: Molecular Vortices, Displacement Current, and Light* (http://books.google.com/?id=AbQq85U8K0gC&pg=PA97&dq=Ampere's+circuital+law+"displacement+current"). Cambridge University Press. pp. 96–98. ISBN 0521533295. .

[10] James C. Maxweel (1961). "On Physical Lines of Force" (http://upload.wikimedia.org/wikipedia/commons/b/b8/On_Physical_Lines_of_Force.pdf). *Philosophical Magazine and Journal of Science*. .

[11] For example, see David J. Griffiths (1999). op. cit.. Upper Saddle River, NJ: Prentice Hall. p. 323. ISBN 013805326X. and Tai L. Chow (2006). *Introduction to Electromagnetic Theory* (http://books.google.com/?id=dpnpMhw1zo8C&pg=PA153&dq=isbn=0763738271#PPA204,M1). Jones & Bartlett. p. 204. ISBN 0763738271. .

[12] Mircea S. Rogalski, Stuart B. Palmer (2006). *Advanced University Physics* (http://books.google.com/?id=rW95PWh3YbkC&pg=PA266&dq=displacement+"Ampere's+circuital+law"). CRC Press. p. 267. ISBN 1584885114. .

[13] Stuart B. Palmer & Mircea S. Rogalski (2006). *Advanced University Physics* (http://books.google.com/?id=rW95PWh3YbkC&printsec=frontcover#PPA251,M1). CRC Press. p. 251. ISBN 1584885114. .

[14] The magnetization current can be expressed as the *curl* of the magnetization, so its divergence is zero and it does not contribute to the continuity equation. See magnetization current.

Further reading

- Griffiths, David J. (1998). *Introduction to Electrodynamics (3rd ed.)*. Prentice Hall. ISBN 013805326X.
- Tipler, Paul (2004). *Physics for Scientists and Engineers: Electricity, Magnetism, Light, and Elementary Modern Physics (5th ed.)*. W. H. Freeman. ISBN 0716708108.

External links

- *Simple Nature* by Benjamin Crowell (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html#Section11.3) Ampere's law from an online textbook
- MISN-0-138 *Ampere's Law* (http://35.9.69.219/home/modules/pdf_modules/m138.pdf) (PDF file) by Kirby Morgan for Project PHYSNET (http://www.physnet.org).

- MISN-0-145 *The Ampere–Maxwell Equation; Displacement Current* (http://35.9.69.219/home/modules/pdf_modules/m145.pdf) (PDF file) by J.S. Kovacs for Project PHYSNET.
- *The Ampère's Law Song* (http://www.haverford.edu/physics-astro/songs/ampere.PDF) (PDF file) by Walter Fox Smith; Main page (http://www.haverford.edu/physics-astro/songs/), with recordings of the song.
- *A Dynamical Theory of the Electromagnetic Field* (http://upload.wikimedia.org/wikipedia/commons/1/19/A_Dynamical_Theory_of_the_Electromagnetic_Field.pdf) Maxwell's paper of 1864

Faraday's law of induction

Faraday's law of induction dates from the 1830s, and is a basic law of electromagnetism relating to the operating principles of transformers, inductors, and many types of electrical motors and generators.[1] Faraday's law is applicable to a closed circuit made of thin wire and states that:

> The induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.[1]

Or alternatively:

> The EMF generated is proportional to the rate of change of the magnetic flux.

The law strictly holds only when the closed circuit is an infinitely-thin wire;[2] for example, a spinning homopolar generator has a constant magnetically-induced EMF, but its magnetic flux does not rise perpetually higher and higher, as would be implied by a naive interpretation of the statements above.[2]

EMF is defined as the energy available per unit charge that travels once around the wire loop (the unit of EMF is the volt).[2] [3] [4] [5] Equivalently, it is the voltage that would be measured by cutting the wire to create an open circuit, and attaching a voltmeter to the leads. According to the Lorentz force law, the EMF on a wire loop is:

$$EMF = \oint (\mathbf{E} + \mathbf{v} \times \mathbf{B})\, \mathrm{d}\ell$$

Faraday's law of induction is closely related to the **Maxwell-Faraday equation**:[2] [3]

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

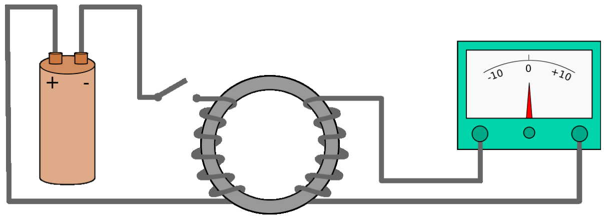
where:

> $\nabla \times$ denotes curl
>
> **E** is the electric field
>
> **B** is the magnetic flux density.

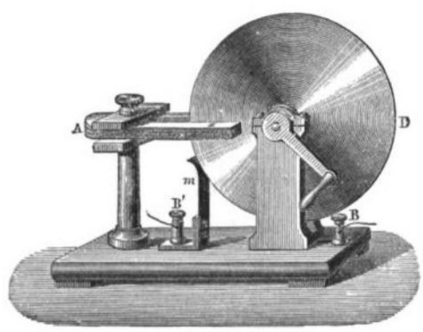
The Maxwell-Faraday equation is one of the four Maxwell's equations, and therefore plays a fundamental role in the theory of classical electromagnetism.

History

Electromagnetic induction was discovered independently by Michael Faraday and Joseph Henry in 1831; however, Faraday was the first to publish the results of his experiments.[6] [7]



A diagram of Faraday's iron ring apparatus. Change in the magnetic flux of the left coil induces a current in the right coil.[8]

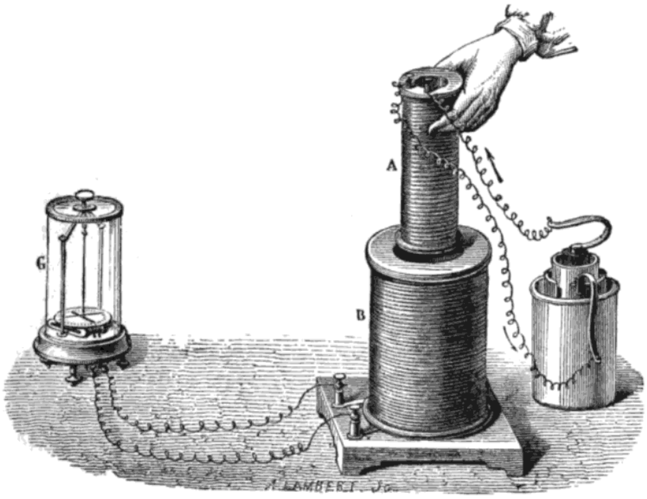


Faraday's disk (see homopolar generator)

In Faraday's first experimental demonstration of electromagnetic induction (August 29, 1831[9]), he wrapped two wires around opposite sides of an iron torus (an arrangement similar to a modern transformer). Based on his assessment of recently-discovered properties of electromagnets, he expected that when current started to flow in one wire, a sort of wave would travel through the ring and cause some electrical effect on the opposite side. He plugged one wire into a galvanometer, and watched it as he connected the other wire to a battery. Indeed, he saw a transient current (which he called a "wave of electricity") when he connected the wire to the battery, and another when he disconnected it.[10] This induction was due to the change in magnetic flux that occurred when the battery was connected and disconnected.[8] Within two months, Faraday had found several other manifestations of electromagnetic induction. For example, he saw transient currents when he quickly slid a bar magnet in and out of a coil of wires, and he generated a steady (DC) current by rotating a copper disk near a bar magnet with a sliding electrical lead ("Faraday's disk").[11]

Faraday explained electromagnetic induction using a concept he called lines of force. However, scientists at the time widely rejected his theoretical ideas, mainly because they were not formulated mathematically.[12] An exception was Maxwell, who used Faraday's ideas as the basis of his quantitative electromagnetic theory.[12] [13] [14] In Maxwell's papers, the time varying aspect of electromagnetic induction is expressed as a differential equation which Oliver Heaviside referred to as Faraday's law even though it is slightly different in form from the original version of Faraday's law, and does not describe motional EMF. Heaviside's version (see Maxwell-Faraday equation below) is the form recognized today in the group of equations known as Maxwell's equations.

Lenz's law, formulated by Heinrich Lenz in 1834, describes "flux through the circuit", and gives the direction of the induced electromotive force and current resulting from electromagnetic induction (elaborated upon in the examples below).



Faraday's experiment showing induction between coils of wire: The liquid battery *(right)* provides a current which flows through the small coil *(A)*, creating a magnetic field. When the coils are stationary, no current is induced. But when the small coil is moved in or out of the large coil *(B)*, the magnetic flux through the large coil changes, inducing a current which is detected by the galvanometer *(G)*.[15]

Faraday's law as two different phenomena

Some physicists have remarked that Faraday's law is a single equation describing two different phenomena: the *motional EMF* generated by a magnetic force on a moving wire (see Lorentz force), and the *transformer EMF* generated by an electric force due to a changing magnetic field (due to the Maxwell-Faraday equation). James Clerk Maxwell drew attention to this fact in his 1861 paper *On Physical Lines of Force*. In the latter half of part II of that paper, Maxwell gives a separate physical explanation for each of the two phenomena. A reference to these two aspects of electromagnetic induction is made in some modern textbooks.[16] As Richard Feynman states:[2]

> So the "flux rule" that the emf in a circuit is equal to the rate of change of the magnetic flux through the circuit applies whether the flux changes because the field changes or because the circuit moves (or both).... Yet in our explanation of the rule we have used two completely distinct laws for the two cases – $\mathbf{v}\times\mathbf{B}$ for "circuit moves" and $\nabla\times\mathbf{E}=-\partial_t\mathbf{B}$ for "field changes".
>
> We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of *two different phenomena*.
>
> — Richard P. Feynman, *The Feynman Lectures on Physics*

Reflection on this apparent dichotomy was one of the principal paths that led Einstein to develop special relativity:

> It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric

currents of the same path and intensity as those produced by the electric forces in the former case.

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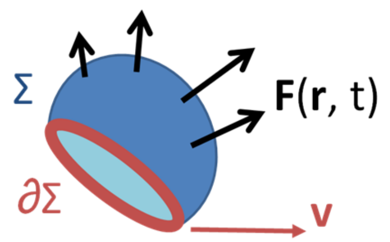
   Examples of this sort, together with unsuccessful attempts to
discover any motion of the earth relative to the "light medium,"
suggest that the phenomena of electrodynamics as well as of mechanics
possess no properties corresponding to the idea of absolute rest.
```

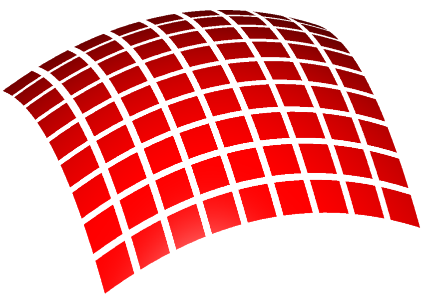
— *Albert Einstein, On the Electrodynamics of Moving Bodies*[17]

Flux through a surface and EMF around a loop

Faraday's law of induction makes use of the magnetic flux Φ_B through a hypothetical surface Σ whose boundary is a wire loop. Since the wire loop may be moving, we write Σ(t) for the surface. The magnetic flux is defined by a surface integral:



The wire loop (red) forms the boundary of a surface Σ (blue). The black arrows denote any vector field **F**(**r**, t) defined throughout space; in the case of Faraday's law, the relevant vector field is the magnetic flux density **B**, and it is integrated over the blue surface. The red arrow represents the fact that the wire loop may be moving and/or deforming.



The definition of surface integral relies on splitting the surface Σ into small surface elements. Each element is associated with a vector d**A** of magnitude equal to the area of the element and with direction normal to the element and pointing outward.

$$\Phi_B = \iint\limits_{\Sigma(t)} \mathbf{B}(\mathbf{r}, t) \cdot d\mathbf{A} \ ,$$

where d**A** is an element of surface area of the moving surface Σ(t), **B** is the magnetic field, and **B**·d**A** is a vector dot product. In more visual terms, the magnetic flux through the wire loop is proportional to the number of magnetic flux lines that pass through the loop.

When the flux changes—because **B** changes, or because the wire loop is moved or deformed, or both—Faraday's law of induction says that the wire loop acquires an EMF $\mathcal{E}$, defined as the energy available per unit charge that travels once around the wire loop (the unit of EMF is the volt). The EMF is given by the rate of change of the magnetic flux:

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| ,$$

where $|\mathcal{E}|$ is the magnitude of the electromotive force (EMF) in volts and Φ_B is the magnetic flux in webers. The direction of the electromotive force is given by Lenz's law.

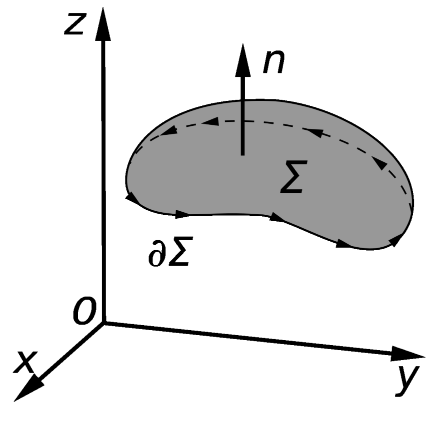
For a tightly-wound coil of wire, composed of *N* identical loops, each with the same Φ_B, Faraday's law of induction states that

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$ [18]

where *N* is the number of turns of wire and Φ_B is the magnetic flux in webers through a *single* loop.

The Maxwell-Faraday equation

A changing magnetic field creates an electric field; this phenomenon is described by the Maxwell-Faraday equation:[19]



An illustration of Kelvin-Stokes theorem with surface **Σ** its boundary **∂Σ** and orientation ***n*** set by the right-hand rule.

$$\nabla \times \mathbf{E}(\mathbf{r},\ t) = -\frac{\partial \mathbf{B}(\mathbf{r},\ t)}{\partial t}$$

where:

$\nabla \times$ denotes curl

E is the electric field

B is the magnetic flux density.

This equation appears in modern sets of Maxwell's equations and is often referred to as Faraday's law. It can also be written in an **integral form** by the Kelvin-Stokes theorem:[20]

$$\oint_{\partial \Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\int_{\Sigma} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$$

where, as indicated in the figure:

Σ is a surface bounded by the closed contour **∂Σ**,

E is the electric field,

dℓ is an infinitesimal vector element of the contour **∂Σ**,

B is the magnetic field.

d**A** is an infinitesimal vector element of surface **Σ**. If its direction is orthogonal to that surface patch, the magnitude is the area of an infinitesimal patch of surface.

Both dℓ and d**A** have a sign ambiguity; to get the correct sign, the right-hand rule is used, as explained in the article Kelvin-Stokes theorem. For a planar surface Σ, a positive path element $d\ell$ of curve ∂Σ is defined by the right-hand rule as one that points with the fingers of the right hand when the thumb points in the direction of the normal **n** to the surface Σ.

The integral around **∂Σ** is called a *path integral* or *line integral*. The surface integral at the right-hand side of the Maxwell-Faraday equation is the explicit expression for the magnetic flux Φ_B through **Σ**.

Notice that a nonzero path integral for **E** is different from the behavior of the electric field generated by charges. A charge-generated **E**-field can be expressed as the gradient of a scalar field that is a solution to Poisson's equation, and has a zero path integral. See gradient theorem.

The integral equation is true for *any* path **∂Σ** through space, and any surface **Σ** for which that path is a boundary.

If the path **Σ** is not changing in time, the equation can be rewritten:

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \int_{\Sigma} \mathbf{B} \cdot d\mathbf{A}$$

Proof of Faraday's law

The four Maxwell's equations (including the Maxwell-Faraday equation), along with the Lorentz force law, are a sufficient foundation to derive *everything* in classical electromagnetism.[2] [3] Therefore it is possible to "prove" Faraday's law starting with these equations.[21] [22] Click "show" in the box below for an outline of this proof. (In an alternative approach, not shown here but equally valid, Faraday's law could be taken as the starting point and used to "prove" the Maxwell-Faraday equation and/or other laws.)

Outline of proof of Faraday's law from Maxwell's equations and the Lorentz force law.

Consider the time-derivative of flux through a possibly-moving loop, with area $\Sigma(t)$:

$$\frac{d\Phi_B}{dt} = \frac{d}{dt} \int_{\Sigma(t)} \mathbf{B}(t) \cdot d\mathbf{A}$$

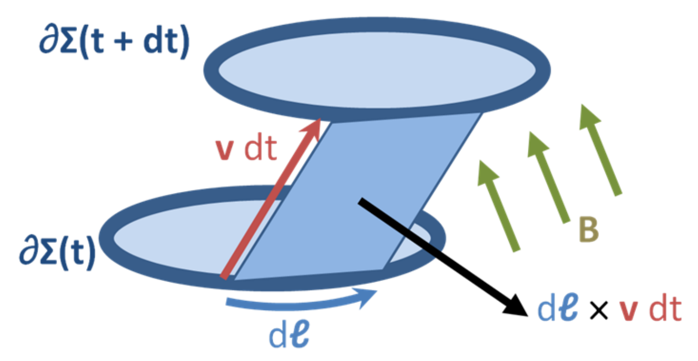
The integral can change over time for two reasons: The integrand can change, or the integration region can change. These add linearly, therefore:

$$\left.\frac{d\Phi_B}{dt}\right|_{t=t_0} = \left(\int_{\Sigma(t_0)} \left.\frac{\partial \mathbf{B}}{\partial t}\right|_{t=t_0} \cdot d\mathbf{A} \right) + \left(\frac{d}{dt} \int_{\Sigma(t)} \mathbf{B}(t_0) \cdot d\mathbf{A} \right)$$

where t_0 is any given fixed time. We will show that the first term on the right-hand side corresponds to transformer EMF, the second to motional EMF (see above). The first term on the right-hand side can be rewritten using the integral form of the Maxwell-Faraday equation:

$$\int_{\Sigma(t_0)} \left.\frac{\partial \mathbf{B}}{\partial t}\right|_{t=t_0} \cdot d\mathbf{A} = -\oint_{\partial\Sigma(t_0)} \mathbf{E}(t_0) \cdot d\boldsymbol{\ell}$$

Next, we analyze the second term on the right-hand side:



Area swept out by vector element $d\ell$ of curve **∂Σ** in time *dt* when moving with velocity **v**.

$$\frac{d}{dt}\int_{\Sigma(t)} \mathbf{B}(t_0)\cdot d\mathbf{A}$$

This is the most difficult part of the proof; more details and alternate approaches can be found in references.[21] [22] [23] As the loop moves and/or deforms, it sweeps out a surface (see figure on right). The magnetic flux through this swept-out surface corresponds to the magnetic flux that is either entering or exiting the loop, and therefore this is the magnetic flux that contributes to the time-derivative. (This step implicitly uses Gauss's law for magnetism: Since the flux lines have no beginning or end, they can only get into the loop by getting cut through by the wire.) As a small part of the loop $d\boldsymbol{\ell}$ moves with velocity $\mathbf{v}$ for a short time dt, it sweeps out a vector area vector $d\mathbf{A} = \mathbf{v}\,dt \times d\boldsymbol{\ell}$. Therefore, the change in magnetic flux through the loop here is

$$\mathbf{B}\cdot(\mathbf{v}\,dt\times d\boldsymbol{\ell}) = -dt\,d\boldsymbol{\ell}\cdot(\mathbf{v}\times\mathbf{B})$$

Therefore:

$$\frac{d}{dt}\int_{\Sigma(t)} \mathbf{B}(t_0)\cdot d\mathbf{A} = -\oint_{\partial\Sigma(t_0)} (\mathbf{v}(t_0)\times\mathbf{B}(t_0))\cdot d\boldsymbol{\ell}$$

where **v** is the velocity of a point on the loop $\partial\Sigma$.

Putting these together,

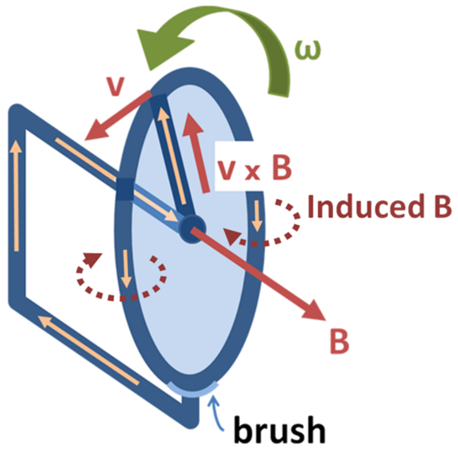
$$\left.\frac{d\Phi_B}{dt}\right|_{t=t_0} = \left(-\oint_{\partial\Sigma(t_0)} \mathbf{E}(t_0)\cdot d\boldsymbol{\ell}\right) + \left(-\oint_{\partial\Sigma(t_0)} (\mathbf{v}(t_0)\times\mathbf{B}(t_0))\cdot d\boldsymbol{\ell}\right)$$

Meanwhile, EMF is defined as the energy available per unit charge that travels once around the wire loop. Therefore, by the Lorentz force law,

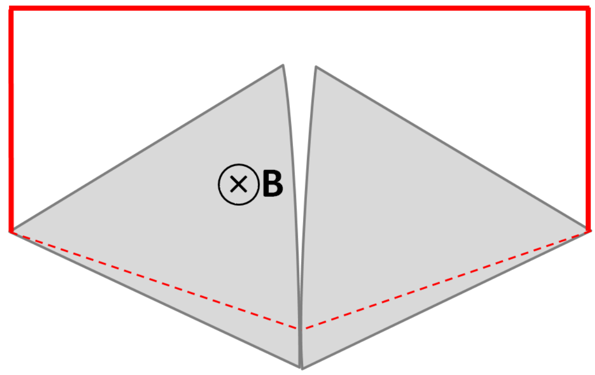
$$EMF = \oint (\mathbf{E} + \mathbf{v}\times\mathbf{B})\,\mathrm{d}\ell$$

Combining these, $\dfrac{d\Phi_B}{dt} = -EMF$

"Counterexamples" to Faraday's law



Faraday's disc electric generator. The disc rotates with angular rate ω, sweeping the conducting radius circularly in the static magnetic field **B**. The magnetic Lorentz force **v** ✕ **B** drives the current along the conducting radius to the conducting rim, and from there the circuit completes through the lower brush and the axle supporting the disc. Thus, current is generated from mechanical motion.



A counterexample to Faraday's Law when over-broadly interpreted. A wire (solid red lines) connects to two touching metal plates (silver) to form a circuit. The whole system sits in a uniform magnetic field, normal to the page. If the word "circuit" is interpreted as "primary path of current flow" (marked in red), then the magnetic flux through the "circuit" changes dramatically as the plates are rotated, yet the EMF is almost zero, which contradicts Faraday's Law. After *Feynman Lectures on Physics* Vol. II page 17-3

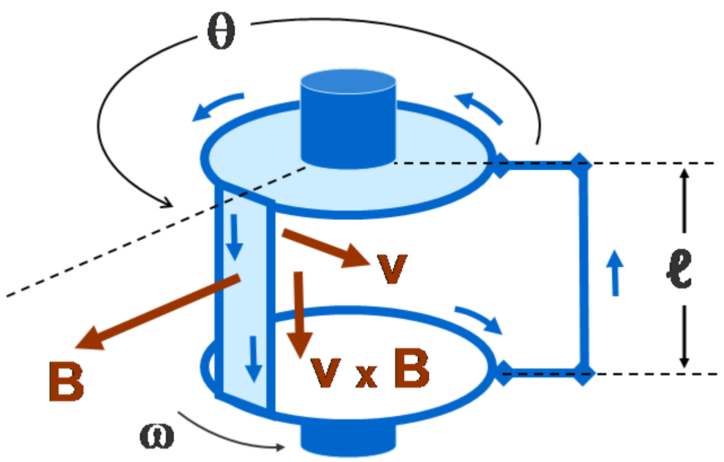
Although Faraday's law is always true for loops of thin wire, it can give the wrong result if naively extrapolated to other contexts.[2] One example is the homopolar generator (above left): A spinning circular metal disc in a homogeneous magnetic field generates a DC (constant in time) EMF. In Faraday's law, EMF is the time-derivative of flux, so a DC EMF is only possible if the magnetic flux is getting uniformly larger and larger perpetually. But in the generator, the magnetic field is constant and the disc stays in the same position, so no magnetic fluxes are growing larger and larger. So this example cannot be analyzed directly with Faraday's law.

Another example, due to Feynman,[2] has a dramatic change in flux through a circuit, even though the EMF is arbitrarily small. See figure and caption above right.

In both these examples, the changes in the current path are different from the motion of the material making up the circuit. The electrons in a material tend to follow the motion of the atoms that make up the material, due to scattering in the bulk and work function confinement at the edges. Therefore, motional EMF is generated when a material's atoms are moving through a magnetic field, dragging the electrons with them, thus subjecting the electrons to the Lorentz force. In the homopolar generator, the material's atoms are moving, even though the overall geometry of the circuit is staying the same. In the second example, the material's atoms are almost stationary, even though the overall geometry of the circuit is changing dramatically. On the other hand, Faraday's law always holds for thin wires, because there the geometry of the circuit always changes in a direct relationship to the motion of the material's atoms.

Although Faraday's law does not apply to all situations, the Maxwell-Faraday equation and Lorentz force law are always correct and can always be used directly.[2]

Electrical generator



Rectangular wire loop rotating at angular velocity ω in radially outward pointing magnetic field **B** of fixed magnitude. Current is collected by brushes attached to top and bottom discs, which have conducting rims. This is a simplified version of the *drum generator*

The EMF generated by Faraday's law of induction due to relative movement of a circuit and a magnetic field is the phenomenon underlying electrical generators. When a permanent magnet is moved relative to a conductor, or vice versa, an electromotive force is created. If the wire is connected through an electrical load, current will flow, and thus electrical energy is generated, converting the mechanical energy of motion to electrical energy. For example, the *drum generator* is based upon the figure to the right. A different implementation of this idea is the Faraday's disc, shown in simplified form on the right.

In the Faraday's disc example, the disc is rotated in a uniform magnetic field perpendicular to the disc, causing a current to flow in the radial arm due to the Lorentz force. It is interesting to understand how it arises that mechanical work is necessary to drive this current. When the generated current flows through the conducting rim, a magnetic field is generated by this current through Ampère's circuital law (labeled "induced B" in the figure). The rim thus becomes an electromagnet that resists rotation of the disc (an example of Lenz's law). On the far side of the figure, the return current flows from the rotating arm through the far side of the rim to the bottom brush. The B-field induced by this return current opposes the applied B-field, tending to *decrease* the flux through that side of the circuit, opposing the *increase* in flux due to rotation. On the near side of the figure, the return current flows from the rotating arm through the near side of the rim to the bottom brush. The induced B-field *increases* the flux on this side of the circuit, opposing the *decrease* in flux due to rotation. Thus, both sides of the circuit generate an emf opposing the rotation. The energy required to keep the disc moving, despite this reactive force, is exactly equal to the electrical energy generated (plus energy wasted due to friction, Joule heating, and other inefficiencies). This behavior is common to all generators converting mechanical energy to electrical energy.

Electrical motor

An electrical generator can be run "backwards" to become a motor. For example, with the Faraday disc, suppose a DC current is driven through the conducting radial arm by a voltage. Then by the Lorentz force law, this traveling charge experiences a force in the magnetic field *B* that will turn the disc in a direction given by Fleming's left hand rule. In the absence of irreversible effects, like friction or Joule heating, the disc turns at the rate necessary to make $d\Phi_B / dt$ equal to the voltage driving the current.

Electrical transformer

The EMF predicted by Faraday's law is also responsible for electrical transformers. When the electric current in a loop of wire changes, the changing current creates a changing magnetic field. A second wire in reach of this magnetic field will experience this change in magnetic field as a change in its coupled magnetic flux, a $d\,\Phi_B / d\,t$. Therefore, an electromotive force is set up in the second loop called the **induced EMF** or **transformer EMF**. If the two ends of this loop are connected through an electrical load, current will flow.

Magnetic flow meter

Faraday's law is used for measuring the flow of electrically conductive liquids and slurries. Such instruments are called magnetic flow meters. The induced voltage $\mathcal{E}$ generated in the magnetic field B due to a conductive liquid moving at velocity v is thus given by:

$$\mathcal{E} = B\ell v,$$

where ℓ is the distance between electrodes in the magnetic flow meter.

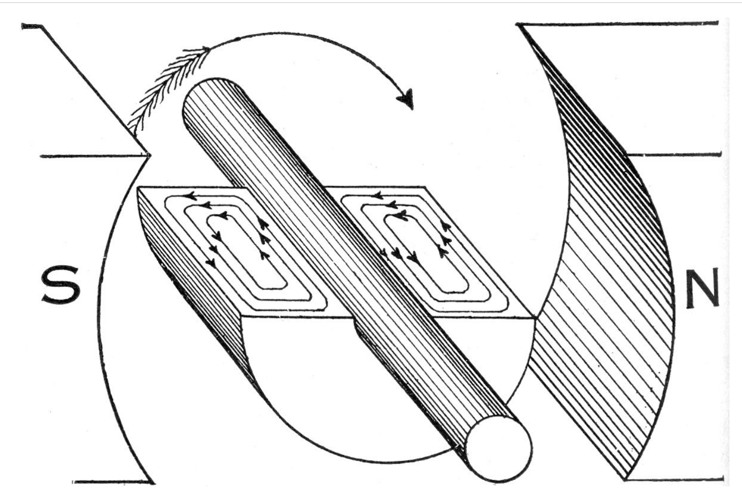
Parasitic induction and waste heating

All metal objects moving in relation to a static magnetic field will experience inductive power flow, as do all stationary metal objects in relation to a moving magnetic field. These power flows are occasionally undesirable, resulting in flowing electric current at very low voltage and heating of the metal.

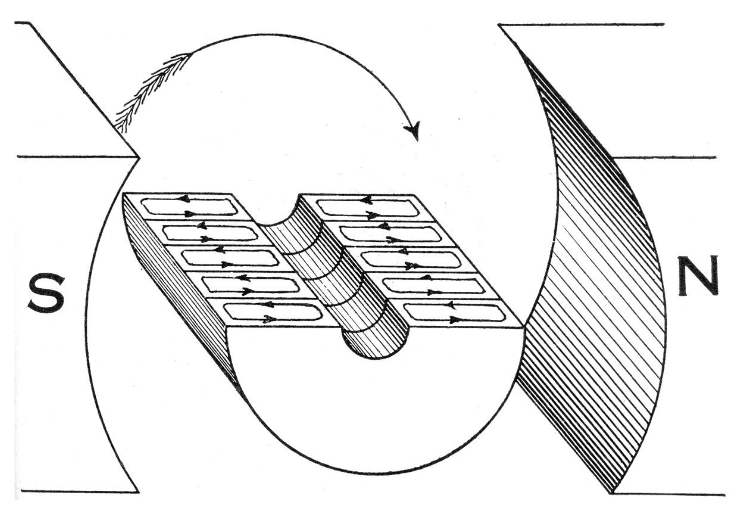
There are a number of methods employed to control these undesirable inductive effects.

- Electromagnets in electric motors, generators, and transformers do not use solid metal, but instead use thin sheets of metal plate, called *laminations*. These thin plates reduce the parasitic eddy currents, as described below.
- Inductive coils in electronics typically use magnetic cores to minimize parasitic current flow. They are a mixture of metal powder plus a resin binder that can hold any shape. The binder prevents parasitic current flow through the powdered metal.

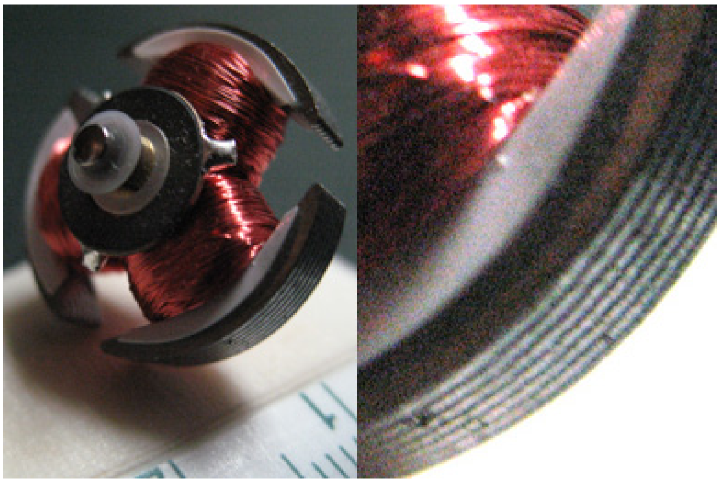
Electromagnet laminations



Eddy currents occur when a solid metallic mass is rotated in a magnetic field, because the outer portion of the metal cuts more lines of force than the inner portion, hence the induced electromotive force not being uniform, tends to set up currents between the points of greatest and least potential. Eddy currents consume a considerable amount of energy and often cause a harmful rise in temperature.[24]

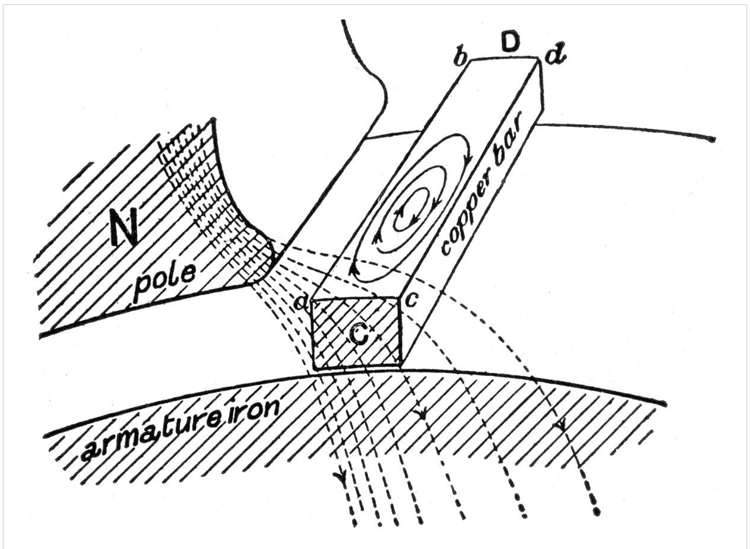


Only five laminations or plates are shown in this example, so as to show the subdivision of the eddy currents. In practical use, the number of laminations or punchings ranges from 40 to 66 per inch, and brings the eddy current loss down to about one percent. While the plates can be separated by insulation, the voltage is so low that the natural rust/oxide coating of the plates is enough to prevent current flow across the laminations.[24]



This is a rotor approximately 20mm in diameter from a DC motor used in a CD player. Note the laminations of the electromagnet pole pieces, used to limit parasitic inductive losses.

Parasitic induction within inductors



In this illustration, a solid copper bar inductor on a rotating armature is just passing under the tip of the pole piece N of the field magnet. Note the uneven distribution of the lines of force across the bar inductor. The magnetic field is more concentrated and thus stronger on the left edge of the copper bar (a,b) while the field is weaker on the right edge (c,d). Since the two edges of the bar move with the same velocity, this difference in field strength across the bar creates whorls or current eddies within the copper bar.[25]

This is one reason high voltage devices tend to be more efficient than low voltage devices. High voltage devices use many turns of small-gauge wire in motors, generators, and transformers. These many small turns of inductor wire in the electromagnet break up the eddy flows that can form within the large, thick inductors of low voltage, high current devices.

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[5] Note that different textbooks may give different definitions. The set of equations used throughout the text was chosen to be compatible with the special relativity theory.

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[11] *Michael Faraday*, by L. Pearce Williams, p. 191-5

[12] *Michael Faraday*, by L. Pearce Williams, p. 510

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[16] Griffiths, David J. (1999). *Introduction to Electrodynamics* (http://www.amazon.com/gp/reader/013805326X/ref=sib_dp_pt/104-2951702-6987112#reader-link) (Third ed.). Upper Saddle River NJ: Prentice Hall. pp. 301–3. ISBN 0-13-805326-X. . Note that the law relating flux to EMF, which this article calls "Faraday's law", is referred to in Griffiths' terminology as the "universal flux rule". Griffiths uses the term "Faraday's law" to refer to what article calls the "Maxwell-Faraday equation". So in fact, in the textbook, Griffiths' statement is about the "universal flux rule".

[17] A. Einstein, On the Electrodynamics of Moving Bodies (http://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf)

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[24] Images and reference text are from the public domain book: Hawkins Electrical Guide, Volume 1, Chapter 19: Theory of the Armature, pp. 272-273, Copyright 1917 by Theo. Audel & Co., Printed in the United States

[25] Images and reference text are from the public domain book: Hawkins Electrical Guide, Volume 1, Chapter 19: Theory of the Armature, pp. 270-271, Copyright 1917 by Theo. Audel & Co., Printed in the United States

Further reading

- Maxwell, James Clerk (1881), *A treatise on electricity and magnetism, Vol. II*, Chapter III, §530, p. 178. (http://books.google.com/books?id=vAsJAAAAIAAJ&printsec=frontcover&dq=intitle:a+intitle:treatise+intitle:on+intitle:electricity+intitle:an+intitle:magnetism&cad=0_1#v=onepage&q&f=false) Oxford, UK: Clarendon Press. ISBN 0486606376.

External links

- A simple interactive Java tutorial on electromagnetic induction (http://www.magnet.fsu.edu/education/tutorials/java/electromagneticinduction/index.html) National High Magnetic Field Laboratory
- R. Vega *Induction: Faraday's law and Lenz's law* - Highly animated lecture (http://www.physics.smu.edu/~vega/em1304/lectures/lect13/lect13_f03.ppt)
- Notes from Physics and Astronomy HyperPhysics at Georgia State University (http://hyperphysics.phy-astr.gsu.edu/HBASE/hframe.html)
- Faraday's Law for EMC Engineers (http://www.learnemc.com/tutorials/Faraday/Faradays_Law.html)
- Tankersley and Mosca: *Introducing Faraday's law* (http://www.nadn.navy.mil/Users/physics/tank/Public/FaradaysLaw.pdf)

Gauss's law

In physics, **Gauss's law**, also known as **Gauss's flux theorem**, is a law relating the distribution of electric charge to the resulting electric field. Gauss's law states that:

The electric flux through any closed surface is proportional to the enclosed electric charge.[1]

The law was formulated by Carl Friedrich Gauss in 1835, but was not published until 1867.[2] It is one of the four Maxwell's equations which form the basis of classical electrodynamics, the other three being Gauss's law for magnetism, Faraday's law of induction, and Ampère's law with Maxwell's correction. Gauss's law can be used to derive Coulomb's law,[3] and vice versa.

Gauss's law may be expressed as:

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

where Φ_E is the electric flux through a closed surface *S*, *Q* is the total charge enclosed within *S*, and ε_0 is the electric constant. The electric flux Φ_E is defined as a surface integral of the electric field:

$$\Phi_E \equiv \oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A}$$

Because the flux is defined as an *integral* of the electric field, this expression of Gauss's law is called the *integral form*.

Gauss's law can alternatively be written in the *differential form*:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

where $\nabla \cdot \mathbf{E}$ is the divergence of the electric field, and ρ is the charge density.

The integral and differential forms are related by the divergence theorem, also called Gauss's theorem. Each of these forms can also be expressed two ways: In terms of a relation between the electric field **E** and the total electric charge, or in terms of the electric displacement field **D** and the *free* electric charge.

Gauss's law has a close mathematical similarity with a number of laws in other areas of physics, such as Gauss's law for magnetism and Gauss's law for gravity. In fact, any "inverse-square law" can be formulated in a way similar to Gauss's law: For example, Gauss's law itself is essentially equivalent to the inverse-square Coulomb's law, and Gauss's law for gravity is essentially equivalent to the inverse-square Newton's law of gravity.

Gauss's law can be used to demonstrate that all electric fields inside a Faraday cage have an electric charge. Gauss's law is something of an electrical analogue of Ampère's law, which deals with magnetism.

In terms of total charge

Integral form

For a volume *V* with surface *S*, Gauss's law states that

$$\Phi_{E,S} = \frac{Q}{\varepsilon_0}$$

where $\Phi_{E,S}$ is the electric flux through *S*, *Q* is total charge inside *V*, and ε_0 is the electric constant. The electric flux is given by a surface integral over a closed surface *S*:

$$\oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A}$$

where **E** is the electric field, d**A** is a vector representing an infinitesimal element of area,[4] and · represents the dot product.

Applying the integral form

If the electric field is known everywhere, Gauss's law makes it quite easy, in principle, to find the distribution of electric charge: The charge in any given region can be deduced by integrating the electric field to find the flux.

However, much more often, it is the reverse problem that needs to be solved: The electric charge distribution is known, and the electric field needs to be computed. This is much more difficult, since if you know the total flux through a given surface, that gives almost no information about the electric field, which (for all you know) could go in and out of the surface in arbitrarily complicated patterns.

An exception is if there is some symmetry in the situation, which mandates that the electric field passes through the surface in a uniform way. Then, if the total flux is known, the field itself can be deduced at every point. Common examples of symmetries which lend themselves to Gauss's law include cylindrical symmetry, planar symmetry, and spherical symmetry. See the article Gaussian surface for examples where these symmetries are exploited to compute electric fields.

Differential form

In differential form, Gauss's law states:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

where $\nabla \cdot$ denotes divergence, **E** is the electric field, and ρ is the total electric charge density (including both free and bound charge), and ε_0 is the electric constant. This is mathematically equivalent to the integral form, because of the divergence theorem.

Equivalence of integral and differential forms

The integral and differential forms are mathematically equivalent, by the divergence theorem. Here is the argument more specifically:

The integral form of Gauss's law is:

$$\oint_S \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \frac{Q}{\varepsilon_0}$$

for any closed surface *S* containing charge *Q*. By the divergence theorem, this equation is equivalent to:

$$\iiint_V \nabla\cdot\mathbf{E}\ \mathrm{d}V = \frac{Q}{\varepsilon_0}$$

for any volume *V* containing charge *Q*. By the relation between charge and charge density, this equation is equivalent to:

$$\iiint_V \nabla\cdot\mathbf{E}\ \mathrm{d}V = \iiint_V \frac{\rho}{\varepsilon_0}\ \mathrm{d}V$$

for any volume *V*. In order for this equation to be *simultaneously true* for *every* possible volume *V*, it is necessary (and sufficient) for the integrands to be equal everywhere. Therefore, this equation is equivalent to:

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}.$$

Thus the integral and differential forms are equivalent.

In terms of free charge

Free versus bound charge

The electric charge that arises in the simplest textbook situations would be classified as "free charge"—for example, the charge which is transferred in static electricity, or the charge on a capacitor plate. In contrast, "bound charge" arises only in the context of dielectric (polarizable) materials. (All materials are polarizable to some extent.) When such materials are placed in an external electric field, the electrons remain bound to their respective atoms, but shift a microscopic distance in response to the field, so that they're more on one side of the atom than the other. All these microscopic displacements add up to give a macroscopic net charge distribution, and this constitutes the "bound charge".

Although microscopically, all charge is fundamentally the same, there are often practical reasons for wanting to treat bound charge differently from free charge. The result is that the more "fundamental" Gauss's law, in terms of **E**, is sometimes put into the equivalent form below, which is in terms of **D** and the free charge only.

Integral form

This formulation of Gauss's law states that, for any volume *V* in space, with surface *S*, the following equation holds:

$$\Phi_{D,S} = Q_{\mathrm{free}},$$

where $\Phi_{D,S}$ is the flux of the electric displacement field **D** through *S*, and Q_{free} is the free charge contained in *V*. The flux $\Phi_{D,S}$ is defined analogously to the flux $\Phi_{E,S}$ of the electric field **E** through *S*. Specifically, it is given by the surface integral

$$\Phi_{D,S} = \oint_S \mathbf{D}\cdot\mathrm{d}\mathbf{A}.$$

Differential form

The differential form of Gauss's law, involving free charge only, states:

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}$$

where $\nabla \cdot \mathbf{D}$ is the divergence of the electric displacement field, and ϱ_{free} is the free electric charge density.

The differential form and integral form are mathematically equivalent. The proof primarily involves the divergence theorem.

Equivalence of total and free charge statements

Proof that the formulations of Gauss's law in terms of free charge are equivalent to the formulations involving total charge.
In this proof, we will show that the equation $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ is equivalent to the equation $\nabla \cdot \mathbf{D} = \rho_{\text{free}}$ Note that we're only dealing with the differential forms, not the integral forms, but that is sufficient since the differential and integral forms are equivalent in each case, by the divergence theorem. We introduce the polarization density **P**, which has the following relation to **E** and **D**: $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ and the following relation to the bound charge: $\rho_{\text{bound}} = -\nabla \cdot \mathbf{P}$ Now, consider the three equations: $\rho_{\text{bound}} = \nabla \cdot (-\mathbf{P})$ $\rho_{\text{free}} = \nabla \cdot \mathbf{D}$ $\rho = \nabla \cdot (\epsilon_0 \mathbf{E})$ The key insight is that the sum of the first two equations is the third equation. This completes the proof: The first equation is true by definition, and therefore the second equation is true if and only if the third equation is true. So the second and third equations are equivalent, which is what we wanted to prove.

In linear materials

In homogeneous, isotropic, nondispersive, linear materials, there is a nice, simple relationship between **E** and **D**:

$$\varepsilon \mathbf{E} = \mathbf{D}$$

where ε is the permittivity of the material. Under these circumstances, there is yet another pair of equivalent formulations of Gauss's law:

$$\Phi_{E,S} = \frac{Q_{\text{free}}}{\varepsilon}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_{\text{free}}}{\varepsilon}$$

Relation to Coulomb's law

Deriving Gauss's law from Coulomb's law

Gauss's law can be derived from Coulomb's law, which states that the electric field due to a stationary point charge is:

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{e_r}}{r^2}$$

where

$\mathbf{e_r}$ is the radial unit vector,

r is the radius, $|\mathbf{r}|$,

ϵ_0 is the electric constant,

q is the charge of the particle, which is assumed to be located at the origin.

Using the expression from Coulomb's law, we get the total field at **r** by using an integral to sum the field at **r** due to the infinitesimal charge at each other point **s** in space, to give

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{s})(\mathbf{r}-\mathbf{s})}{|\mathbf{r}-\mathbf{s}|^3}\, d^3\mathbf{s}$$

where ρ is the charge density. If we take the divergence of both sides of this equation with respect to **r**, and use the known theorem[5]

$$\nabla\cdot\left(\frac{\mathbf{s}}{|\mathbf{s}|^3}\right) = 4\pi\delta(\mathbf{s})$$

where δ(**s**) is the Dirac delta function, the result is

$$\nabla\cdot\mathbf{E}(\mathbf{r}) = \frac{1}{\varepsilon_0}\int \rho(\mathbf{s})\ \delta(\mathbf{r}-\mathbf{s})\, d^3\mathbf{s}$$

Using the "sifting property" of the Dirac delta function, we arrive at

$$\nabla\cdot\mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\varepsilon_0},$$

which is the differential form of Gauss's law, as desired.

Note that since Coulomb's law only applies to *stationary* charges, there is no reason to expect Gauss's law to hold for moving charges *based on this derivation alone*. In fact, Gauss's law *does* hold for moving charges, and in this respect Gauss's law is more general than Coulomb's law.

Deriving Coulomb's law from Gauss's law

Strictly speaking, Coulomb's law cannot be derived from Gauss's law alone, since Gauss's law does not give any information regarding the curl of **E** (see Helmholtz decomposition and Faraday's law). However, Coulomb's law *can* be proven from Gauss's law if it is assumed, in addition, that the electric field from a point charge is spherically-symmetric (this assumption, like Coulomb's law itself, is exactly true if the charge is stationary, and approximately true if the charge is in motion).

Taking S in the integral form of Gauss's law to be a spherical surface of radius r, centered at the point charge Q, we have

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

By the assumption of spherical symmetry, the integrand is a constant which can be taken out of the integral. The result is

$$4\pi r^2 \hat{\mathbf{r}} \cdot \mathbf{E}(\mathbf{r}) = \frac{Q}{\varepsilon_0}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially away from the charge. Again by spherical symmetry, **E** points in the radial direction, and so we get

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2}$$

which is essentially equivalent to Coulomb's law. Thus the inverse-square law dependence of the electric field in Coulomb's law follows from Gauss's law.

Notes

[1] Serway, Raymond A. (1996). *Physics for Scientists and Engineers with Modern Physics, 4th edition*. pp. 687.
[2] Bellone, Enrico (1980). *A World on Paper: Studies on the Second Scientific Revolution*.
[3] Halliday, David; Resnick, Robert (1970). *Fundamentals of Physics*. John Wiley & Sons, Inc. pp. 452–53.
[4] More specifically, the infinitesimal area is thought of as planar and with area d*A*. The vector d**A** is normal to this area element and has magnitude d*A*.Matthews, Paul (1998). *Vector Calculus*. Springer. ISBN 3540761802.
[5] See, for example, Griffiths, David J. (1998). *Introduction to Electrodynamics (3rd ed.)*. Prentice Hall. p. 50. ISBN 0-13-805326-X.

References

Jackson, John David (1999). Classical Electrodynamics, 3rd ed., New York: Wiley. ISBN 0-471-30932-X.

External links

- MIT Video Lecture Series (30 x 50 minute lectures)- Electricity and Magnetism (http://ocw.mit.edu/OcwWeb/Physics/8-02Electricity-and-MagnetismSpring2002/VideoAndCaptions/) Taught by Professor Walter Lewin.
- section on Gauss's law in an online textbook (http://www.lightandmatter.com/html_books/0sn/ch10/ch10.html#Section10.6)
- MISN-0-132 *Gauss's Law for Spherical Symmetry* (http://physnet2.pa.msu.edu/home/modules/pdf_modules/m132.pdf) (PDF file) by Peter Signell for Project PHYSNET (http://www.physnet.org).
- MISN-0-133 *Gauss's Law Applied to Cylindrical and Planar Charge Distributions* (http://physnet2.pa.msu.edu/home/modules/pdf_modules/m133.pdf) (PDF file) by Peter Signell for Project PHYSNET.

Gauss's law for magnetism

In physics, **Gauss's law for magnetism** is one of Maxwell's equations, the four equations that underlie classical electrodynamics. It states that the magnetic field **B** has divergence equal to zero, in other words, that it is a solenoidal vector field. It is equivalent to the statement that magnetic monopoles do not exist. Rather than "magnetic charges", the basic entity for magnetism is the magnetic dipole. (Of course, if monopoles were ever found, the law would have to be modified, as elaborated below.)

Gauss's law for magnetism can be written in two forms, a *differential form* and an *integral form*. These forms are equivalent due to the divergence theorem.

The name "Gauss's law for magnetism"[1] is not universally used. The law is also called "Absence of free magnetic poles".[2] (or some variant); one reference even explicitly says the law has "no name".[3] It is also referred to as the "transversality requirement"[4] because for plane waves it requires that the polarization be transverse to the direction of propagation.

Differential form

The differential form for Gauss's law for magnetism is the following:

$$\nabla \cdot \mathbf{B} = 0$$

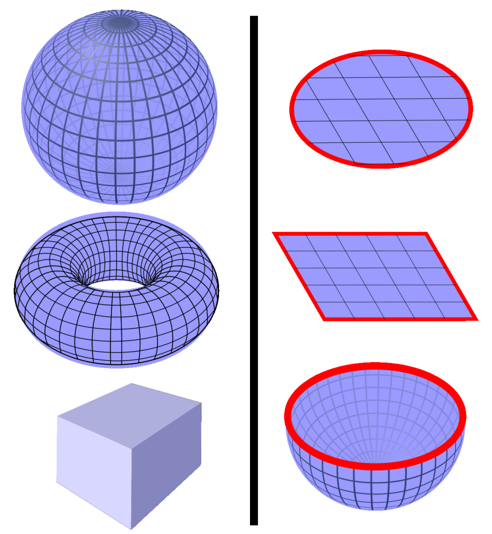
where

$\nabla \cdot$ denotes divergence,

B is the magnetic field.

Integral form

The integral form of Gauss's law for magnetism states:



Definition of a closed surface. Left: Some examples of closed surfaces include the surface of a sphere, surface of a torus, and surface of a cube. The magnetic flux through any of these surfaces is zero. Right: Some examples of non-closed surfaces include the disk surface, square surface, or hemisphere surface. They all have boundaries (red lines) and they do not fully enclose a 3D volume. The magnetic flux through these surfaces is *not necessarily zero*.

$$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$$

where

S is any closed surface (a "closed surface" is the boundary of some three-dimensional volume; the surface of a sphere or cube is a "closed surface", but a disk is not),

d**A** is a vector, whose magnitude is the area of an infinitesimal piece of the surface S, and whose direction is the outward-pointing surface normal (see surface integral for more details).

The left-hand side of this equation is called the net flux of the magnetic field out of the surface, and Gauss's law for magnetism states that it is always zero.

The integral and differential forms of Gauss's law for magnetism are mathematically equivalent, due to the divergence theorem. That said, one or the other might be more convenient to use in a particular computation.

The law in this form states that for each volume element in space, there are exactly the same number of "magnetic field lines" entering and exiting the volume. No total "magnetic charge" can build up in any point in space. For example, the south pole of the magnet is exactly as strong as the north pole, and free-floating south poles without accompanying north poles (magnetic monopoles) are not allowed. In contrast, this is not true for other fields such as electric fields or gravitational fields, where total electric charge or mass can build up in a volume of space.

In terms of vector potential

Due to the Helmholtz decomposition theorem, Gauss's law for magnetism is equivalent to the following statement:[5][6]

There exists a vector field **A** such that $\mathbf{B} = \nabla \times \mathbf{A}$

The vector field $\mathbf{A}$ is called the magnetic vector potential.

Note that there is more than one possible **A** which satisfies this equation for a given **B** field. In fact, there are infinitely many: Any field of the form ∇ φ can be added onto **A** to get an alternative choice for **A**, by the identity (see Vector calculus identities):

$$\nabla \times \mathbf{A} = \nabla \times (\mathbf{A} + \nabla\phi)$$

This arbitrariness in **A** is called gauge freedom.

In terms of field lines

The magnetic field **B**, like any vector field, can be depicted via field lines (also called *flux lines*)-- that is, a set of curves whose direction corresponds to the direction of **B**, and whose areal density is proportional to the magnitude of **B**. Gauss's law for magnetism is equivalent to the statement that the field lines have neither a beginning nor an end: Each one either forms a closed loop, winds around forever without ever quite joining back up to itself exactly, or extends to infinity.

Modification if magnetic monopoles exist

If magnetic monopoles were ever discovered to exist, then Gauss's law for magnetism would be disproved. Instead, the divergence of **B** would be proportional to the "magnetic charge density" ρ_m, as follows:

$\nabla \cdot \mathbf{B} = \rho_m$ (SI units, weber convention)[7]

$\nabla \cdot \mathbf{B} = \mu_0 \rho_m$ (SI units, ampere·meter convention)[8]

$\nabla \cdot \mathbf{B} = 4\pi \rho_m$ (cgs units)[9] where μ_0 is the vacuum permeability.

So far, despite extensive search no magnetic monopoles have been found.

History

The equation $\mathbf{B} = \nabla \times \mathbf{A}$ was one of Maxwell's original eight equations. However, the interpretation was somewhat different: Maxwell's **A** field directly corresponded to an important physical quantity which he believed corresponded to Faraday's *electrotonic state*,[10] while the modern interpretation emphasizes gauge freedom, the idea that there are many possible **A** fields, all equally valid.[10]

Notes

[1] Tai L. Chow (2006). *Electromagnetic Theory: A modern perspective* (http://books.google.com/books?id=dpnpMhw1zo8C&pg=PA153&dq=isbn:0763738271#PPA134,M1). Jones and Bartlett. p. 134. ISBN 0-7637-3827-1. .

[2] John David Jackson (1999). *Classical Electrodynamics* (3rd ed.). Wiley. p. 237. ISBN 0-471-30932-X.

[3] David J. Griffiths (1998). *Introduction to Electrodynamics* (3rd ed.). Prentice Hall. p. 321. ISBN 0-13-805326-X.

[4] John D. Joannopoulos, Steve G. Johnson, Joshua N. Winn, Robert D. Meade (2008). *Photonic Crystals: Molding the Flow of Light* (2nd ed.). Princeton University Press. p. 9. ISBN 978-0-691-12456-8.

[5] W.H.A. Schilders *et al.* (2005-05-23). *Handbook of Numerical Analysis* (http://books.google.com/books?id=F_E9SAe6ny0C&pg=PA13). p. 13. ISBN 9780444513755. .

[6] John David Jackson (1999). *Classical Electrodynamics* (3rd ed.). Wiley. p. 180. ISBN 0-471-30932-X.

[7] John David Jackson (1999). *Classical Electrodynamics* (3rd ed.). Wiley. p. 273, eq. (6.150).

[8] See for example equation (4) in M. Nowakowski, N. G. Kelkar (2005). "Faraday's law in the presence of magnetic monopoles". *Europhysics Letters* **71** (3): 346. arXiv:physics/0508099. Bibcode 2005EL.....71..346N. doi:10.1209/epl/i2004-10545-2.

[9] F. Moulin (2001). "Magnetic monopoles and Lorentz force". *Il Nuovo Cimento B* **116** (8): 869–877. arXiv:math-ph/0203043. Bibcode 2001NCimB.116..869M.

[10] Paul G. Hurray (2010). *Maxwell's Equations* (http://books.google.com/books?id=0QsDgdd0MhMC&pg=PA22). p. 22. ISBN 9780470542767. .

Related equations

Biot–Savart law

The **Biot–Savart law** (/ˈbiːoʊsəˈvɑr/ or /ˈbjoʊsəˈvɑr/)[1] is an equation in electromagnetism that describes the magnetic field **B** generated by an electric current. The vector field **B** depends on the magnitude, direction, length, and proximity of the electric current, and also on a fundamental constant called the magnetic constant. The law is valid in the magnetostatic approximation, and results in a **B** field consistent with both Ampère's circuital law and Gauss's law for magnetism.[2]

Introduction

The Biot–Savart law is used to compute the magnetic field generated by a *steady current*, i.e. a continual flow of charges, for example through a wire, which is constant in time and in which charge is neither building up nor depleting at any point. The equation in SI units is

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{\hat{r}}}{|r|^2},$$

or, equivalently,

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{r}}{|r|^3},$$

where

I is the current,

$d\mathbf{l}$ is a vector, whose magnitude is the length of the differential element of the wire, and whose direction is the direction of conventional current,

B is the net magnetic field,

μ_0 is the magnetic constant,

$\mathbf{\hat{r}}$ is the displacement unit vector in the direction pointing from the wire element towards the point at which the field is being computed, and

$\mathbf{r} = r\mathbf{\hat{r}}$ is the full displacement vector from the wire element to the point at which the field is being computed.

The symbols in boldface denote vector quantities.

To apply the equation, you choose a point in space at which you want to compute the magnetic field. Holding that point fixed, you integrate over the path of the current(s) to find the total magnetic field at that point. The application of this law implicitly relies on the superposition principle for magnetic fields, i.e. the fact that the magnetic field is a vector sum of the field created by each infinitesimal section of the wire individually.[3]

The formulations given above work well when the current can be approximated as running through an infinitely-narrow wire. If the current has some thickness, the proper formulation of the Biot–Savart law (again in SI units) is:

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{(\mathbf{J}\, dV) \times \mathbf{\hat{r}}}{r^2}, \text{ or (equivalently), } \mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{(\mathbf{J}\, dV) \times \mathbf{r}}{r^3},$$

where dV is the differential element of volume and **J** is the current density vector in that volume.

The Biot–Savart law is fundamental to magnetostatics, playing a similar role to Coulomb's law in electrostatics. When magnetostatics does not apply, the Biot–Savart law should be replaced by Jefimenko's equations.

Forms

General

In the magnetostatic approximation, the magnetic field can be determined if the current density **J** is known:

$$\mathbf{B} = K_m \int \frac{\mathbf{J} \times \hat{\mathbf{r}}}{r^2} dV$$

where:

dV is the differential element of volume.

$K_m = \frac{\mu_0}{4\pi}$ is the magnetic constant

Constant uniform current

In the special case of a constant, uniform current **I**, the magnetic field **B** is

$$\mathbf{B} = K_m I \int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

Point charge at constant velocity

In the case of a charged point particle **q** moving at a constant velocity **v**, then Maxwell's equations give the following expression for the electric field and magnetic field:[4]

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{1 - v^2/c^2}{(1 - v^2 \sin^2\theta/c^2)^{3/2}} \frac{\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{B} = \mathbf{v} \times \frac{1}{c^2}\mathbf{E}$$

where $\hat{\mathbf{r}}$ is the vector pointing from the current (non-retarded) position of the particle to the point at which the field is being measured, and θ is the angle between $\mathbf{v}$ and $\mathbf{r}$.

When $v^2 \ll c^2$, the electric field and magnetic field can be approximated as[4]

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}}}{r^2} \ \grave{}$$

$$\mathbf{B} = \frac{\mu_0 q \mathbf{v}}{4\pi} \times \frac{\hat{\mathbf{r}}}{r^2}$$

These equations are called the "Biot–Savart law for a point charge"[5] due to its closely analogous form to the "standard" Biot–Savart law given previously. These equations were first derived by Oliver Heaviside in 1888.

Magnetic responses applications

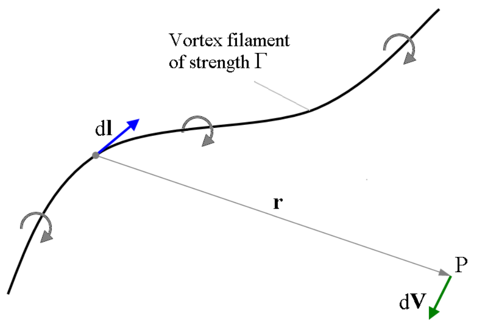
The Biot–Savart law can be used in the calculation of magnetic responses even at the atomic or molecular level, e.g. chemical shieldings or magnetic susceptibilities, provided that the current density can be obtained from a quantum mechanical calculation or theory.

Aerodynamics applications

The Biot–Savart law is also used in aerodynamic theory to calculate the velocity induced by vortex lines.

In the aerodynamic application, the roles of vorticity and current are reversed as when compared to the magnetic application.

In Maxwell's 1861 paper 'On Physical Lines of Force [6]', magnetic field strength **H** was directly equated with pure vorticity (spin), whereas **B** was a weighted vorticity that was weighted for the density of the vortex sea. Maxwell considered magnetic permeability μ to be a measure of the density of the vortex sea. Hence the relationship,



The figure shows the velocity (dV) induced at a point P by an element of vortex filament (dL) of strength Γ.

(1) **Magnetic induction current**

$$\mathbf{B} = \mu\mathbf{H}$$

was essentially a rotational analogy to the linear electric current relationship,

(2) **Electric convection current**

$$\mathbf{J} = \rho\mathbf{v}$$

where **ρ** is electric charge density. **B** was seen as a kind of magnetic current of vortices aligned in their axial planes, with **H** being the circumferential velocity of the vortices.

The electric current equation can be viewed as a convective current of electric charge that involves linear motion. By analogy, the magnetic equation is an inductive current involving spin. There is no linear motion in the inductive current along the direction of the **B** vector. The magnetic inductive current represents lines of force. In particular, it represents lines of inverse square law force.

In aerodynamics the induced air currents are forming solenoidal rings around a vortex axis that is playing the role that electric current plays in magnetism. This puts the air currents of aerodynamics into the equivalent role of the magnetic induction vector **B** in electromagnetism.

In electromagnetism the **B** lines form solenoidal rings around the source electric current, whereas in aerodynamics, the air currents form solenoidal rings around the source vortex axis.

Hence in electromagnetism, the vortex plays the role of 'effect' whereas in aerodynamics, the vortex plays the role of 'cause'. Yet when we look at the **B** lines in isolation, we see exactly the aerodynamic scenario in so much as that **B** is the vortex axis and **H** is the circumferential velocity as in Maxwell's 1861 paper.

For a vortex line of infinite length, the induced velocity at a point is given by

$$v = \frac{\Gamma}{2\pi r}$$

where

Γ is the strength of the vortex

r is the perpendicular distance between the point and the vortex line.

This is a limiting case of the formula for vortex segments of finite length:

$$v = \frac{\Gamma}{4\pi r}\left[\cos A - \cos B\right]$$

where A and B are the (signed) angles between the line and the two ends of the segment.

The Biot–Savart law, Ampère's circuital law, and Gauss's law for magnetism

Here is a demonstration that the magnetic field **B** as computed from the Biot–Savart law will always satisfy Ampère's circuital law and Gauss's law for magnetism.[7] Click "show" in the box below for an outline of the proof.

Outline of proof that a magnetic field calculated by the Biot–Savart law will always satisfy Gauss's law for magnetism and Ampère's law.[7]

Starting with the Biot–Savart law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r'\mathbf{J}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

Plugging in the easily-derived relation

$$\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} = -\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right)$$

and using the product rule for curls, as well as the fact that **J** does not depend on the unprimed coordinates, this equation can be rewritten as[7]

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\nabla\times\int d^3r'\frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Since the divergence of a curl is always zero, this establishes Gauss's law for magnetism. Next, taking the curl of both sides, using the formula for the curl of a curl (see the article Curl (mathematics)), and again using the fact that **J** does not depend on the unprimed coordinates, we eventually get the result[7]

$$\nabla\times\mathbf{B} = \frac{\mu_0}{4\pi}\nabla\int d^3r'\mathbf{J}(\mathbf{r}')\cdot\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) - \frac{\mu_0}{4\pi}\int d^3r'\mathbf{J}(\mathbf{r}')\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right)$$

Finally, plugging in the relations[7]

$$\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -\nabla'\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right),$$

$$\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta(\mathbf{r}-\mathbf{r}')$$

(where δ is the Dirac delta function), using the fact that the divergence of **J** is zero (due to the assumption of magnetostatics), and performing an integration by parts, the result turns out to be[7]

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J}$$

i.e. Ampère's law.

Notes

[1] (http://dictionary.reference.com/browse/biot+savart+law?qsrc=2446)

[2] Jackson, John David (1999). *Classical Electrodynamics* (3rd ed. ed.). New York: Wiley. Chapter 5. ISBN 0-471-30932-X.

[3] The superposition principle holds for the electric and magnetic fields because they are the solution to a set of linear differential equations, namely Maxwell's equations, where the current is one of the "source terms".

[4] Griffiths, David J. (1998). *Introduction to Electrodynamics (3rd ed.)*. Prentice Hall. pp. 222–224, 435–440. ISBN 0-13-805326-X.

[5] http://maxwell.ucdavis.edu/~electro/magnetic_field/pointcharge.html

[6] http://upload.wikimedia.org/wikipedia/commons/b/b8/On_Physical_Lines_of_Force.pdf

[7] See Jackson, page 178–79 or Griffiths p. 222–24. The presentation in Griffiths is particularly thorough, with all the details spelled out.

References

- Griffiths, David J. (1998). *Introduction to Electrodynamics* (3rd ed. ed.). Prentice Hall. ISBN 0-13-805326-X.
- Feynman, Richard (1966). *The Feynman Lectures on Physics* (2nd ed. ed.). Addison-Wesley. ISBN 0-63-20717.

External links

- Electromagnetism (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html), B. Crowell, Fullerton College
- MISN-0-125 *The Ampère–Laplace–Biot–Savart Law* (http://physnet2.pa.msu.edu/home/modules/pdf_modules/m125.pdf) by Orilla McHarris and Peter Signell for Project PHYSNET (http://www.physnet.org).

Electromagnetic wave equation

The **electromagnetic wave equation** is a second-order partial differential equation that describes the propagation of electromagnetic waves through a medium or in a vacuum. The homogeneous form of the equation, written in terms of either the electric field **E** or the magnetic field **B**, takes the form:

$$\left(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}\right)\mathbf{E} \ = \ \mathbf{0}$$
$$\left(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}\right)\mathbf{B} \ = \ \mathbf{0}$$

where

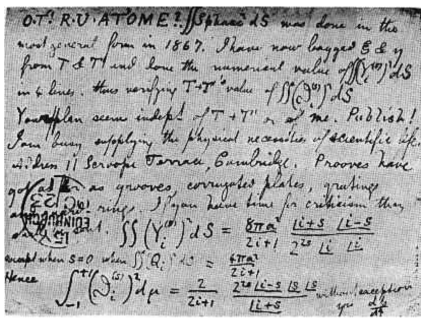
$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

is the speed of light in the medium, and ∇^2 is the Laplace operator. In a vacuum, $c = c_0$ = 299,792,458 meters per second, which is the speed of light in free space.[1] The electromagnetic wave equation derives from Maxwell's equations. It should also be noted that in most older literature, **B** is called the *magnetic flux density* or *magnetic induction*.

The origin of the electromagnetic wave equation

In his 1864 paper titled A Dynamical Theory of the Electromagnetic Field, Maxwell utilized the correction to Ampère's circuital law that he had made in part III of his 1861 paper On Physical Lines of Force. In *Part VI* of his 1864 paper titled *Electromagnetic Theory of Light*,[2] Maxwell combined displacement current with some of the other equations of electromagnetism and he obtained a wave equation with a speed equal to the speed of light. He commented:

> *The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.*[3]



A postcard from Maxwell to Peter Tait.

Maxwell's derivation of the electromagnetic wave equation has been replaced in modern physics by a much less cumbersome method involving combining the corrected version of Ampère's circuital law with Faraday's law of induction.

To obtain the electromagnetic wave equation in a vacuum using the modern method, we begin with the modern 'Heaviside' form of Maxwell's equations. In a vacuum and charge free space, these equations are:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 0\\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}\\ \nabla \cdot \mathbf{B} &= 0\\ \nabla \times \mathbf{B} &= \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

where ρ = 0 because there's no charge density in free space.

Taking the curl of the curl equations gives:

$$\begin{aligned}\nabla \times \nabla \times \mathbf{E} &= -\frac{\partial}{\partial t}\nabla \times \mathbf{B} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}\\ \nabla \times \nabla \times \mathbf{B} &= \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\nabla \times \mathbf{E} = -\mu_o \varepsilon_o \frac{\partial^2 \mathbf{B}}{\partial t^2}\end{aligned}$$

By using the vector identity

$$\nabla \times \left(\nabla \times \mathbf{V}\right) = \nabla\left(\nabla \cdot \mathbf{V}\right) - \nabla^2 \mathbf{V}$$

where $\mathbf{V}$ is any vector function of space, it turns into the wave equations:

$$\begin{aligned}\frac{\partial^2 \mathbf{E}}{\partial t^2} - {c_0}^2 \cdot \nabla^2 \mathbf{E} &= 0\\ \frac{\partial^2 \mathbf{B}}{\partial t^2} - {c_0}^2 \cdot \nabla^2 \mathbf{B} &= 0\end{aligned}$$

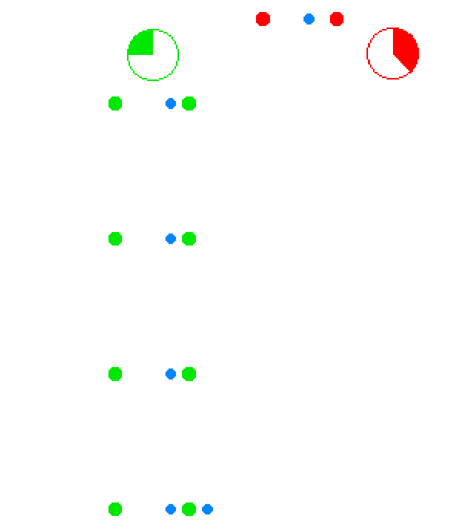
where

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \ \mathrm{m/s}$$

is the speed of light in free space.

Covariant form of the homogeneous wave equation

These relativistic equations can be written in contravariant form as



Time dilation in transversal motion. The requirement that the speed of light is constant in every inertial reference frame leads to the theory of Special Relativity.

$$\Box A^{\mu} = 0$$

where the electromagnetic four-potential is

$$A^{\mu} = (\phi/c, \mathbf{A})$$

with the Lorenz gauge condition:

$$\partial_{\mu} A^{\mu} = 0.$$

Where

$\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is the d'Alembertian operator. (The square box is not a typographical error; it is the correct symbol for this operator.)

Homogeneous wave equation in curved spacetime

The electromagnetic wave equation is modified in two ways, the derivative is replaced with the covariant derivative and a new term that depends on the curvature appears.

$$-{A^{\alpha;\beta}}_{;\beta} + {R^{\alpha}}_{\beta} A^{\beta} = 0$$

where ${R^{\alpha}}_{\beta}$ is the Ricci curvature tensor and the semicolon indicates covariant differentiation.

The generalization of the Lorenz gauge condition in curved spacetime is assumed:

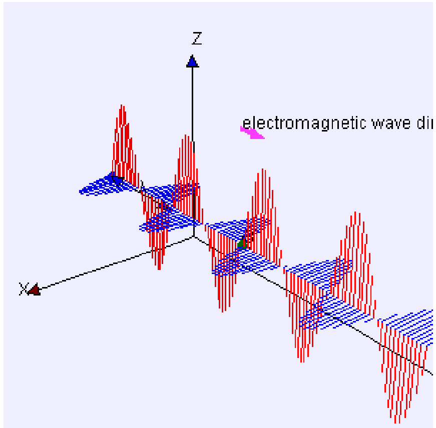
$${A^{\mu}}_{;\mu} = 0.$$

Inhomogeneous electromagnetic wave equation

Localized time-varying charge and current densities can act as sources of electromagnetic waves in a vacuum. Maxwell's equations can be written in the form of a wave equation with sources. The addition of sources to the wave equations makes the partial differential equations inhomogeneous.

Solutions to the homogeneous electromagnetic wave equation

The general solution to the electromagnetic wave equation is a linear superposition of waves of the form



This 3D diagram shows a plane linearly polarized wave propagating from left to right with the same wave equations where $\mathbf{E} = E_0 \sin(-\omega t + \mathbf{k} \cdot \mathbf{r})$ and $\mathbf{B} = B_0 \sin(-\omega t + \mathbf{k} \cdot \mathbf{r})$

$$\mathbf{E}(\mathbf{r}, t) = g(\phi(\mathbf{r}, t)) = g(\omega t - \mathbf{k} \cdot \mathbf{r})$$

and

$$\mathbf{B}(\mathbf{r}, t) = g(\phi(\mathbf{r}, t)) = g(\omega t - \mathbf{k} \cdot \mathbf{r})$$

for virtually *any* well-behaved function g of dimensionless argument φ, where

ω is the angular frequency (in radians per second), and

$\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector (in radians per meter).

Although the function g can be and often is a monochromatic sine wave, it does not have to be sinusoidal, or even periodic. In practice, g cannot have infinite periodicity because any real electromagnetic wave must always have a finite extent in time and space. As a result, and based on the theory of Fourier decomposition, a real wave must consist of the superposition of an infinite set of sinusoidal frequencies.

In addition, for a valid solution, the wave vector and the angular frequency are not independent; they must adhere to the dispersion relation:

$$k = |\mathbf{k}| = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

where k is the wavenumber and λ is the wavelength.

Monochromatic, sinusoidal steady-state

The simplest set of solutions to the wave equation result from assuming sinusoidal waveforms of a single frequency in separable form:

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{i\omega t}\}$$

where

- i is the imaginary unit,
- $\omega = 2\pi f$ **is the angular frequency in radians per second,**
- f is the **frequency in hertz, and**
- $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ is Euler's formula.

Plane wave solutions

Consider a plane defined by a unit normal vector

$$\mathbf{n} = \frac{\mathbf{k}}{k}.$$

Then planar traveling wave solutions of the wave equations are

$$\mathbf{E}(\mathbf{r}) = E_0 e^{-i\mathbf{k}\cdot\mathbf{r}}$$

and

$$\mathbf{B}(\mathbf{r}) = B_0 e^{-i\mathbf{k}\cdot\mathbf{r}}$$

where

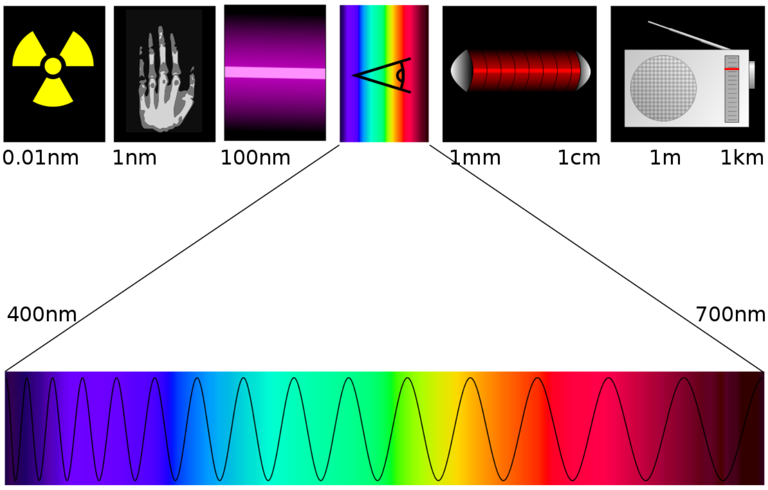
$\mathbf{r} = (x, y, z)$ is the position vector (in meters).

These solutions represent planar waves traveling in the direction of the normal vector $\mathbf{n}$. If we define the z direction as the direction of $\mathbf{n}$ and the x direction as the direction of $\mathbf{E}$, then by Faraday's Law the magnetic field lies in the y direction and is related to the electric field by the relation $c^2\frac{\partial B}{\partial z} = \frac{\partial E}{\partial t}$. Because the divergence of the electric and magnetic fields are zero, there are no fields in the direction of propagation.

This solution is the linearly polarized solution of the wave equations. There are also circularly polarized solutions in which the fields rotate about the normal vector.

Spectral decomposition

Because of the linearity of Maxwell's equations in a vacuum, solutions can be decomposed into a superposition of sinusoids. This is the basis for the Fourier transform method for the solution of differential equations. The sinusoidal solution to the electromagnetic wave equation takes the form



Electromagnetic spectrum illustration.

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$

and

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$

where

t is time (in seconds),

ω is the angular frequency (in radians per second),

$\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector (in radians per meter), and

ϕ_0 is the phase angle (in radians).

The wave vector is related to the angular frequency by

$$k = |\mathbf{k}| = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

where k is the wavenumber and λ is the wavelength.

The electromagnetic spectrum is a plot of the field magnitudes (or energies) as a function of wavelength.

Multipole Expansion

Assuming monochromatic fields varying in time as $e^{-i\omega t}$, if one uses Maxwell's Equations to eliminate **B**, the electromagnetic wave equation reduces to the Helmholtz Equation for **E**:

$$(\nabla^2 + k^2)\mathbf{E} = 0,\ \mathbf{B} = -\frac{i}{k}\nabla \times \mathbf{E},$$

with $k = \omega/c$ as given above. Alternatively, one can eliminate **E** in favor of **B** to obtain:

$$(\nabla^2 + k^2)\mathbf{B} = 0,\ \mathbf{E} = -\frac{i}{k}\nabla \times \mathbf{B}.$$

A generic electromagnetic field with frequency ω can be written as a sum of solutions to these two equations. The three-dimensional solutions of the Helmholtz Equation can be expressed as expansions in spherical harmonics with coefficients proportional to the spherical Bessel functions. However, applying this expansion to each vector component of **E** or **B** will give solutions that are not generically divergence-free ($\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{B} = 0$), and therefore require additional restrictions on the coefficients.

The multipole expansion circumvents this difficulty by expanding not **E** or **B**, but $\mathbf{r} \cdot \mathbf{E}$ or $\mathbf{r} \cdot \mathbf{B}$ into spherical harmonics. These expansions still solve the original Helmholtz equations for **E** and **B** because for a divergence-free field **F**, $\nabla^2 (\mathbf{r} \cdot \mathbf{F}) = \mathbf{r} \cdot (\nabla^2 \mathbf{F})$. The resulting expressions for a generic electromagnetic field are:

$$\mathbf{E} = e^{-i\omega t} \sum_{l,m} \sqrt{l(l+1)} \left[a_E(l,m)\mathbf{E}^{(E)}_{l,m} + a_M(l,m)\mathbf{E}^{(M)}_{l,m}\right]$$

$$\mathbf{B} = e^{-i\omega t} \sum_{l,m} \sqrt{l(l+1)} \left[a_E(l,m)\mathbf{B}^{(E)}_{l,m} + a_M(l,m)\mathbf{B}^{(M)}_{l,m}\right],$$

where $\mathbf{E}^{(E)}_{l,m}$ and $\mathbf{B}^{(E)}_{l,m}$ are the *electric multipole fields of order (l, m)*, and $\mathbf{E}^{(M)}_{l,m}$ and $\mathbf{B}^{(M)}_{l,m}$ are the corresponding *magnetic multipole fields*, and $a_E(l,m)$ and $a_M(l,m)$ are the coefficients of the expansion. The multipole fields are given by

$$\mathbf{B}^{(E)}_{l,m} = \sqrt{l(l+1)} \left[B^{(1)}_l h^{(1)}_l(kr) + B^{(2)}_l h^{(2)}_l(kr)\right] \mathbf{\Phi}_{l,m}$$

$$\mathbf{E}^{(E)}_{l,m} = \frac{i}{k}\nabla \times \mathbf{B}^{(E)}_{l,m}$$

$$\mathbf{E}^{(M)}_{l,m} = \sqrt{l(l+1)} \left[E^{(1)}_l h^{(1)}_l(kr) + E^{(2)}_l h^{(2)}_l(kr)\right] \mathbf{\Phi}_{l,m}$$

$$\mathbf{B}^{(M)}_{l,m} = -\frac{i}{k}\nabla \times \mathbf{E}^{(M)}_{l,m},$$

where $h_l^{(1,2)}(x)$ are the spherical Hankel functions, $E_l^{(1,2)}$ and $B_l^{(1,2)}$ are determined by boundary conditions, and $\mathbf{\Phi}_{l,m} = \frac{1}{\sqrt{l(l+1)}}(\mathbf{r} \times \nabla)Y_{l,m}$ are vector spherical harmonics normalized so that

$$\int \mathbf{\Phi}^*_{l,m} \cdot \mathbf{\Phi}_{l',m'} d\Omega = \delta_{l,l'}\delta m, m'$$

The multipole expansion of the electromagnetic field finds application in a number of problems involving spherical symmetry, for example antennae radiation patterns, or nuclear gamma decay. In these applications, one is often interested in the power radiated in the far-field. In this regions, the **E** and **B** fields asymptote to

$$\mathbf{B} \approx \frac{e^{i(kr-\omega t)}}{kr} \sum_{l,m} (-i)^{l+1} \left[a_E(l,m)\mathbf{\Phi}_{l,m} + a_M(l,m)\hat{\mathbf{r}} \times \mathbf{\Phi}_{l,m}\right]$$

$$\mathbf{E} \approx \mathbf{B} \times \hat{\mathbf{r}}.$$

The angular distribution of the time-averaged radiated power is then given by

$$\frac{dP}{d\Omega} \approx \frac{1}{2k^2}\left|\sum_{l,m}(-i)^{l+1}\left[a_E(l,m)\mathbf{\Phi}_{l,m}\times\hat{\mathbf{r}} + a_M(l,m)\mathbf{\Phi}_{l,m}\right]\right|^2$$

Other solutions

Other spherically and cylindrically symmetric analytic solutions to the electromagnetic wave equations are also possible.

In spherical coordinates the solutions to the wave equation can be written as follows:

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{r}\mathbf{E}_0\cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0)\,,\ \mathbf{E}(\mathbf{r},t) = \frac{1}{r}\mathbf{E}_0\sin(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0)$$

and

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{r}\mathbf{B}_0\cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0),$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{r}\mathbf{B}_0\sin(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0).$$

These can be rewritten in terms of the spherical bessel function.

In cylindrical coordinates, the solutions to the wave equation are the ordinary bessel function of integer order.

Notes

[1] Current practice is to use c_0 to denote the speed of light in vacuum according to ISO 31. In the original Recommendation of 1983, the symbol c was used for this purpose. See NIST *Special Publication 330*, Appendix 2, p. 45 (http://physics.nist.gov/Pubs/SP330/sp330.pdf)

[2] Maxwell 1864, page 497.

[3] See Maxwell 1864, page 499.

Further reading

Electromagnetism

Journal articles

- Maxwell, James Clerk, "*A Dynamical Theory of the Electromagnetic Field*", Philosophical Transactions of the Royal Society of London 155, 459-512 (1865). (This article accompanied a December 8, 1864 presentation by Maxwell to the Royal Society.)

Undergraduate-level textbooks

- Griffiths, David J. (1998). *Introduction to Electrodynamics (3rd ed.)*. Prentice Hall. ISBN 0-13-805326-X.
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- H. M. Schey, *Div Grad Curl and all that: An informal text on vector calculus*, 4th edition (W. W. Norton & Company, 2005) ISBN 0-393-92516-1.

Biographies

- André-Marie Ampère
- Albert Einstein
- Michael Faraday
- Heinrich Hertz
- Oliver Heaviside
- James Clerk Maxwell

Electromotive force

In physics, **electromotive force**, **emf** (seldom capitalized), or **electromotance** (denoted ε and measured in volts) refers to voltage generated by a battery or by the magnetic force according to Faraday's Law, which states that a time varying magnetic field will induce an electric current.[1]

It is important to note that the electromotive "force" is not a force in the classical physics sense—as can be seen in the fact that it is measured in volts and not newtons. Formally, emf is the external work expended per unit of charge to produce an electric potential difference across two open-circuited terminals.[2] [3] The electric potential difference produced is created by separating positive and negative charges, thereby generating an electric field.[4] [5] The created electrical potential difference drives current flow if a circuit is attached to the source of emf. When current flows, however, the voltage across the terminals of the source of emf is no longer the open-circuit value, due to voltage drops inside the device due to its internal resistance.

Devices that can provide emf include electrochemical cells, thermoelectric devices, solar cells, electrical generators, transformers, and even Van de Graaff generators.[2] [6] In nature, emf is generated whenever magnetic field fluctuations occur through a surface. An example for this is the varying Earth magnetic field during a geomagnetic storm, acting on anything on the surface of the planet, like an extended electrical grid.

In the case of a battery, charge separation that gives rise to a voltage difference is accomplished by chemical reactions at the electrodes;[5] a voltaic cell can be thought of as having a "charge pump" of atomic dimensions at each electrode, that is:[7]

> A source of emf can be thought of as a kind of *charge pump* that acts to move positive charge from a point of low potential through its interior to a point of high potential. … By chemical, mechanical or other means, the source of emf performs work dW on that charge to move it to the high potential terminal. The emf $\mathcal{E}$ of the source is defined as the work dW done per charge dq: $\mathcal{E} = dW/dq$.

Around 1830 Faraday established that the reactions at each of the two electrode–electrolyte interfaces provide the "seat of emf" for the voltaic cell, that is, these reactions drive the current.[8] In the open-circuit case, charge separation continues until the electrical field from the separated charges is sufficient to arrest the reactions. Years earlier, Volta, who had measured a contact potential difference at the metal-metal (electrode-electrode) interface of his cells, held the incorrect opinion that this contact potential was the origin of the seat of emf.

In the case of an electrical generator, a time-varying magnetic field inside the generator creates an electric field via electromagnetic induction, which in turn creates an energy difference between generator terminals. Charge separation takes place within the generator, with electrons flowing away from one terminal and toward the other, until, in the open-circuit case, sufficient electric field builds up to make further movement unfavorable. Again the emf is countered by the electrical voltage due to charge separation. If a load is attached, this voltage can drive a current. The general principle governing the emf in such electrical machines is Faraday's law of induction.

A solar cell or photodiode is another source of emf, with light energy as the external power source.

Notation and units of measurement

Electromotive force is often denoted by $\mathcal{E}$ or ℰ (script capital E, Unicode U+2130).

In a device without internal resistance, if an electric charge Q passes through that device, and gains an energy W, the net emf for that device is the energy gained per unit charge, or W/Q. Like other measures of energy per charge, emf has SI units of volts, equivalent to joules per coulomb.[9]

Electromotive force in electrostatic units is the statvolt (in the centimeter gram second system of units equal in amount to an erg per electrostatic unit of charge).

Terminology

The term electromotive force is due to Alessandro Volta (1745–1827), who invented the battery, or voltaic pile. "Electromotive force" originally referred to the 'force' with which positive and negative charges could be separated (that is, moved, hence "electromotive"), and was also called "electromotive power" (although it is not a power in the modern sense). Maxwell's 1865 explanation of what are now called Maxwell's equations used the term "electromotive force" for what is now called the electric field strength.[10] But, in his later textbook[11] he uses the term "electromotive force" both for "voltage-like" causes of current flow in an electric circuit, and (inconsistently) for contact potential difference (which is a form of electrostatic potential difference). Given that Maxwell's textbook was written before the discovery of the electron, it is understandable that Maxwell exhibits what (in terms of modern knowledge) is inconsistency in the use of the term "electromotive force".

The word "force" in "electromotive force" is a misnomer:[12]

> [Electromotive force] has turned out to be an unfortunate choice of words which is still with us 160 years later. In all of physics except electromagnetic induction, the term 'force' is reserved for mechanical action on ponderable matter and is measured in units called newtons. In contrast electromotive force is measured in units of volts and causes charge separation.[12]

Nonetheless, the term "electromotive force" has resisted change. "Electromotance", meaning (literally) tendency to move ("-motance") electrical charge, is semantically more accurate, but not widely adopted. Both terms are less common than the abbreviation *emf*.

These terms (*emf*, *voltage*, etc.) have many interpretations and applications, not all necessarily consistent with each other. The emf is typically considered to be the work done per unit charge by a source in creating a separation of positive from negative charges, thereby creating a voltage difference; the work done per unit charge in pushing charge through a battery creating the battery's voltage difference, for example. However, there is not complete unanimity upon this usage. As Sydney Ross says, in excusing himself for avoiding the term emf:[13]

> We have refrained from using the term 'electromotive force' or 'e.m.f.' for short; for there is no consistency between different authors in the meaning of the term. … To some authors it is synonymous with 'voltage.' To others it means the open-circuit voltage of a battery. To a third group of authors it means the open-circuit voltage of any two-terminal device. This use is met most often in connection with Thevenin's theorem in circuit theory. To a fourth group it means the work accounted for by agencies other than differences of the (not measurable) Galvani potentials. Such authors equate the current–resistance product of a circuit branch to the sum of voltage plus e.m.f. A fifth group extends this use to field theory. The authors of this group equate the product of current density and resistivity to the sum of electric-field strength plus an e.m.f. gradient. A sixth group applies the term to electromagnetic induction. These authors define e.m.f. as the spatial line integral of the electric-field strength taken over a complete loop. To them the term 'counter e.m.f.' means something.

It is common in some fields, such as circuit theory, to refer to the *voltage created by the emf* as the *emf*.[14] [15] [16] Some authors do not distinguish between the emf and the voltage it creates.[17] [18] Some use *emf* to refer to the open-circuit voltage and *voltage* to the potential difference when current is drawn.[19] Here is a quotation describing emf as an open-circuit voltage difference:[20]

> This buildup of charge on the electrodes tends to oppose the current flow with a 'back voltage' ΔV. On an open circuit, $I = 0$ the value of ΔV for which $I = 0$ is defined as the emf $\mathscr{E}$ of the cell. That is, $\mathscr{E} = \Delta V_{I=0}$.

This usage does not identify the work done per unit charge in creating the charge build-up as emf, but rather identifies emf with the consequent "back voltage" that arrests current flow in the open-circuit condition.[21] One emphasizes the conversion of energy from other forms to electrical energy, the other emphasizes the resulting electrical potential. This article focuses upon the conversion of other forms of energy to electrical potential energy, and describes some examples of how this conversion comes about.

Formal definitions of electromotive force

Inside a source of emf that is open-circuited, the conservative electrostatic field created by separation of charge exactly cancels the forces producing the emf. Thus, the emf has the same value but opposite sign as the integral of the electric field aligned with an internal path between two terminals *A* and *B* of a source of emf in open-circuit condition (the path is taken from the negative terminal to the positive terminal to yield a positive emf, indicating work done on the electrons moving in the circuit).[22] Mathematically:

$$\mathcal{E} = -\int_A^B \boldsymbol{E_{cs}} \cdot d\boldsymbol{\ell} \ ,$$

where $\boldsymbol{E}_{cs}$ is the conservative electrostatic field created by the charge separation associated with the emf, $d\boldsymbol{\ell}$ is an element of the path from terminal *A* to terminal *B*, and '·' denotes the vector dot product.[23] This equation applies only to locations *A* and *B* that are terminals, and does not apply to paths between points *A* and *B* with portions outside the source of emf. This equation involves the electrostatic electric field due to charge separation $\boldsymbol{E}_{cs}$ and does not involve (for example) any non-conservative component of electric field due to Faraday's law of induction.

In the case of a closed path in the presence of a varying magnetic field, the integral of the electric field around a closed loop may be nonzero; one common application of the concept of emf, known as "*induced emf*" is the voltage induced in a such a loop.[24] The "*induced emf*" around a stationary closed path *C* is:

$$\mathcal{E} = \oint_C \boldsymbol{E} \cdot d\boldsymbol{\ell} \ ,$$

where now $\boldsymbol{E}$ is the entire electric field, conservative and non-conservative, and the integral is around an arbitrary but stationary closed curve *C* through which there is a varying magnetic field. Note that the electrostatic field does not contribute to the net emf around a circuit because the electrostatic portion of the electric field is conservative (that is, the work done against the field around a closed path is zero).

This definition can be extended to arbitrary sources of emf and moving paths C:[25]

$$\mathcal{E} = \oint_C \left[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right] \cdot d\boldsymbol{\ell} + \frac{1}{q}\oint_C \textbf{effective chemical forces} \cdot d\boldsymbol{\ell} + \frac{1}{q}\oint_C \textbf{effective thermal forces} \cdot d\boldsymbol{\ell} \ ,$$

which is a conceptual equation mainly, because the determination of the "effective forces" is difficult.

Electromotive force in thermodynamics

When multiplied by an amount of charge dZ the emf $\mathcal{E}$ yields a thermodynamic work term $\mathcal{E}dZ$ that is used in the formalism for the change in Gibbs free energy when charge is passed in a battery:

$$dG = -SdT + VdP + \mathcal{E}dZ \ ,$$

where G is the Gibb's free energy, S is the entropy, V is the system volume, P is its pressure and T is its absolute temperature.

The combination ($\mathcal{E}$, Z) is an example of a conjugate pair of variables. At constant pressure the above relationship produces a Maxwell relation that links the change in open cell voltage with temperature T (a measurable quantity) to the change in entropy S when charge is passed isothermally and isobarically. The latter is closely related to the reaction entropy of the electrochemical reaction that lends the battery its power. This Maxwell relation is:[26]

$$\left(\frac{\partial \mathcal{E}}{\partial T}\right)_Z = -\left(\frac{\partial S}{\partial Z}\right)_T$$

If a mole of ions goes into solution (for example, in a Daniell cell, as discussed below) the charge through the external circuit is:

$$\Delta Z = -n_0 F_0 \ ,$$

where n_0 is the number of electrons/ion, and F_0 is the Faraday constant and the minus sign indicates discharge of the cell. Assuming constant pressure and volume, the thermodynamic properties of the cell are related strictly to the behavior of its emf by:[26]

$$\Delta H = -n_0 F_0 \left(\mathcal{E} - T\frac{d\mathcal{E}}{dT}\right) \ ,$$

where ΔH is the heat of reaction. The quantities on the right all are directly measurable.

Electromotive force and voltage difference

An electrical voltage difference is sometimes called an emf.[14] [15] [16] [17] [19] The points below illustrate the more formal usage, in terms of the distinction between emf and the voltage it generates:

1. For a circuit as a whole, such as one containing a resistor in series with a voltaic cell, electrical voltage does not contribute to the overall emf, because the voltage difference on going around a circuit is zero. (The ohmic *IR* drop plus the applied electrical voltage is zero. See Kirchhoff's Law). The emf is due solely to the chemistry in the battery that causes charge separation, which in turn creates an electrical voltage that drives the current.
2. For a circuit consisting of an electrical generator that drives current through a resistor, the emf is due solely to a time-varying magnetic field that generates an electrical voltage that in turn drives the current. (The ohmic *IR* drop plus the applied electrical voltage again is zero. See Kirchhoff's Law)

3. A transformer coupling two circuits may be considered a source of emf for one of the circuits, just as if it were caused by an electrical generator; this example illustrates the origin of the term "transformer emf".
4. A photodiode or solar cell may be considered as a source of emf, similar to a battery, resulting in an electrical voltage generated by charge separation driven by light rather than chemical reaction.[27]
5. Other devices that produce emf are fuel cells, thermocouples, and thermopiles.[28]

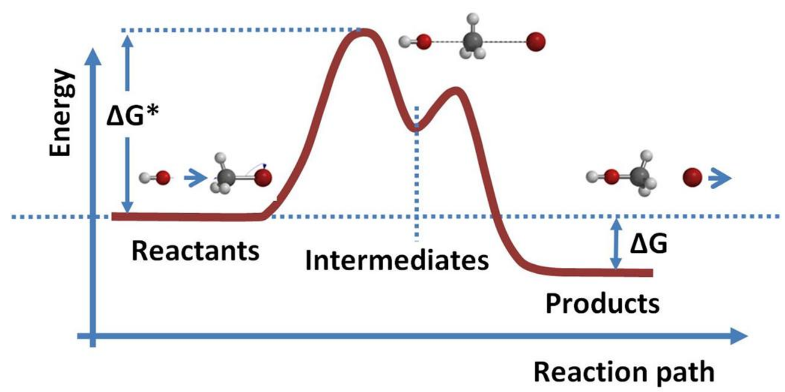
In the case of an open circuit, the electric charge that has been separated by the mechanism generating the emf creates an electric field opposing the separation mechanism. For example, the chemical reaction in a voltaic cell stops when the opposing electric field at each electrode is strong enough to arrest the reactions. A larger opposing field can reverse the reactions in what are called *reversible* cells.[29] [30]

The electric charge that has been separated creates an electric potential difference that can be measured with a voltmeter between the terminals of the device. The magnitude of the emf for the battery (or other source) is the value of this 'open circuit' voltage. When the battery is charging or discharging, the emf itself cannot be measured directly using the external voltage because some voltage is lost inside the source.[] It can, however, be inferred from a measurement of the current I and voltage difference V, provided that the internal resistance r already has been measured: $I = (\mathscr{E} - V)/\mathrm{r}$.

Electromotive force generation

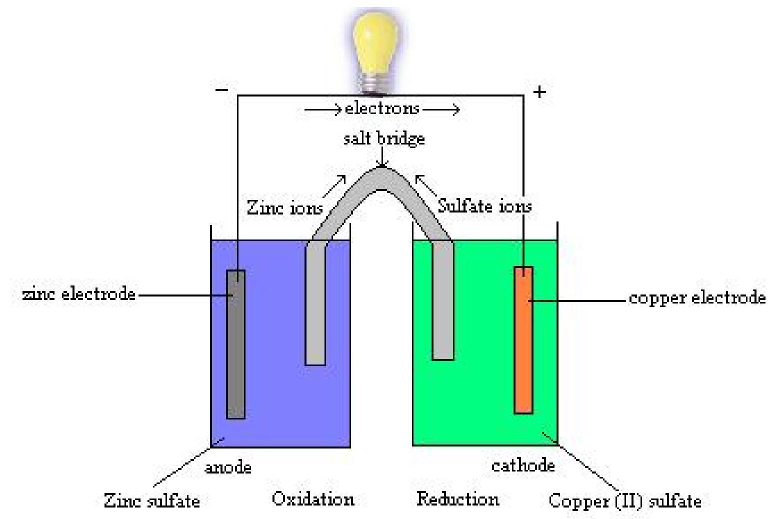
Chemical sources

The question of how batteries (galvanic cells) generate an emf is one that occupied scientists for most of the 19th century. The "seat of the electromotive force" was eventually determined by Walther Nernst to be primarily at the interfaces between the electrodes and the electrolyte.[8]



A typical reaction path requires the initial reactants to cross an energy barrier, enter an intermediate state and finally emerge in a lower energy configuration. If charge separation is involved, this energy difference can result in an emf. See Bergmann *et al.*[31] and Transition state.

Molecules are groups of atoms held together by chemical bonds, and these bonds consist of electrical forces between electrons (negative) and protons (positive). The molecule in isolation is a stable entity, but when different molecules are brought together, some types of molecules are able to steal electrons from others, resulting in charge separation. This redistribution of charge is accompanied by a change in energy of the system, and a reconfiguration of the atoms in the molecules.[32] The gain of an electron is termed "reduction" and the loss of an electron is termed "oxidation". Reactions in



Galvanic cell using a salt bridge

which such electron exchange occurs (which are the basis for batteries) are called reduction-oxidation reactions or redox reactions. In a battery, one electrode is composed of material that gains electrons from the solute, and the other electrode loses electrons, because of these fundamental molecular attributes. The same behavior can be seen in atoms themselves, and their ability to steal electrons is referred to as their electronegativity.[33]

As an example, a Daniell cell consists of a zinc anode (an electron collector), which dissolves into a zinc sulfate solution, the dissolving zinc leaving behind its electrons in the electrode according to the oxidation reaction (*s* = solid electrode; *aq* = aqueous solution):

$$\mathrm{Zn}(s) \rightarrow \mathrm{Zn}^{2+}(aq) + 2\mathrm{e}^- \ .$$

The zinc sulfate is an electrolyte, that is, a solution in which the components consist of ions, in this case zinc ions Zn^{2+}, and sulfate ions SO_4^{2-} .

At the cathode, the copper ions in a copper sulfate electrolyte adopt electrons from the electrode by the reduction reaction:

$$\mathrm{Cu}^{2+}(aq) + 2e^- \rightarrow \mathrm{Cu}(s) \ ,$$

and the thus-neutralized copper plates onto the electrode. (A detailed discussion of the microscopic process of electron transfer between an electrode and the ions in an electrolyte may be found in Conway.)[34]

The electrons pass through the external circuit (light bulb in figure), while the ions pass through the salt bridge to maintain charge balance. In the process the zinc anode is dissolved while the copper electrode is plated with copper.[35] If the light bulb is removed (open circuit) the emf between the electrodes is opposed by the electric field due to charge separation, and the reactions stop.

At 273 K, the emf $\mathcal{E}$ = 1.0934 V, with a temperature coefficient of $d\mathcal{E}/dT = -4.53\times10^{-4}$ V/K.[26]

Voltaic cells

Volta developed the voltaic cell about 1792, and presented his work March 20, 1800.[36] Volta correctly identified the role of dissimilar electrodes in producing the voltage, but incorrectly dismissed any role for the electrolyte.[37] Volta ordered the metals in a 'tension series', “that is to say in an order such that any one in the list becomes positive when in contact with any one that succeeds, but negative by contact with any one that precedes it.”[38] A typical symbolic convention in a schematic of this circuit (—|⊢) would have a long electrode 1 and a short electrode 2, to indicate that electrode 1 dominates. Volta's law about opposing electrode emfs means that, given ten electrodes (for example, zinc and nine other materials), which can be used to produce 45 types of voltaic cells (10 × 9/2), only nine relative measurements (for example, copper and each of the nine others) are needed to get all 45 possible emfs that these ten electrodes can produce.

Electromotive force of cells

The electromotive force produced by primary and secondary cells is usually of the order of a few volts. The figures quoted below are nominal, because emf varies according to the size of the load and the state of exhaustion of the cell.

Emf	Cell chemistry
1.2 V	nickel-cadmium
1.2 V	nickel–metal hydride
1.5 V	zinc–carbon
2.1 V	lead–acid
3.6 V to 3.7 V	lithium-ion

Electromagnetic induction

The principle of electromagnetic induction, noted above, states that a time-dependent magnetic field produces a circulating electric field. A time-dependent magnetic field can be produced either by motion of a magnet relative to a circuit, by motion of a circuit relative to another circuit (at least one of these must be carrying a current), or by changing the current in a fixed circuit. The effect on the circuit itself, of changing the current, is known as self-induction; the effect on another circuit is known as mutual induction.

For a given circuit, the electromagnetically induced emf is determined purely by the rate of change of the magnetic flux through the circuit according to Faraday's law of induction.

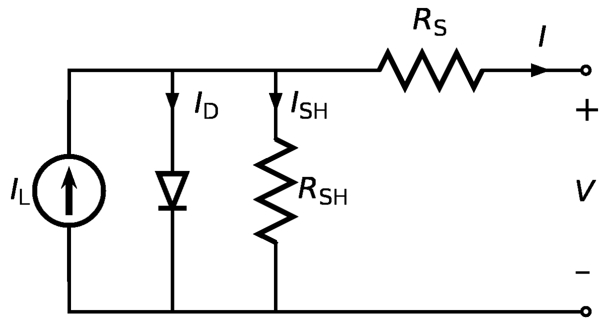
An emf is induced in a coil or conductor whenever there is change in the flux linkages. Depending on the way in which the changes are brought about, there are two types: When the conductor is moved in a stationary magnetic field to procure a change in the flux linkage, the emf is *statically induced*. The electromotive force generated by motion is often referred to as *motional emf*. When the change in flux linkage arises from a change in the magnetic field around the stationary conductor, the emf is *dynamically induced.* The electromotive force generated by a time-varying magnetic field is often referred to as *transformer emf*.

Contact potentials

When two different solids are in contact, it is common that thermodynamic equilibrium requires one of the solids assume a higher electrical potential than the other, the *contact potential*.[39] For example, dissimilar metals in contact produce what is known also as a contact electromotive force or Galvani potential. The magnitude of this potential difference often is expressed as a difference in Fermi levels in the two solids, where the Fermi level (a name for the chemical potential of an electron system[40] [41]) describes the energy necessary to remove an electron from the body.[42] Evidently, if there is an energy advantage in taking an electron from one body to the other, this transfer will occur, thereby causing a charge separation, one body gaining electrons and the other losing electrons. This charge transfer causes a potential difference between the bodies, and therefore, charge transfer becomes more difficult as the charge separation increases. At thermodynamic equilibrium, the gain in energy due to Fermi level difference is matched by the work needed to surmount this potential difference, and at this point no more transfer occurs, and the potential difference has the value called the contact potential. The difference in Fermi levels, on the other hand, is referred to as the emf.[43] The contact potential cannot drive current through a load attached to its terminals because that current would involve a charge transfer. No mechanism exists to continue such transfer and, hence, maintain a current, once equilibrium is attained.

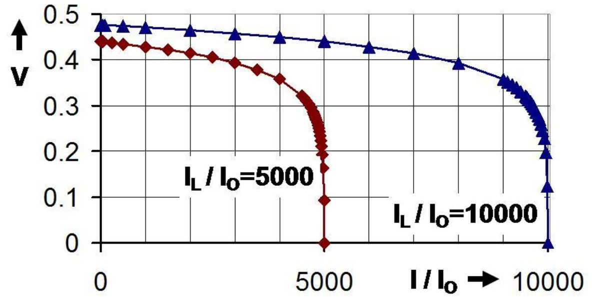
One might inquire why the contact potential does not appear in Kirchhoff's law of voltages as one contribution to the sum of potential drops. The customary answer is that any circuit involves not only a particular diode or junction, but also all the contact potentials due to wiring and so forth around the entire circuit. The sum of *all* the contact potentials is zero, and so they may be ignored in Kirchhoff's law.[44] [45]

Solar cell



The equivalent circuit of a solar cell; parasitic resistances are ignored in the discussion of the text.

Operation of a solar cell can be understood from the equivalent circuit at right. Light, if it includes photons of sufficient energy (greater than the bandgap of the material), creates mobile electron–hole pairs in a semiconductor. Charge separation occurs because of a pre-existing electric field associated with the p-n junction in thermal equilibrium (a contact potential creates the field). This charge separation between positive holes and negative electrons across a p-n junction (a diode), yields a *forward voltage*, the *photo voltage*, between the illuminated diode terminals.[47] As has been noted earlier in the terminology section, the photo *voltage* is sometimes referred to as the photo *emf*, rather than distinguishing between the effect and the cause.



Solar cell voltage as a function of solar cell current delivered to a load for two light-induced currents I_L; currents as a ratio with reverse saturation current I_0. Compare with Fig. 1.4 in Nelson.[46]

The light-induced charge separation creates a reverse current through the cell's junction (that is, not in the direction that a diode normally conducts current), and the charge separation causes a photo voltage that drives current through any attached load. However, a side effect of this voltage is that it tends to forward bias the junction. At high enough levels, this forward bias of the junction will cause a forward current in the diode that subtracts from the current created by the light. Consequently, the greatest current is obtained under short-circuit conditions, and is denoted as I_L (for light-induced current) in the equivalent circuit.[48] Approximately this same current is obtained for forward voltages up to the point where the diode conduction becomes significant.

With this notation, the current-voltage relation for the illuminated diode is:

$$I = I_L - I_0 \left(e^{qV/(mkT)} - 1 \right) \ ,$$

where I is the current delivered to the load, I_0 is the reverse saturation current, and m the ideality factor, two parameters that depend on the solar cell construction and to some degree upon the voltage itself,[48] and where kT/q is the thermal voltage (about 0.026 V at room temperature). This relation is plotted in the figure using a fixed value $m = 2$.[49] Under open-circuit conditions (that is, as $I \rightarrow 0$), the open-circuit voltage is the voltage at which forward bias of the junction is enough that the forward current completely balances the photocurrent. Rearrangement of the I–V equation provides the open-circuit voltage as:

$$V_{oc} = m \ \frac{kT}{q} \ \ln \left(\frac{I_L}{I_0} + 1 \right) \ ,$$

which is useful in indicating a logarithmic dependence of V_{oc} upon the light-induced current. Typically, the open-circuit voltage is not more than about 0.5 V.[50]

The value of the photo voltage when driving a load is variable. As shown in the figure, for a load resistance R_L, the cell develops a voltage between the short-circuit value $V = 0$, $I = I_L$ and the open-circuit value V_{oc}, $I = 0$, a value given by Ohm's law $V = I\,R_L$, where the current I is the difference between the short-circuit current and current due to forward bias of the junction, as indicated by the equivalent circuit (neglecting the parasitic resistances).[46]

In contrast to the battery, at current levels near I_L, the solar cell acts more like a *current source* rather than a voltage source.[46] The current drawn is nearly fixed over a range of load voltages, at one electron per converted photon. The quantum efficiency, or probability of getting an electron of photocurrent per incident photon, depends not only upon the solar cell itself, but upon the spectrum of the light.

The diode possesses a "built-in potential" due to the contact potential difference between the two different materials on either side of the junction. This built-in potential is established when the junction is formed as a by-product of thermodynamic equilibrium. Once established, this potential difference cannot drive a current, however, as connecting a load does not upset this equilibrium. In contrast, the accumulation of excess electrons in one region and of excess holes in another due to illumination results in a photo voltage that does drive a current when a load is attached to the illuminated diode. As noted above, this photo voltage also forward biases the junction, and so *reduces* the pre-existing field in the depletion region.

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External links

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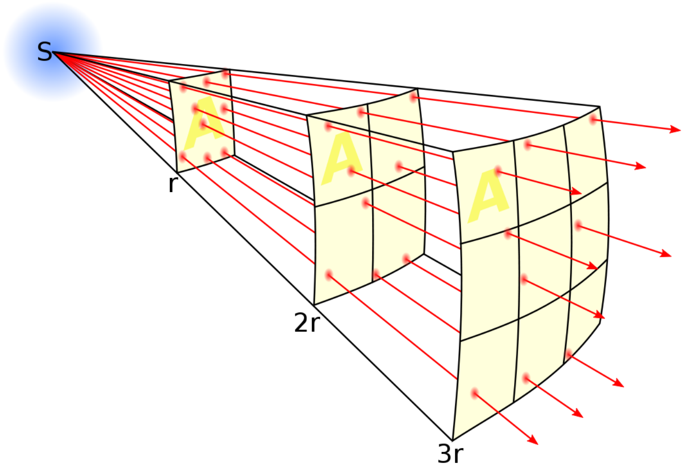
Inverse-square law

In physics, an **inverse-square law** is any physical law stating that a specified physical quantity or strength is inversely proportional to the square of the distance from the source of that physical quantity.

The divergence of a vector field which is the resultant of radial inverse-square law fields with respect to one or more sources is everywhere proportional to the strength of the local sources, and hence zero outside sources.



The lines represent the flux emanating from the source. The total number of flux lines depends on the strength of the source and is constant with increasing distance. A greater density of flux lines (lines per unit area) means a stronger field. The density of flux lines is inversely proportional to the square of the distance from the source because the surface area of a sphere increases with the square of the radius. Thus the strength of the field is inversely proportional to the square of the distance from the source.

Justification

The inverse-square law generally applies when some force, energy, or other conserved quantity is radiated outward radially in three-dimensional space from a point source. Since the surface area of a sphere (which is $4\pi r^2$) is proportional to the square of the radius, as the emitted radiation gets farther from the source, it is spread out over an area that is increasing in proportion to the square of the distance from the source. Hence, the intensity of radiation passing through any unit area (directly facing the point source) is inversely proportional to the square of the distance from the point source. Gauss's law applies to and can be used with any physical quantity that acts in accord to the inverse-square relationship.

Occurrences

Gravitation

Gravitation is the attraction of two objects with mass. This law states:

> *The gravitational attraction force between two **point masses** is directly proportional to the product of their masses and inversely proportional to the square of their separation distance. The force is always attractive and acts along the line joining them from their center.*

If the distribution of matter in each body is spherically symmetric, then the objects can be treated as point masses without approximation, as shown in the shell theorem. Otherwise, if we want to calculate the attraction between massive bodies, we need to add all the point-point attraction forces vectorially and the net attraction might not be exact inverse square. However, if the separation between the massive bodies is much larger compared to their sizes, then to a good approximation, it is reasonable to treat the masses as point mass while calculating the gravitational force.

As the law of gravitation, this law was suggested in 1645 by Ismael Bullialdus. But Bullialdus did not accept Kepler's second and third laws, nor did he appreciate Christiaan Huygens's solution for circular motion (motion in a straight line pulled aside by the central force). Indeed, Bullialdus maintained the sun's force was attractive at aphelion and repulsive at perihelion. Robert Hooke and Giovanni Alfonso Borelli both expounded gravitation in 1666 as an attractive force[1] (Hooke's lecture "On gravity" at the Royal Society, London, on 21 March; Borelli's "Theory of the Planets", published later in 1666). Hooke's 1670 Gresham lecture explained that gravitation applied to "all celestiall bodys" and added the principles that the gravitating power decreases with distance and that in the absence of any such power bodies move in straight lines. By 1679, Hooke thought gravitation had inverse square dependence and communicated this in a letter to Isaac Newton. Hooke remained bitter about Newton claiming the invention of this principle, even though Newton's "Principia" acknowledged that Hooke, along with Wren and Halley, had separately appreciated the inverse square law in the solar system,[2] as well as giving some credit to Bullialdus.

Electrostatics

The force of attraction or repulsion between two electrically charged particles, in addition to being directly proportional to the product of the electric charges, is inversely proportional to the square of the distance between them; this is known as Coulomb's law. The deviation of the exponent from 2 is less than one part in 10^{15}.[3]

Light and other electromagnetic radiation

The intensity (or illuminance or irradiance) of light or other linear waves radiating from a point source (energy per unit of area perpendicular to the source) is inversely proportional to the square of the distance from the source; so an object (of the same size) twice as far away, receives only one-quarter the energy (in the same time period).

More generally, the irradiance, *i.e.,* the intensity (or power per unit area in the direction of propagation), of a spherical wavefront varies inversely with the square of the distance from the source (assuming there are no losses caused by absorption or scattering).

For example, the intensity of radiation from the Sun is 9126 watts per square meter at the distance of Mercury (0.387 AU); but only 1367 watts per square meter at the distance of Earth (1 AU)—an approximate threefold increase in distance results in an approximate ninefold decrease in intensity of radiation.

In photography and theatrical lighting, the inverse-square law is used to determine the "fall off" or the difference in illumination on a subject as it moves closer to or further from the light source. For quick approximations, it is enough to remember that doubling the distance reduces illumination to one quarter;[4] or similarly, to halve the illumination increase the distance by a factor of 1.4 (the square root of 2), and to double illumination, reduce the distance to 0.7 (square root of 1/2). When the illuminant is not a point source, the inverse square rule is often still a useful approximation; when the size of the light source is less than one-fifth of the distance to the subject, the calculation error is less than 1%.[5]

The fractional reduction in electromagnetic fluence (Φ) for indirectly ionizing radiation with increasing distance from a point source can be calculated using the inverse-square law. Since emissions from a point source have radial directions, they intercept at a perpendicular incidence. The area of such a shell is $4\pi r^2$ where *r* is the radial distance from the center. The law is particularly important in diagnostic radiography and radiotherapy treatment planning, though this proportionality does not hold in practical situations unless source dimensions are much smaller than the distance.

Example

Let the total power radiated from a point source, for example, an omnidirectional isotropic antenna, be *P*. At large distances from the source (compared to the size of the source), this power is distributed over larger and larger spherical surfaces as the distance from the source increases. Since the surface area of a sphere of radius *r* is $A = 4\pi r^2$, then intensity *I* (power per unit area) of radiation at distance *r* is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}.$$

The energy or intensity decreases (divided by 4) as the distance *r* is doubled; measured in dB it would decrease by 6.02 dB per doubling of distance.

Acoustics

In acoustics one usually measures the sound pressure at a given distance *r* from the source using the 1/r law.[6] Since intensity is proportional to the square of pressure amplitude, this is just a variation on the inverse-square law.

Example

In acoustics, the sound pressure of a spherical wavefront radiating from a point source decreases by 50% as the distance *r* is doubled; measured in dB, the decreases is still 6.02 dB, since dB represents an intensity ratio. The behaviour is not inverse-square, but is inverse-proportional (inverse distance law):

$$p \ \propto \ \frac{1}{r}$$

The same is true for the component of particle velocity v that is in-phase with the instantaneous sound pressure p:

$$v \ \propto \frac{1}{r}$$

In the near field is a quadrature component of the particle velocity that is 90° out of phase with the sound pressure and does not contribute to the time-averaged energy or the intensity of the sound. The sound intensity is the product of the RMS sound pressure and the *in-phase* component of the RMS particle velocity, both of which are inverse-proportional. Accordingly, the intensity follows an inverse-square behaviour:

$$I \ = \ pv \ \propto \ \frac{1}{r^2}.$$

Field theory interpretation

For an irrotational vector field in three-dimensional space the inverse-square law corresponds to the property that the divergence is zero outside the source. This can be generalized to higher dimensions. Generally, for an irrotational vector field in *n*-dimensional Euclidean space, the intensity "I" of the vector field falls off with the distance "r" following the inverse $(n-1)^{\text{th}}$ power law

$$I \propto \frac{1}{r^{n-1}},$$

given that the space outside the source is divergence free.

Notes

[1] Hooke's gravitation was also not yet universal, though it approached universality more closely than previous hypotheses: See page 239 in Curtis Wilson (1989), "The Newtonian achievement in astronomy", ch.13 (pages 233–274) in "Planetary astronomy from the Renaissance to the rise of astrophysics: 2A: Tycho Brahe to Newton", CUP 1989.

[2] Newton acknowledged Wren, Hooke and Halley in this connection in the Scholium to Proposition 4 in Book 1 (in all editions): See for example the 1729 English translation of the 'Principia', at page 66 (http://books.google.com/books?id=Tm0FAAAAQAAJ&pg=PA66#v=onepage&q=&f=false).

[3] Williams, Faller, Hill, E.; Faller, J.; Hill, H. (1971), "New Experimental Test of Coulomb's Law: A Laboratory Upper Limit on the Photon Rest Mass", *Physical Review Letters* **26** (12): 721–724, Bibcode 1971PhRvL..26..721W, doi:10.1103/PhysRevLett.26.721

[4] Millerson,G. (1991) *Lighting for Film and Television - 3rd Edition* p.27

[5] Ryer,A. (1997) "The Light Measurement Handbook", ISBN 0-9658356-9-3 p.26

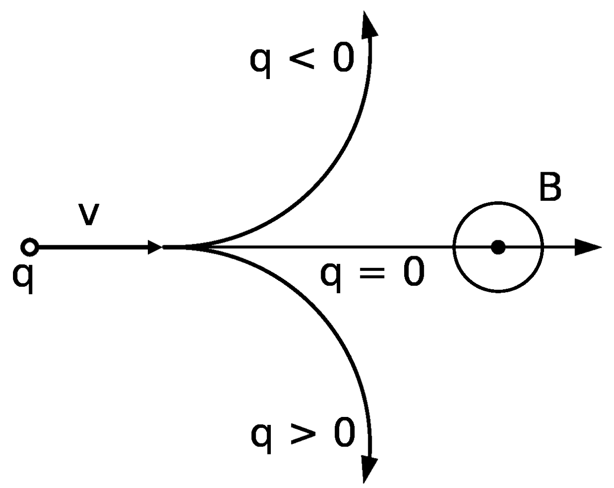
[6] Inverse-Square law for sound (http://hyperphysics.phy-astr.gsu.edu/hbase/acoustic/invsqs.html)

External links

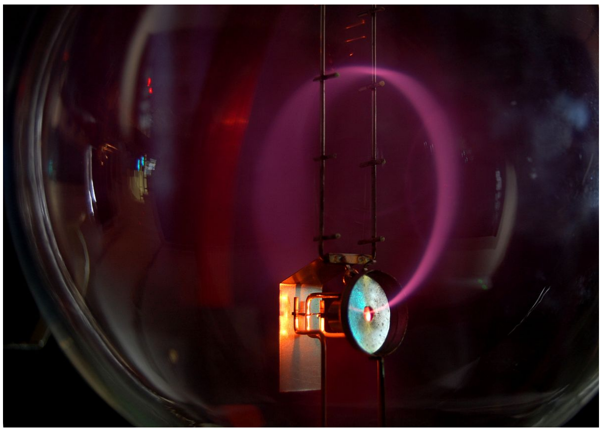
- Damping of sound level with distance (http://www.sengpielaudio.com/calculator-distance.htm)
- Sound pressure p and the inverse distance law 1/r (http://www.sengpielaudio.com/calculator-distancelaw.htm)
- Inverse Square Law & Radiation Protection by Ionactive (Animation) (http://www.ionactive.co.uk/multi-media_video.html?m=6)

Lorentz force

In physics, the **Lorentz force** is the force on a point charge due to electromagnetic fields. It is given by the following equation in terms of the electric and magnetic fields:[1]



Trajectory of a particle with a positive or negative charge *q* under the influence of a magnetic field *B*, which is directed perpendicularly out of the screen.



Beam of electrons moving in a circle, due to the presence of a magnetic field. Purple light is emitted along the electron path, due to the electrons colliding with gas molecules in the bulb.

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})],$$

where

- **F** is the force (in newtons)
- **E** is the electric field (in volts per metre)
- **B** is the magnetic field (in teslas)
- *q* is the electric charge of the particle (in coulombs)
- **v** is the instantaneous velocity of the particle (in metres per second)
- × is the vector cross product operator

All the quantities written in boldface are vectors.

The Lorentz force law has a close relationship with Faraday's law of induction.

A positively charged particle will be accelerated in the *same* linear orientation as the **E** field, but will curve perpendicularly to both the instantaneous velocity vector **v** and the **B** field according to the right-hand rule (in detail, if the thumb of the right hand points along **v** and the index finger along **B**, then the middle finger points along **F**).

The term $q\mathbf{E}$ is called the **electric force**, while the term $q\mathbf{v} \times \mathbf{B}$ is called the **magnetic force**.[2] According to some definitions, the term "Lorentz force" refers specifically to the formula for the magnetic force,[3] with the *total* electromagnetic force (including the electric force) given some other (nonstandard) name. This article will *not* follow this nomenclature: In what follows, the term "Lorentz force" will refer only to the expression for the total force.

The magnetic force component of the Lorentz force manifests itself as the force that acts on a current-carrying wire in a magnetic field. In that context, it is also called the **Laplace force**.

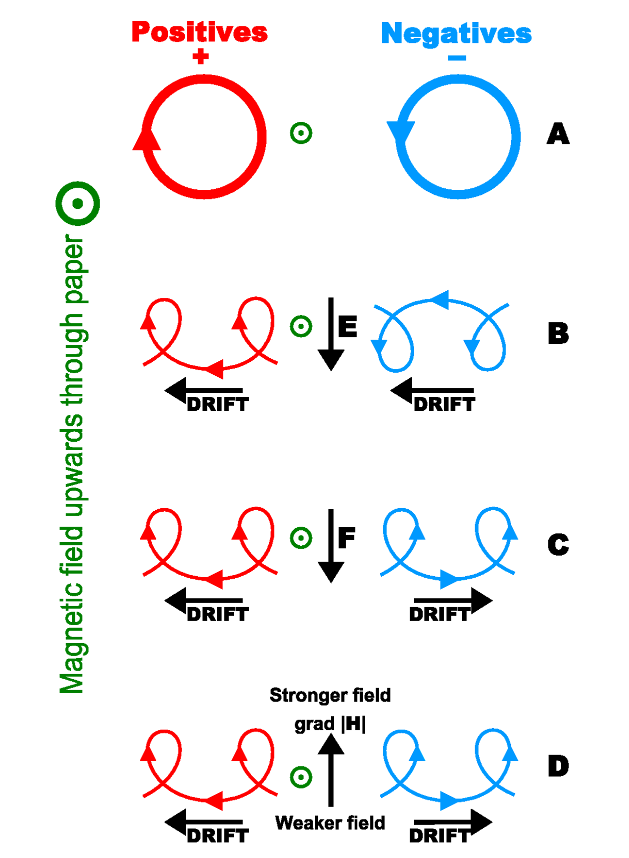
History

Early attempts to quantitatively describe the electromagnetic force were made in the mid-18th century. It was proposed that the force on magnetic poles, by Johann Tobias Mayer and others in 1760, and electrically charged objects, by Henry Cavendish in 1762, obeyed an inverse-square law. However, in both cases the experimental proof was neither complete nor conclusive. It was not until 1784 when Charles-Augustin de Coulomb, using a torsion balance, was able to definitively show through experiment that this was true.[4] Soon after the discovery in 1820 by H. C. Ørsted that a magnetic needle is acted on by a voltaic current, André-Marie Ampère that same year was able to devise through experimentation the formula for the angular dependence of the force between two current elements.[5][6] In all these descriptions, the force was always given in terms of the properties of the objects involved and the distances between them rather than in terms of electric and magnetic fields.[7]

The modern concept of electric and magnetic fields first arose in the theories of Michael Faraday, particularly his idea of lines of force, later to be given full mathematical description by Lord Kelvin and James Clerk Maxwell.[8] From a modern perspective it is possible to identify in Maxwell's 1865 formulation of his field equations a form of the Lorentz force equation in relation to electric currents,[9] however, in the time of Maxwell it was not evident how his equations related to the forces on moving charged objects. J. J. Thomson was the first to attempt to derive from Maxwell's field equations the electromagnetic forces on a moving charged object in terms of the object's properties and external fields. Interested in determining the electromagnetic behavior of the charged particles in cathode rays, Thomson published a paper in 1881 wherein he gave the force on the particles due to an external magnetic field as $\frac{1}{2}q\mathbf{v} \times \mathbf{B}$. Thomson was able to arrive at the correct basic form of the formula, but, because of some miscalculations and an incomplete description of the displacement current, included an incorrect scale-factor of a half in front of the formula. It was Oliver Heaviside, who had invented the modern vector notation and applied them to Maxwell's field equations, that in 1885 and 1889 fixed the mistakes of Thomson's derivation and arrived at the correct form of the magnetic force on a moving charged object.[10] Finally, in 1892, Hendrik Lorentz derived the modern day form of the formula for the electromagnetic force which includes the contributions to the total force from both the electric and the magnetic fields. Lorentz began by abandoning the Maxwellian descriptions of the ether and conduction. Instead, Lorentz made a distinction between matter and the luminiferous aether and sought to apply the Maxwell equations at a microscopic scale. Using the Heaviside's version of the Maxwell equations for a stationary ether and applying Lagrangian mechanics, Lorentz arrived at the correct and complete form of the force law that now bears his name.[11] [12]

Trajectories of particles in a Lorentz force

In many cases of practical interest, the motion in a magnetic field of an electrically charged particle (such as an electron or ion in a plasma) can be treated as the superposition of a relatively fast circular motion around a point called the **guiding center** and a relatively slow **drift** of this point. The drift speeds may differ for various species depending on their charge states, masses, or temperatures, possibly resulting in electric currents or chemical separation.



Charged particle drifts in a homogeneous magnetic field. (A) No disturbing force (B) With an electric field, E (C) With an independent force, F (e.g. gravity) (D) In an inhomogeneous magnetic field, grad H

Significance of the Lorentz force

While the modern Maxwell's equations describe how electrically charged particles and currents or moving charged particles give rise to electric and magnetic fields, the Lorentz force law completes that picture by describing the force acting on a moving point charge q in the presence of electromagnetic fields.[1][13] The Lorentz force law describes the effect of **E** and **B** upon a point charge, but such electromagnetic forces are not the entire picture. Charged particles are possibly coupled to other forces, notably gravity and nuclear forces. Thus, Maxwell's equations do not stand separate from other physical laws, but are coupled to them via the charge and current densities. The response of a point charge to the Lorentz law is one aspect; the generation of **E** and **B** by currents and charges is another.

In real materials the Lorentz force is inadequate to describe the behavior of charged particles, both in principle and as a matter of computation. The charged particles in a material medium both respond to the **E** and **B** fields and generate these fields. Complex transport equations must be solved to determine the time and spatial response of charges, for example, the Boltzmann equation or the Fokker–Planck equation or the Navier–Stokes equations. For example, see magnetohydrodynamics, fluid dynamics, electrohydrodynamics, superconductivity, stellar evolution. An entire physical apparatus for dealing with these matters has developed. See for example, Green–Kubo relations and Green's function (many-body theory).

Lorentz force law as the definition of *E* and *B*

In many textbook treatments of classical electromagnetism, the Lorentz Force Law is used as the *definition* of the electric and magnetic fields **E** and **B**.[14] To be specific, the Lorentz Force is understood to be the following empirical statement:

> The electromagnetic force on a test charge at a given point and time is a certain function of its charge and velocity, which can be parameterized by exactly two vectors **E** and **B**, in the functional form:
>
> $$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})].$$

If this empirical statement is valid (and, of course, countless experiments have shown that it is), then two vector fields **E** and **B** are thereby defined throughout space and time, and these are called the "electric field" and "magnetic field".

Note that the fields are defined everywhere in space and time, regardless of whether or not a charge is present to experience the force. In particular, the fields are defined with respect to what force a test charge *would* feel, if it *were* hypothetically placed there.

Note also that as a definition of **E** and **B**, the Lorentz force is only a definition *in principle* because a *real* particle (as opposed to the hypothetical "test charge" of infinitesimally-small mass and charge) would generate its own finite **E** and **B** fields, which would alter the electromagnetic force that it experiences. In addition, if the charge experiences acceleration, for example, if forced into a curved trajectory by some external agency, it emits radiation that causes braking of its motion. See, for example, Bremsstrahlung and synchrotron light. These effects occur through both a direct effect (called the radiation reaction force) and indirectly (by affecting the motion of nearby charges and currents).

Moreover, the electromagnetic force is not in general the same as the *net* force, due to gravity, electroweak and other forces, and any extra forces would have to be taken into account in a real measurement.

Lorentz force and Faraday's law of induction

Given a loop of wire in a magnetic field, Faraday's law of induction states:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where:

Φ_B is the magnetic flux through the loop,

$\mathcal{E}$ is the electromotive force (EMF) experienced,

t is time

The sign of the EMF is determined by Lenz's law.

Note that this is valid for not only a stationary wire but also for a moving wire. From the Faraday Law (that is valid for a moving wire, for instance in a motor) and the Maxwell Equations, the Lorentz Force can be deduced. The reverse is also true, the Lorentz force and the Maxwell Equations can be used to derive the Faraday Law.

Let $\partial\boldsymbol{\Sigma}(t)$ be the moving wire, moving together without rotation and with constant velocity $\mathbf{v}$ and $\boldsymbol{\Sigma}(t)$ be the internal surface of the wire. The EMF around the closed path $\partial\boldsymbol{\Sigma}(t)$ is given by:[15]

$$\mathcal{E} = \oint_{\partial\Sigma(t)} d\boldsymbol{\ell} \cdot \mathbf{F}/q$$

where $d\boldsymbol{\ell}$ is an element of the curve $\partial\boldsymbol{\Sigma}(t)$. The flux Φ_B in Faraday's law of induction can be expressed explicitly as:

$$\Phi_B = \iint_{\Sigma(t)} d\boldsymbol{A} \cdot \mathbf{B}(\mathbf{r}, t)$$

where

$\boldsymbol{\Sigma}(t)$ is a surface bounded by the closed contour $\partial\boldsymbol{\Sigma}(t)$

E is the electric field,

$d\boldsymbol{\ell}$ is an infinitesimal vector element of the contour $\partial\boldsymbol{\Sigma}(t)$,

v is the velocity of the infinitesimal contour element $d\boldsymbol{\ell}$,

B is the magnetic field.

d**A** is an infinitesimal vector element of surface $\partial\boldsymbol{\Sigma}(t)$, whose magnitude is the area of an infinitesimal patch of surface, and whose direction is orthogonal to that surface patch.

Both d and d**A** have a sign ambiguity; to get the correct sign, the right-hand rule is used, as explained in the article Kelvin-Stokes theorem.

The above result can be compared with the version of Faraday's law of induction that appears in the modern Maxwell's equations, called here the *Maxwell-Faraday equation*:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \ .$$

The Maxwell-Faraday equation also can be written in an *integral form* using the Kelvin-Stokes theorem:.[16]

So we have, the Maxwell Faraday equation:

$$\oint_{\partial \Sigma(t)} d\boldsymbol{\ell} \cdot \mathbf{E}(\mathbf{r},\ t) = -\ \iint_{\Sigma(t)} d\boldsymbol{A} \cdot \frac{d\,\mathbf{B}(\mathbf{r},\ t)}{dt}$$

and the Faraday Law,

$$\oint_{\partial \Sigma(t)} d\boldsymbol{\ell} \cdot \mathbf{F}/q(\mathbf{r},\ t) = -\frac{d}{dt} \iint_{\Sigma(t)} d\boldsymbol{A} \cdot \mathbf{B}(\mathbf{r},\ t)$$

The two are equivalent if the wire is not moving. Using the Leibniz integral rule and that *div* **B** = 0, results in,

$$\oint_{\partial \Sigma(t)} d\boldsymbol{\ell} \cdot \mathbf{F}/q(\mathbf{r},t) = -\iint_{\Sigma(t)} d\boldsymbol{A} \cdot \frac{d}{dt}\mathbf{B}(\mathbf{r},t) + \oint_{\partial \Sigma(t)} \mathbf{v} \times \mathbf{B}\, d\boldsymbol{\ell}$$

and using the Maxwell Faraday equation,

$$\oint_{\partial \Sigma(t)} d\boldsymbol{\ell} \cdot \mathbf{F}/q(\mathbf{r},\ t) = \oint_{\partial \Sigma(t)} d\boldsymbol{\ell} \cdot \mathbf{E}(\mathbf{r},\ t) + \oint_{\partial \Sigma(t)} \mathbf{v} \times \mathbf{B}(\mathbf{r},\ t) d\boldsymbol{\ell}$$

since this is valid for any wire position it implies that,

$$\mathbf{F} = q\,\mathbf{E}(\mathbf{r},\ t) + q\,\mathbf{v} \times \mathbf{B}(\mathbf{r},\ t)$$

Faraday's law of induction holds whether the loop of wire is rigid and stationary, or in motion or in process of deformation, and it holds whether the magnetic field is constant in time or changing. However, there are cases where Faraday's law is either inadequate or difficult to use, and application of the underlying Lorentz force law is necessary. See inapplicability of Faraday's law.

If the magnetic field is fixed in time and the conducting loop moves through the field, the flux magnetic flux Φ_B linking the loop can change in several ways. For example, if the **B**-field varies with position, and the loop moves to a location with different **B**-field, Φ_B will change. Alternatively, if the loop changes orientation with respect to the **B**-field, the **B**•d**A** differential element will change because of the different angle between **B** and d*A*, also changing Φ_B. As a third example, if a portion of the circuit is swept through a uniform, time-independent **B**-field, and another portion of the circuit is held stationary, the flux linking the entire closed circuit can change due to the shift in relative position of the circuit's component parts with time (surface $\partial \boldsymbol{\Sigma}(t)$ time-dependent). In all three cases, Faraday's law of induction then predicts the EMF generated by the change in Φ_B.

Note that the Maxwell Faraday's equation implies that the Electric Field (E) is non conservative when the Magnetic Field (B) varies in time, and is not expressible as the gradient of a scalar field, and not subject to the gradient theorem since its rotational is not zero.

See Landau, L. D., Lifshits, E. M., & Pitaevskiĭ, L. P. (1984). *Electrodynamics of continuous media; Volume 8* [[Course of Theoretical Physics [17]]] (Second ed.). Oxford: Butterworth-Heinemann. p. §63 (§49 pp. 205–207 in 1960 edition). ISBN 0750626348. M N O Sadiku (2007). *Elements of elctromagnetics* [18] (Fourth ed.). NY/Oxford: Oxford University Press. p. 391. ISBN 0-19-530048-3.

Relativistic form of the Lorentz force

Because the electric and magnetic fields are dependent on the velocity of an observer, the relativistic form of the Lorentz force law can best be exhibited starting from a coordinate-independent expression for the electromagnetic and magnetic fields,[19] $\mathcal{F}$, and an arbitrary time-direction, γ_0, where

$$\mathbf{E} = (\mathcal{F} \cdot \gamma_0)$$

and

$$\mathbf{B} = (\mathcal{F} \wedge \gamma_0)^*.$$

$\mathcal{F}$ is a space-time plane (bivector), which has six degrees of freedom corresponding to translations (rotations in space-time planes) and rotations (rotations in space-space planes). The dot product with the vector γ_0 pulls a vector from the translational part, while the wedge-product creates a space-time trivector, whose dot product with the volume element (the dual above) creates the magnetic field vector from the spatial rotation part. Only the parts of the above two formulas perpendicular to γ_0 are relevant. The relativistic velocity is given by the (time-like) changes in a time-position vector $\dot{x}$, where

$$\dot{x}^2 = 1,$$

(which shows our choice for the metric) and the velocity is

$$\mathbf{v} = \dot{x} \wedge \gamma_0 / (\dot{x} \cdot \gamma_0).$$

Then the Lorentz force law is simply (note that the order is important)

$$m\ddot{x} = q\mathcal{F} \cdot \dot{x}.$$

Lorentz force in terms of potentials

If the scalar potential and vector potential replace **E** and **B** (see Helmholtz decomposition), the force becomes:

$$\mathbf{F} = q(-\nabla\phi - \frac{\partial \mathbf{A}}{\partial \mathbf{t}} + \mathbf{v} \times (\nabla \times \mathbf{A}))$$

or, equivalently (making use of the fact that **v** is a constant; see triple product),

$$\mathbf{F} = q(-\nabla\phi - \frac{\partial \mathbf{A}}{\partial \mathbf{t}} + \nabla(\mathbf{v} \cdot \mathbf{A}) - (\mathbf{v} \cdot \nabla)\mathbf{A})$$

where

A is the magnetic vector potential

ϕ is the electrostatic potential

The symbols ∇, $(\nabla\times)$, $(\nabla\cdot)$ denote gradient, curl, and divergence, respectively.

The potentials are related to **E** and **B** by

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force in cgs units

The above-mentioned formulae use SI units which are the most common among experimentalists, technicians, and engineers. In cgs-Gaussian units, which are somewhat more common among theoretical physicists, one has instead

$$\mathbf{F} = q_{cgs} \cdot (\mathbf{E}_{cgs} + \frac{\mathbf{v}}{c} \times \mathbf{B}_{cgs}).$$

where c is the speed of light. Although this equation looks slightly different, it is completely equivalent, since one has the following relations:

$q_{cgs} = \frac{q_{SI}}{\sqrt{4\pi\epsilon_0}}$, $\mathbf{E}_{cgs} = \sqrt{4\pi\epsilon_0}\,\mathbf{E}_{SI}$, and $\mathbf{B}_{cgs} = \sqrt{4\pi/\mu_0}\,\mathbf{B}_{SI}$

where ε_0 and μ_0 are the vacuum permittivity and vacuum permeability, respectively. In practice, unfortunately, the subscripts "cgs" and "SI" are always omitted, and the unit system has to be assessed from context.

Covariant form of the Lorentz force

Newton's law of motion can be written in covariant form in terms of the field strength tensor.

$$\frac{dp^\alpha}{d\tau} = qU_\beta F^{\alpha\beta}$$

where

τ is c times the proper time of the particle,

q is the charge,

U is the covariant 4-velocity of the particle, defined as:

$U_\beta = (U_0, U_1, U_2, U_3) = \gamma\left(-c, u_x, u_y, u_z\right)$ under the metric signature (-1,1,1,1)

with γ = Lorentz factor defined above, and F is the contravariant electromagnetic tensor written in terms of fields as:

$$F^{\alpha\beta} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{bmatrix}.$$

The fields are transformed to a frame moving with constant relative velocity by:

$$\acute{F}^{\mu\nu} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta F^{\alpha\beta},$$

where $\Lambda^\mu{}_\alpha$ is a Lorentz transformation. Alternatively, using the four vector:

$$A^\alpha = (\phi/c,\ A_x,\ A_y,\ A_z)\ ,$$

related to the electric and magnetic fields by:

$$\mathbf{E} = -\nabla\phi - \partial_t\mathbf{A}\ \mathbf{B} = \nabla\times\mathbf{A}\ ,$$

the field tensor becomes:[20]

$$F^{\alpha\beta} = \frac{\partial A^\beta}{\partial x_\alpha} - \frac{\partial A^\alpha}{\partial x_\beta}\ ,$$

where:

$$x_\alpha = (-ct,\ x,\ y,\ z)\ .$$

Translation to vector notation

The $\alpha = 1$ component (x-component) of the force is

$$\frac{dp^1}{d\tau} = qU_\beta F^{1\beta} = q\left(U_0F^{10} + U_1F^{11} + U_2F^{12} + U_3F^{13}\right).$$

Here, τ is the proper time of the particle. Substituting the components of the covariant electromagnetic tensor *F* yields

$$\frac{dp^1}{d\tau} = q\left(U_0\left(\frac{-E_x}{c}\right) + U_2(B_z) + U_3(-B_y)\right)$$

Using the components of covariant four-velocity yields

$$\frac{dp^1}{d\tau} = q\gamma\left(-c\left(\frac{-E_x}{c}\right) + u_yB_z + u_z(-B_y)\right)$$

$$\frac{dp^1}{d\tau} = q\gamma\left(E_x + u_yB_z - u_zB_y\right)$$

$$\frac{dp^1}{d\tau} = q\gamma\left(E_x + \left[\mathbf{u}\times\mathbf{B}\right]_x\right).$$

The calculation of the $\alpha = 2$ or $\alpha = 3$ is similar yielding

$$\frac{d\mathbf{p}}{d\tau} = q\gamma\left(\mathbf{E} + \mathbf{u}\times\mathbf{B}\right) \ ,$$

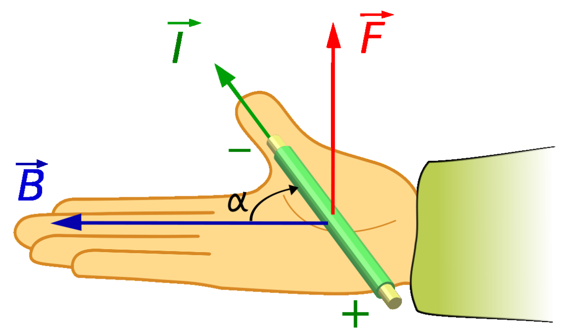
or, in terms of the vector and scalar potentials ***A*** and φ,

$$\frac{d\mathbf{p}}{d\tau} = q\gamma(-\nabla\phi - \frac{\partial\mathbf{A}}{\partial t} + \mathbf{v}\times(\nabla\times\mathbf{A})) \ ,$$

which are the relativistic forms of Newton's law of motion when the Lorentz force is the only force present.

Force on a current-carrying wire

When a wire carrying an electrical current is placed in a magnetic field, each of the moving charges, which comprise the current, experiences the Lorentz force, and together they can create a macroscopic force on the wire (sometimes called the **Laplace force**). By combining the Lorentz force law above with the definition of electrical current, the following equation results, in the case of a straight, stationary wire:



Right hand rule for a current-carrying wire in a magnetic field B

$$\mathbf{F} = I\mathbf{L}\times\mathbf{B}$$

where

F = Force, measured in newtons

I = current in wire, measured in amperes

B = magnetic field vector, measured in teslas

$\times$ = vector cross product

L = a vector, whose magnitude is the length of wire (measured in metres), and whose direction is along the wire, aligned with the direction of conventional current flow.

Alternatively, some authors write

$$\mathbf{F} = L\mathbf{I} \times \mathbf{B}$$

where the vector direction is now associated with the current variable, instead of the length variable. The two forms are equivalent.

If the wire is not straight but curved, the force on it can be computed by applying this formula to each infinitesimal segment of wire $d\boldsymbol{\ell}$, then adding up all these forces via integration. Formally, the net force on a stationary, rigid wire carrying a current *I* is

$$\mathbf{F} = I \int d\boldsymbol{\ell} \times \mathbf{B}$$

(This is the net force. In addition, there will usually be torque, plus other effects if the wire is not perfectly rigid.)

One application of this is Ampère's force law, which describes how two current-carrying wires can attract or repel each other, since each experiences a Lorentz force from the other's magnetic field. For more information, see the article: Ampère's force law.

EMF

The magnetic force ($q\,\mathbf{v} \times \mathbf{B}$) component of the Lorentz force is responsible for *motional* electromotive force (or *motional EMF*), the phenomenon underlying many electrical generators. When a conductor is moved through a magnetic field, the magnetic force tries to push electrons through the wire, and this creates the EMF. The term "motional EMF" is applied to this phenomenon, since the EMF is due to the *motion* of the wire.

In other electrical generators, the magnets move, while the conductors do not. In this case, the EMF is due to the electric force ($q\mathbf{E}$) term in the Lorentz Force equation. The electric field in question is created by the changing magnetic field, resulting in an *induced* EMF, as described by the Maxwell-Faraday equation (one of the four modern Maxwell's equations).[21]

Both of these EMF's, despite their different origins, can be described by the same equation, namely, the EMF is the rate of change of magnetic flux through the wire. (This is Faraday's law of induction, see above.) Einstein's theory of special relativity was partially motivated by the desire to better understand this link between the two effects.[21] In fact, the electric and magnetic fields are different faces of the same electromagnetic field, and in moving from one inertial frame to another, the solenoidal vector field portion of the *E*-field can change in whole or in part to a *B*-field or *vice versa*.[22]

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Numbered footnotes and references

[1] See Jackson page 2. The book lists the four modern Maxwell's equations, and then states, "Also essential for consideration of charged particle motion is the Lorentz force equation, $\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, which gives the force acting on a point charge q in the presence of electromagnetic fields."
[2] See Griffiths page 204.
[3] For example, see the website of the "Lorentz Institute": (http://ilorentz.org/history/lorentz/lorentz.html), or Griffiths.
[4] Meyer, Herbert W. (1972). *A History of Electricity and Magnetism*. Norwalk, CT: Burndy Library. pp. 30–31. ISBN 026213070X.
[5] Verschuur, Gerrit L. (1993). *Hidden Attraction : The History And Mystery Of Magnetism*. New York: Oxford University Press. pp. 78–79. ISBN 0195064887.
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[13] See Griffiths page 326, which states that Maxwell's equations, "together with the [Lorentz] force law...summarize the entire theoretical content of classical electrodynamics".
[14] See, for example, Jackson p777-8.
[15] Landau, L. D., Lifshit͡s͡, E. M., & Pitaevskiĭ, L. P. (1984). *Electrodynamics of continuous media; Volume 8* [[Course of Theoretical Physics (http://worldcat.org/search?q=0750626348&qt=owc_search)]] (Second ed.). Oxford: Butterworth-Heinemann. p. §63 (§49 pp. 205–207 in 1960 edition). ISBN 0750626348. .
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[17] http://worldcat.org/search?q=0750626348&qt=owc_search
[18] http://books.google.com/?id=w2ITHQAACAAJ&dq=isbn:0-19-530048-3
[19] Hestenes, David. "SpaceTime Calculus" (http://geocalc.clas.asu.edu/html/STC.html). .
[20] DJ Griffiths (1999). *Introduction to electrodynamics*. Saddle River NJ: Pearson/Addison-Wesley. p. 541. ISBN 0-13-805326-X.
[21] See Griffiths pages 301–3.
[22] Tai L. Chow (2006). *Electromagnetic theory* (http://books.google.com/?id=dpnpMhw1zo8C&pg=PA153&dq=isbn=0763738271). Sudbury MA: Jones and Bartlett. p. 395. ISBN 0-7637-3827-1. .
[23] http://books.google.com/?id=5OepxIG42B4C&pg=PA315&dq=isbn=9780521864497

Applications

The Lorentz force occurs in many devices, including:

- Cyclotrons and other circular path particle accelerators
- Mass spectrometers
- Velocity Filters
- Magnetrons

In its manifestation as the Laplace force on an electric current in a conductor, this force occurs in many devices including:

- Electric motors
- Railguns
- Linear motors
- Loudspeakers
- Magnetoplasmadynamic thrusters
- Electrical generators
- Homopolar generators
- Linear alternators

External links

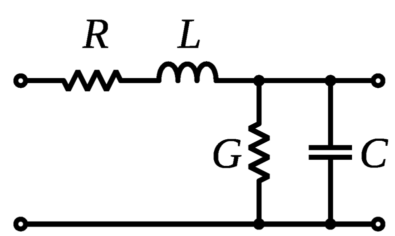
- Interactive Java tutorial on the Lorentz force (http://www.magnet.fsu.edu/education/tutorials/java/lorentzforce/index.html) National High Magnetic Field Laboratory
- Lorentz force (demonstration) (http://www.youtube.com/watch?v=mxMMqNrm598)
- Faraday's law: Tankersley and Mosca (http://www.nadn.navy.mil/Users/physics/tank/Public/FaradaysLaw.pdf)
- Notes from Physics and Astronomy HyperPhysics at Georgia State University (http://hyperphysics.phy-astr.gsu.edu/HBASE/hframe.html); see also home page (http://hyperphysics.phy-astr.gsu.edu/HBASE/hframe.html)
- Interactive Java applet on the magnetic deflection of a particle beam in a homogeneous magnetic field (http://chair.pa.msu.edu/applets/Lorentz/a.htm) by Wolfgang Bauer

Telegrapher's equations

The **telegrapher's equations** (or just **telegraph equations**) are a pair of linear differential equations which describe the voltage and current on an electrical transmission line with distance and time. The equations come from Oliver Heaviside who in the 1880s developed the *transmission line model* which is described in this article. The model demonstrates that the electromagnetic waves can be reflected on the wire, and that wave patterns can appear along the line. The theory applies to transmission lines of all frequencies including high-frequency transmission lines (such as telegraph wires and radio frequency conductors), audio frequency (such as telephone lines), low frequency (such as power lines) and direct current.

The equations

The telegrapher's equations, like all other equations describing electrical phenomena, can be held to result from Maxwell's equations. In a more practical approach, one assumes that the conductors are composed of an infinite series of two-port elementary components, each representing an infinitesimally short segment of the transmission line:



Schematic representation of the elementary components of a transmission line.

- The distributed resistance R of the conductors is represented by a series resistor (expressed in ohms per unit length).
- The distributed inductance L (due to the magnetic field around the wires, self-inductance, etc.) is represented by a series inductor (henries per unit length).
- The capacitance C between the two conductors is represented by a shunt capacitor C (farads per unit length).
- The conductance G of the dielectric material separating the two conductors is represented by a shunt resistor between the signal wire and the return wire (siemens per unit length). This resistor in the model has a resistance of $1/G$ ohms.

It should be repeated for clarity that the model consists of an *infinite series* of the infinitesimal elements shown in the figure, and that the values of the components are specified *per unit length* so the picture of the component can be misleading. An alternative notation is to use R', L', C' and G' to emphasize that the values are derivatives with respect to length. These quantities can also be known as the primary line constants to distinguish from the secondary line constants derived from them, these being the characteristic impedance, the propagation constant, attenuation constant and phase constant. All these constants are constant with respect to time, voltage and current. They may be non-constant functions of frequency.

The Telegrapher's Equations are developed in similar forms in the following references: Kraus,[1] Hayt,[2] Marshall,[3] Sadiku,[4] Harrington,[5] Karakash,[6] Metzger,[7]

Values of Primary Parameters for Telephone Cable

Representative parameter data for 24 gauge PIC telephone cable at 70°F

Frequency	R	L	G	C
Hz	Ω/kft	mH/kft	μS/kft	nF/kft
1	52.50	0.1868	0.000	15.72
1k	52.51	0.1867	0.022	15.72
10k	52.64	0.1859	0.162	15.72
100k	58.41	0.1770	1.197	15.72
1M	141.30	0.1543	8.873	15.72
2M	196.03	0.1482	16.217	15.72
5M	304.62	0.1425	35.989	15.72

More extensive tables and tables for other gauges, temperatures and types are available in Reeve.[8] Chen[9] gives the same data in a parameterized form that he states is usable up to 50 MHz.

The variation of R and L is mainly due to skin effect and proximity effect.

The constancy of the capacitance is a consequence of intentional design.

The variation of G can be inferred from Terman[10] "The power factor ... tends to be independent of frequency, since the fraction of energy lost during each cycle ... is substantially independent of the number of cycles per second, over wide frequency ranges." A function of the form $G(f) = G_1 \left(\frac{f}{f_1} \right)^{ge}$ with **ge** close to 1.0 would fit the statement from Terman. Chen [9] gives an equation of similar form.

G in this table can be modeled well with

f_1= 1MHz

$G_1 = 8.873\mu$ S/kft

ge = 0.87

Usually the resistive losses grow proportionately to $f^{0.5}$ and dielectric losses grow proportionately to f^{ge} with **ge** > 0.5 so at a high enough frequency, dielectric losses will exceed resistive losses. In practice, before that point is reached, a transmission line with a better dielectric is used. The dielectric can be reduced down to air with an occasional plastic spacer.

Lossless transmission

When the elements *R* and *G* are very small, their effects can be neglected, and the transmission line is considered as an idealized, lossless, structure. In this case, the model depends only on the *L* and *C* elements, and we obtain a pair of first-order partial differential equations, one function describing the voltage *V* along the line and the other the current *I*, both function of position *x* and time *t*:

$$\frac{\partial}{\partial x}V(x,t) = -L\frac{\partial}{\partial t}I(x,t)$$
$$\frac{\partial}{\partial x}I(x,t) = -C\frac{\partial}{\partial t}V(x,t)$$

These equations may be combined to form either of two exact wave equations:

$$\frac{\partial^2}{\partial t^2}V = \frac{1}{LC}\frac{\partial^2}{\partial x^2}V$$
$$\frac{\partial^2}{\partial t^2}I = \frac{1}{LC}\frac{\partial^2}{\partial x^2}I$$

In the steady-state case (assuming a sinusoidal wave $E = E_o \cdot e^{-j\omega(\frac{x}{c}-t)}$, these reduce to

$$\frac{\partial^2 V(x)}{\partial x^2} + \omega^2 LC \cdot V(x) = 0$$
$$\frac{\partial^2 I(x)}{\partial x^2} + \omega^2 LC \cdot I(x) = 0$$

where ω is the frequency of the steady-state wave

If the line has infinite length or when it is terminated with its characteristic impedance, these equations indicate the presence of a wave, travelling with a speed $v = \frac{1}{\sqrt{LC}}$.

(Note that this propagation speed applies to the wave phenomenon on the line and has nothing to do with the electron drift velocity. In other words, the electrical impulse travels very close to the speed of light, although the electrons themselves travel only a few centimeters per second.) For a coaxial transmission line, made of perfect conductors with vacuum between them, it can be shown that this speed is equal to the speed of light.

The Lossless line and Distortionless line are discussed in Sadiku,[11] and Marshall,[12]

Lossy transmission line

When the loss elements *R* and *G* are not negligible, the original differential equations describing the elementary segment of line become

$$\frac{\partial}{\partial x}V(x,t) = -L\frac{\partial}{\partial t}I(x,t) - RI(x,t)$$
$$\frac{\partial}{\partial x}I(x,t) = -C\frac{\partial}{\partial t}V(x,t) - GV(x,t)$$

By differentiating the first equation with respect to *x* and the second with respect to *t*, and some algebraic manipulation, we obtain a pair of hyperbolic partial differential equations each involving only one unknown:

$$\frac{\partial^2}{\partial x^2}V = LC\frac{\partial^2}{\partial t^2}V + (RC+GL)\frac{\partial}{\partial t}V + GRV$$
$$\frac{\partial^2}{\partial x^2}I = LC\frac{\partial^2}{\partial t^2}I + (RC+GL)\frac{\partial}{\partial t}I + GRI$$

Note that these equations resemble the homogeneous wave equation with extra terms in *V* and *I* and their first derivatives. These extra terms cause the signal to decay and spread out with time and distance. If the transmission line is only slightly lossy (small *R* and *G* = 0), signal strength will decay over distance as $e^{-\alpha x}$, where $\alpha = R/2Z_0$

Direction of signal propagations

The wave equations above indicate that there are two solutions for the travelling wave: one forward and one reverse. Assuming a simplification of being lossless (requiring both *R*=0 and *G*=0) the solution can be represented as:

$$V(x,t) \;=\; f_1(\omega t - kx) + f_2(\omega t + kx)$$

where:

$$k = \omega\sqrt{LC} = \frac{\omega}{v}$$

k is called the **wavenumber** and has units of radians per meter,

ω is the **angular frequency** (in radians per second),

f_1 and f_2 can be *any* functions whatsoever, and

$$v = \frac{1}{\sqrt{LC}}$$

is the waveform's propagation speed (also known as phase velocity).

f_1 represents a wave traveling from left to right in a positive x direction whilst f_2 represents a wave traveling from right to left. It can be seen that the instantaneous voltage at any point x on the line is the sum of the voltages due to **both** waves.

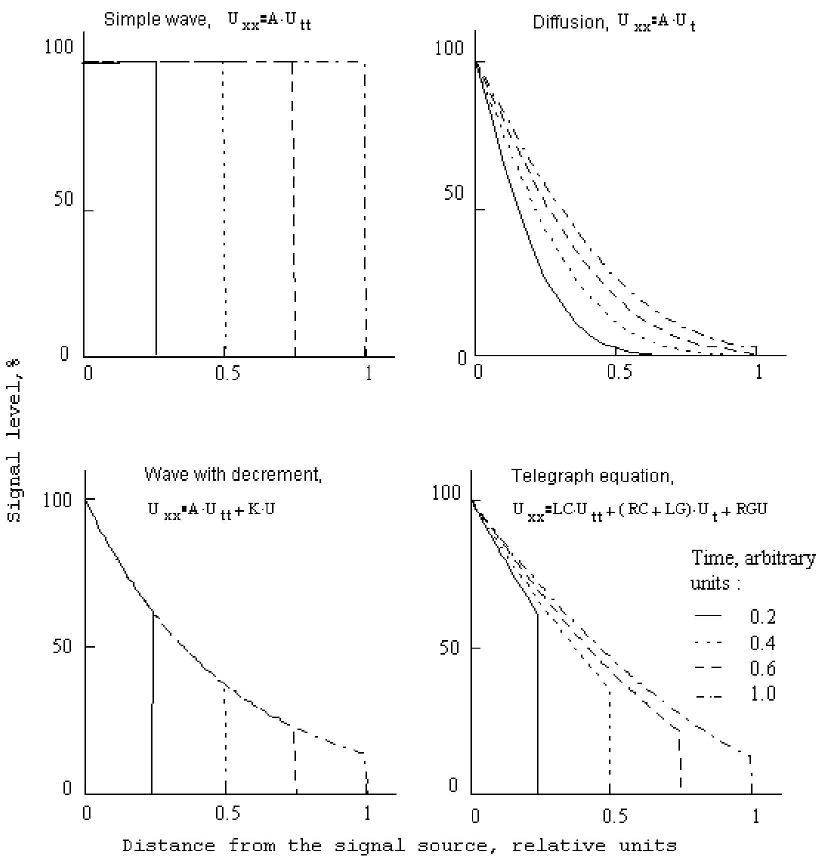
Since the current *I* is related to the voltage *V* by the telegrapher's equations, we can write

$$I(x,t) \;=\; \frac{f_1(\omega t - kx)}{Z_0} - \frac{f_2(\omega t + kx)}{Z_0}$$

where Z_0 is the **characteristic impedance** of the transmission line, which, for a lossless line is given by

$$Z_0 = \sqrt{\frac{L}{C}}$$

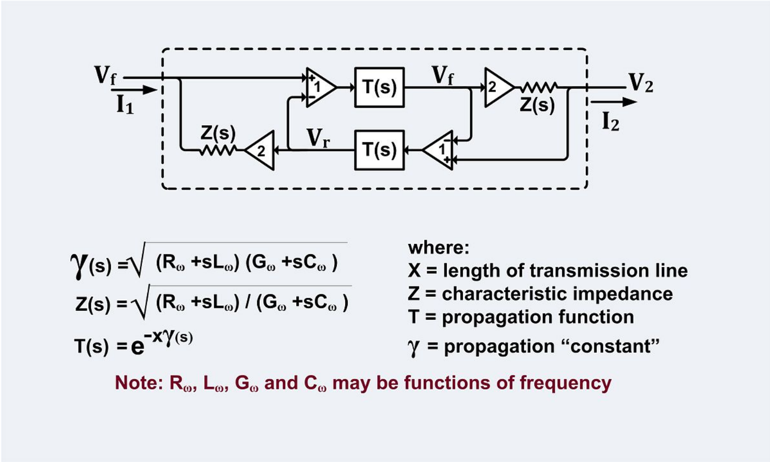
Signal pattern examples



Changes of the signal level distribution along the single dimensional transmission media. Depending on the parameters of the telegraph equation, this equation can reproduce all four patterns.

Depending on the parameters of the telegraph equation, the changes of the signal level distribution along the length of the single-dimensional transmission media may take the shape of the simple wave, wave with decrement, or the diffusion-like pattern of the telegraph equation. The shape of the diffusion-like pattern is caused by the effect of the shunt capacity.

Solutions of the Telegrapher's Equations as Circuit Components



Equivalent circuit of a transmission line described by the Telegrapher's equations.

The solutions of the telegrapher's equations can be inserted into a circuit as components of an equivalent sub-circuit as shown the figure. As drawn, all voltages are with respect to ground and all amplifiers have unshown connections to ground. An example of a transmission line modeled by this circuit would be an unbalanced transmission line such as a strip line on a circuit board. The impedance Z(s), the voltage doubler (the triangle with the number "2") and the difference amplifier (the triangle with the number "1") account for the interaction of the transmission line with the rest of the circuit. The T(s) blocks account for delay, attenuation, dispersion and whatever happens to the signal in transit. One of the T(s) blocks carries the "forward wave" and the other carries the "backward wave". The circuit, as depicted, is fully symmetric, although it is not drawn that way. The circuit depicted is equivalent to a transmission line connected from V1 to V2 in the sense that V1, V2, I1 and I2 would be same whether this circuit or an actual transmission line was connected between V1 and V2. There is no implication that there are actually amplifiers inside the transmission line.

This is not the only possible equivalent circuit. Voltage amplifiers and sensors can be replaced with current, transimpedance or transconductance amplifiers. Series impedances can be replaced with shunt admittances. The circuit can be augmented to account for different "grounds" at each end. The circuit can be made fully differential.

External links

- SPICE Simulation of Transmission Lines [13]

Notes

[1] Kraus (1989, pp. 380–419)
[2] Hayt (1989, pp. 382–392)
[3] Marshall (1987, pp. 359–378)
[4] Sadiku (1989, pp. 497–505)
[5] Harrington (1961, pp. 61–65)
[6] Karakash (1950, pp. 5–14)
[7] Metzger (1969, pp. 1–10)
[8] Reeve (1995, p. 558)
[9] Chen (2004, p. 26)
[10] Terman (1943, p. 112)
[11] Sadiku (1989, pp. 501–503)
[12] Marshall (198y, pp. 369–372)
[13] http://www.eetimes.com/design/microwave-rf-design/4200760/SPICE-Simulation-of-Transmission-Lines-by-the-Telegrapher-s-Method-Part-1-of-3-?Ecosystem=microwave-rf-design

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Physical quantities

E field

In physics, an **electric field** surrounds electrically charged particles and time-varying magnetic fields. The electric field depicts the force exerted on other electrically charged objects by the electrically charged particle the field is surrounding. The concept of an electric field was introduced by Michael Faraday.

The electric field is a vector field with SI units of newtons per coulomb ($N\ C^{-1}$) or, equivalently, volts per metre ($V\ m^{-1}$). The SI base units of the electric field are $kg \cdot m \cdot s^{-3} \cdot A^{-1}$. The strength or magnitude of the field at a given point is defined as the force that would be exerted on a positive test charge of 1 coulomb placed at that point; the direction of the field is given by the direction of that force. Electric fields contain electrical energy with energy density proportional to the square of the field amplitude. The electric field is to charge as gravitational acceleration is to mass and force density is to volume.

An electric field that changes with time, such as due to the motion of charged particles in the field, influences the local magnetic field. That is, the electric and magnetic fields are not completely separate phenomena; what one observer perceives as an electric field, another observer in a different frame of reference perceives as a mixture of electric and magnetic fields. For this reason, one speaks of "electromagnetism" or "electromagnetic fields". In quantum electrodynamics, disturbances in the electromagnetic fields are called photons, and the energy of photons is quantized.

Definition

The electric field intensity is defined as the force per unit positive charge that would be experienced by a stationary point charge, or "test charge", at a given location in the field:[1]

$$\mathbf{E} = \frac{\mathbf{F}}{q_t}$$

where

F is the electric force experienced by the test particle

q_t is the charge of the test particle in the electric field

E is the electric field wherein the particle is located.

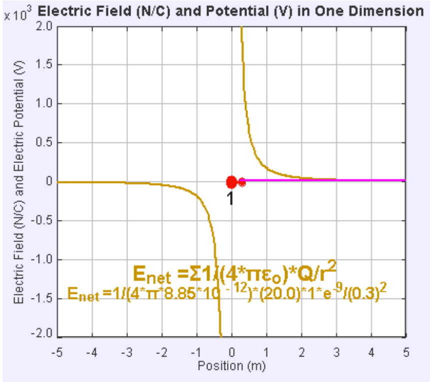
Taken literally, this equation only defines the electric field at a specific location as the force experienced by a stationary test charge at that point(with the sign of q_t, positive or negative, determining the direction of the force). Given that electric fields are generated by electrically charged particles, adding and/or moving a source charge, q_s, will alter the electric field distribution. Therefore, it is important to remember that an electric field is defined with respect to a particular configuration of source charges. In practice, this is achieved by placing test particles with successively smaller electric charge in the vicinity of the source distribution and measuring the force exerted on the test charges as their charge approaches zero.

$$\mathbf{E} = \lim_{q \to 0} \frac{\mathbf{F}}{q}$$

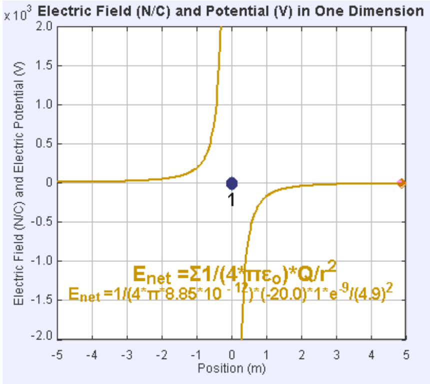
This allows the electric field to be determined from the distribution of its source charges alone.

As is clear from the definition, the direction of the electric field is the same as the direction of the force it would exert on a positively-charged particle, and opposite the direction of the force on a negatively-charged particle. Since like charges repel and opposites attract (as quantified below), the electric field tends to point away from positive charges and towards negative charges.

Based on Coulomb's law for interacting point charges, the contribution to the E-field at a point in space due to a single, discrete charge located at another point in space is given by the following[1] :



Electric field from a positive Q



Electric field from a negative Q

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

where

Q is the charge of the particle creating the electric force,

r is the distance from the particle with charge Q to the E-field evaluation point,

$\hat{\mathbf{r}}$ is the unit vector pointing from the particle with charge Q to the E-field evaluation point,

ε_0 is the electric constant.

The total E-field due to a quantity of point charges, n_q, is simply the superposition of the contribution of each individual point charge[2] :

$$\mathbf{E} = \sum_{i=1}^{n_q} \mathbf{E}_i = \sum_{i=1}^{n_Q} \frac{1}{4\pi\varepsilon_0}\frac{Q_i}{r_i^2}\hat{\mathbf{r}}_i.$$

Alternatively, Gauss's law allows the E-field to be calculated in terms of a continuous distribution of charge density in space, ρ:[3]

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}.$$

Coulomb's law is actually a special case of Gauss's Law, a more fundamental description of the relationship between the distribution of electric charge in space and the resulting electric field. Gauss's law is one of Maxwell's equations, a set of four laws governing electromagnetics.

Uniform fields

A uniform field is one in which the electric field is constant at every point. It can be approximated by placing two conducting plates parallel to each other and maintaining a voltage between them; it is only an approximation because of edge effects. Ignoring such effects, the equation for the magnitude of the electric field is:

$$E = -\frac{V}{d}$$

where

V is the voltage difference between the plates

d is the distance separating the plates

The negative sign arises as positive charges repel, so a positive charge will experience a force away from the positively charged plate, in the opposite direction to that in which the voltage increases.

Time-varying fields

An electric field can be produced, not only by a static charge, but also by a changing magnetic field. The combined electric field is expressed as,

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

where,

$$\mathbf{B} = \nabla \times \mathbf{A}$$

The vector **B** is the magnetic flux density and the vector **A** is the magnetic vector potential. Taking the curl of the electric field equation we obtain,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

which is one of Maxwell's equations, referred to as Faraday's law of induction.[4]

Where electrostatics is the study of the fields surrounding static charges, the study of the electric fields induced by changing magnetic field comes under the domain of electrodynamics or electromagnetics.

Properties (in electrostatics)

According to Coulomb's law the electric field is dependent on position. The electric field due to any single charge falls off as the square of the distance from that charge.

Electric fields follow the superposition principle. If more than one charge is present, the total electric field at any point is equal to the vector sum of the respective electric fields that each object would create in the absence of the others.

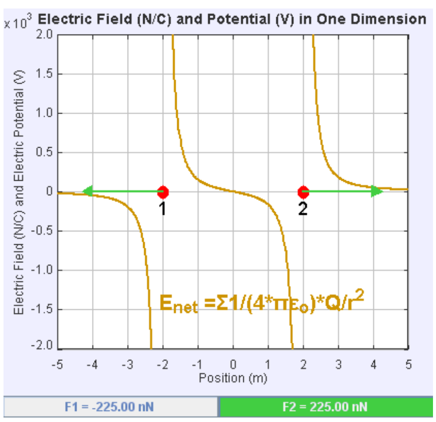


Illustration of the electric field surrounding a positive (red) and a negative (blue) charge in one dimension if the right charge is changing from positive to negative

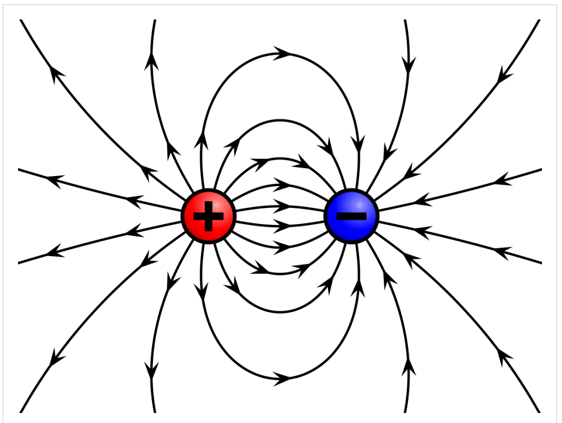


Illustration of the electric field surrounding a positive (red) and a negative (blue) charge.

$$\mathbf{E}_{\rm total} = \sum_i \mathbf{E}_i = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 \ldots$$

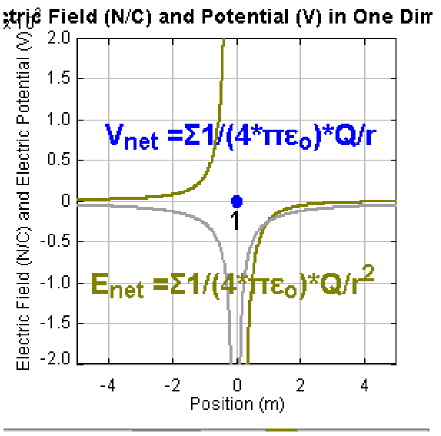
If this principle is extended to an infinite number of infinitesimally small elements of charge, the following formula results:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r^2}\hat{\mathbf{r}}\,\mathrm{d}V$$

where

ρ is the charge density, or the amount of charge per unit volume.

The electric field at a point is equal to the negative gradient of the electric potential there. In symbols,



The electric field at a point is equal to the negative gradient of the electric potential there, :

$$\mathbf{E} = -\nabla\Phi$$

$$\mathbf{E} = -\nabla\Phi$$

where

$\Phi(x, y, z)$ is the scalar field representing the electric potential at a given point.

If several spatially distributed charges generate such an electric potential, e.g. in a solid, an electric field gradient may also be defined.

Considering the permittivity ε of a linear material, which may differ from the permittivity of free space ε_0, the electric displacement field is:

$$\mathbf{D} = \varepsilon\mathbf{E}.$$

Energy in the electric field

The electric field stores energy. The energy density of the electric field is given by

$$u = \frac{1}{2}\varepsilon|\mathbf{E}|^2\,,$$

where ε is the permittivity of the medium in which the field exists, and $\mathbf{E}$ is the electric field vector.

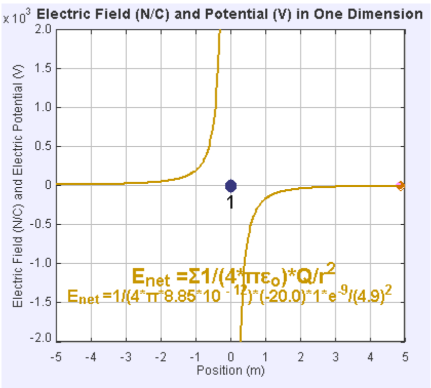
The total energy stored in the electric field in a given volume V is therefore

$$\frac{1}{2}\varepsilon\int_V |\mathbf{E}|^2\,\mathrm{d}V\,,$$

where $\mathrm{d}V$ is the differential volume element.

Parallels between electrostatics and gravity

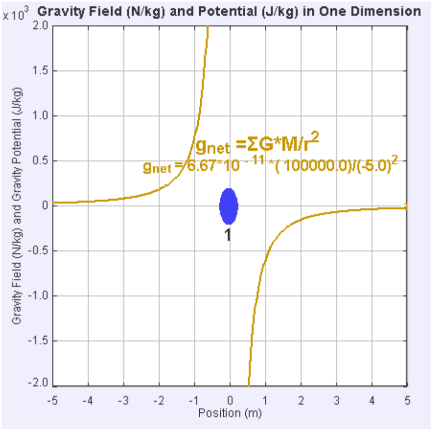
Coulomb's law, which describes the interaction of electric charges:



Electric field from a negative Q where $\mathbf{F} = q(\frac{-1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}) = q\mathbf{E}$

$$\mathbf{F} = q(\frac{-1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}) = q\mathbf{E}$$

is similar to Newton's law of universal gravitation:



Gravitational field determined using Newton's law of universal gravitation. : $F = m(-G\frac{M}{|r|^2})\hat{r} = mg$

$$\mathbf{F} = m(-G\frac{M}{r^2}\hat{\mathbf{r}}) = m\mathbf{g}.$$

This suggests similarities between the electric field **E** and the gravitational field **g**, so sometimes mass is called "gravitational charge".

Similarities between electrostatic and gravitational forces:

1. Both act in a vacuum.
2. Both are central and conservative.
3. Both obey an inverse-square law (both are inversely proportional to square of r).
4. Both propagate with finite speed c, the speed of light.
5. Electric charge and relativistic mass are conserved; note, though, that rest mass is not conserved.

Differences between electrostatic and gravitational forces:

1. Electrostatic forces are much greater than gravitational forces (by about 10^{36} times).
2. Gravitational forces are attractive for like charges, whereas electrostatic forces are repulsive for like charges.
3. There are no negative gravitational charges (no negative mass) while there are both positive and negative electric charges. This difference combined with previous implies that gravitational forces are always attractive, while electrostatic forces may be either attractive or repulsive.

References

[1] Electric field in "Electricity and Magnetism", R Nave (http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefie.html)

[2] 'The Electric Field' - Chapter 23 of Frank Wolfs's lectures (http://teacher.pas.rochester.edu/phy122/Lecture_Notes/Chapter23/Chapter23.html#Heading3) at University of Rochester

[3] 'Gauss's Law' - Chapter 24 of Frank Wolfs's lectures (http://teacher.pas.rochester.edu/phy122/Lecture_Notes/Chapter24/Chapter24.html) at University of Rochester

[4] Huray, Paul G. (2009), *Maxwell's Equations* (http://books.google.com/books?id=0QsDgdd0MhMCp), Wiley-IEEE, p. 205, ISBN 0-470-54276-4, , Chapter 7, p 205 (http://books.google.com/books?id=0QsDgdd0MhMC&pg=PA205)

External links

- (http://www.its.caltech.edu/~phys1/java/phys1/MovingCharge/MovingCharge.html) - An applet that shows the electric field of a moving point charge.
- Fields (http://www.lightandmatter.com/html_books/0sn/ch10/ch10.html) - a chapter from an online textbook
- Learning by Simulations (http://www.vias.org/simulations/simusoft_efield.html) Interactive simulation of an electric field of up to four point charges
- Java simulations of electrostatics in 2-D (http://www.falstad.com/emstatic/) and 3-D (http://www.falstad.com/vector3de/)
- Electric Fields Applet (http://www.physics-lab.net/applets/electric-fields) - An applet that shows electric field lines as well as potential gradients.
- The inverse cube law (http://blazelabs.com/inversecubelaw.pdf) The inverse cube law for dipoles (PDF file) by Eng. Xavier Borg
- Interactive Flash simulation picturing the electric field of user-defined or preselected sets of point charges (http://www.flashphysics.org/electricField.html) by field vectors, field lines, or equipotential lines. Author: David Chappell

D field

In physics, the **electric displacement field**, denoted as $\mathbf{D}$, is a vector field that appears in Maxwell's equations. It accounts for the effects of free charges within materials. "D" stands for "displacement," as in the related concept of displacement current in dielectrics. In free space, the electric displacement field is equivalent to flux density, a concept that lends understanding to Gauss's law.

Definition

In a dielectric material the presence of an electric field **E** causes the bound charges in the material (atomic nuclei and their electrons) to slightly separate, inducing a local electric dipole moment. The electric displacement field **D** is defined as

$$\mathbf{D} \equiv \varepsilon_0 \mathbf{E} + \mathbf{P},$$

where ε_0 is the vacuum permittivity (also called permittivity of free space), and **P** is the (macroscopic) density of the permanent and induced electric dipole moments in the material, called the polarization density. Separating the total volume charge density into free and bound charges:

$$\rho = \rho_f + \rho_b$$

the density can be rewritten as a function of the polarization **P**:

$$\rho = \rho_f - \nabla \cdot \mathbf{P}.$$

P is a vector field whose divergence yields the density of bound charges ρ_b in the material. The electric field satisfies the equation:

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0}(\rho_f + \rho_b) = \frac{1}{\varepsilon_0}(\rho_f - \nabla \cdot \mathbf{P})$$

and hence

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = \rho_f.$$

The displacement field therefore satisfies Gauss's law in a dielectric:

$$\nabla \cdot \mathbf{D} = \rho - \rho_b = \rho_f.$$

D is not determined exclusively by the free charge. Consider the relationship:

$$\nabla \times \mathbf{D} = \varepsilon_0 (\nabla \times \mathbf{E}) + (\nabla \times \mathbf{P})$$

Which, by the fact that **E** has a curl of zero in electrostatic situations, evaluates to:

$$\nabla \times \mathbf{D} = \nabla \times \mathbf{P}$$

Which can be immediately seen in the case of some object with a "frozen in" polarization like a bar electret, the electric analogue to a bar magnet. There is no free charge in such a material, but the inherent polarization gives rise to an electric field. If the wayward student were to assume the **D** field were entirely determined by the free charge, he or she would immediately conclude the electric field were zero in such a material, but this is patently not true. The electric field can be properly determined by using the above relation along with other boundary conditions on the polarization density yielding the bound charges, which will, in turn, yield the electric field.

In a linear, homogeneous, isotropic dielectric with instantaneous response to changes in the electric field, **P** depends linearly on the electric field,

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E},$$

where the constant of proportionality χ is called the electric susceptibility of the material. Thus

$$\mathbf{D} = \varepsilon_0 (1 + \chi) \mathbf{E} = \varepsilon \mathbf{E}$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity, and $\varepsilon_r = 1 + \chi$ the relative permittivity of the material.

In linear, homogeneous, isotropic media ε is a constant. However, in linear anisotropic media it is a matrix, and in nonhomogeneous media it is a function of position inside the medium. It may also depend upon the electric field (nonlinear materials) and have a time dependent response. Explicit time dependence can arise if the materials are physically moving or changing in time (e.g. reflections off a moving interface give rise to Doppler shifts). A different form of time dependence can arise in a time-invariant medium, in that there can be a time delay between the imposition of the electric field and the resulting polarization of the material. In this case, **P** is a convolution of the impulse response susceptibility χ and the electric field **E**. Such a convolution takes on a simpler form in the frequency domain—by Fourier transforming the relationship and applying the convolution theorem, one obtains the following relation for a linear time-invariant medium:

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega),$$

where ω is frequency of the applied field (e.g. in radian/s). The constraint of causality leads to the Kramers–Kronig relations, which place limitations upon the form of the frequency dependence. The phenomenon of a frequency-dependent permittivity is an example of material dispersion. In fact, all physical materials have some material dispersion because they cannot respond instantaneously to applied fields, but for many problems (those concerned with a narrow enough bandwidth) the frequency-dependence of ε ; can be neglected.

At a boundary, $D_{1,\perp} - D_{2,\perp} = \sigma_f$, where σ_f is the free charge density.[1]

Units

In the standard SI system of units, **D** is measured in coulombs per square meter (C/m^2). This choice of units (together with measuring the magnetizing field **H** in amperes per meter (A/m)) is designed to absorb the electric and magnetic constants in the Maxwell's equations expressed in terms of free charge and current, and results in very simple forms for Gauss's law and the Ampère-Maxwell equation:

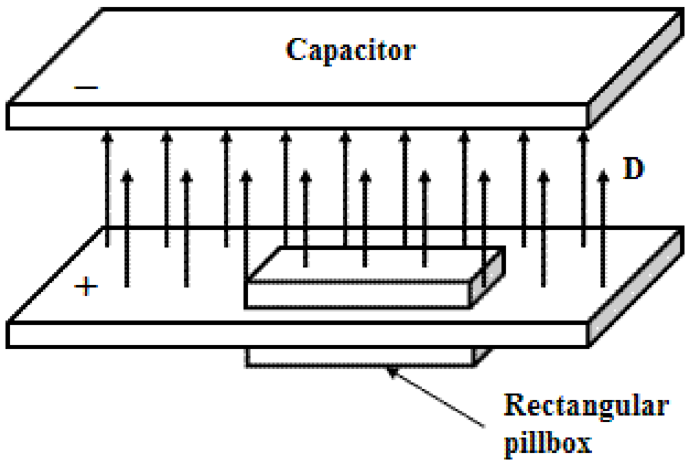
$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

Choice of units has differed in history, for example in the Gaussian CGS system of units the unit of charge is defined so that **E** and **D** are expressed in the same units.

Example: Displacement field in a capacitor

Consider an infinite parallel plate capacitor placed in space (or in a medium) with no free charges present except on the capacitor. In SI units, the charge density on the plates is equal to the value of the **D** field between the plates. This follows directly from Gauss's law, by integrating over a small rectangular pillbox straddling one plate of the capacitor:



A parallel plate capacitor. Using an imaginary pillbox, it is possible to use Gauss's law to explain the relationship between electric displacement and free charge.

$$\oint_A \mathbf{D} \cdot d\mathbf{A} = Q_{free}$$

On the sides of the pillbox, $d\mathbf{A}$ is perpendicular to the field, so that part of the integral is zero, leaving, for the space inside the capacitor where the fields of the two plates add

$$|\mathbf{D}| = Q/A ,$$

where A is surface area of the top face of the small rectangular pillbox and Q/A is just the free surface charge density on the positive plate. Outside the capacitor, the fields of the two plates cancel each other and $|\mathbf{D}| = 0$. If the space between the capacitor plates is filled with a linear homogeneous isotropic dielectric with permittivity ε the electric field between the plates is constant: $|\mathbf{E}| = Q/(\varepsilon A)$.

If the distance d between the plates of a *finite* parallel plate capacitor is much smaller than its lateral dimensions we can approximate it using the infinite case and obtain its capacitance as

$$C = \frac{Q}{V} \approx \frac{Q}{|\mathbf{E}|d} = \varepsilon \frac{A}{d} .$$

References

[1] David Griffiths. *Introduction to Electrodynamics* (3rd 1999 ed.).

- "electric displacement field" at PhysicsForums (http://www.physicsforums.com/library.php?do=view_item&itemid=6)

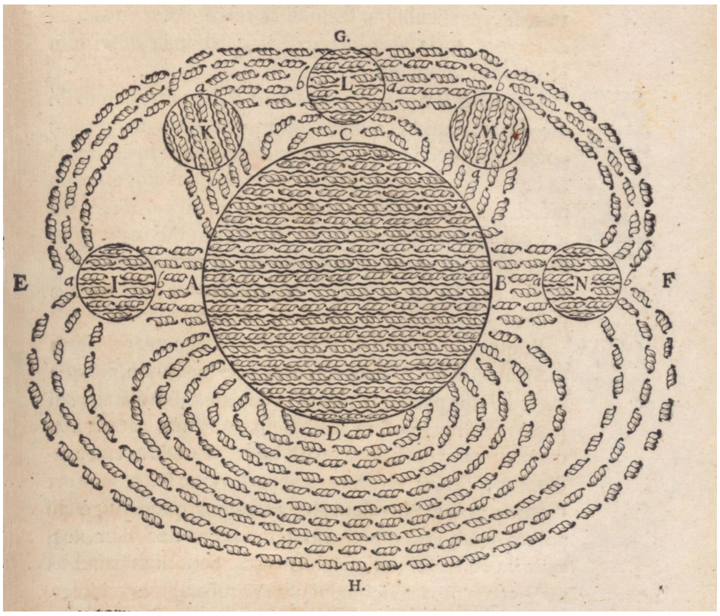
B and *H* fields

A **magnetic field** is a mathematical description of the magnetic influence of electric currents and magnetic materials. The magnetic field at any given point is specified by both a *direction* and a *magnitude* (or strength); as such it is a vector field.[1] The magnetic field is most commonly defined in terms of the Lorentz force it exerts on moving electric charges. There are two separate but closely related fields to which the name 'magnetic field' can refer: a magnetic **B** field and a magnetic **H** field.

Magnetic fields are produced by moving electric charges and the intrinsic magnetic moments of elementary particles associated with a fundamental quantum property, their spin. In special relativity, electric and magnetic fields are two interrelated aspects of a single object, called the electromagnetic field tensor; the aspect of the electromagnetic field that is seen as a magnetic field is dependent on the reference frame of the observer. In quantum physics, the electromagnetic field is quantized and electromagnetic interactions result from the exchange of photons.

Magnetic fields have had many uses in ancient and modern society. The Earth produces its own magnetic field, which is important in navigation. Rotating magnetic fields are utilized in both electric motors and generators. Magnetic forces give information about the charge carriers in a material through the Hall effect. The interaction of magnetic fields in electric devices such as transformers is studied in the discipline of magnetic circuits.

History



One of the first drawings of a magnetic field, by René Descartes, 1644. It illustrated his theory that magnetism was caused by the circulation of tiny helical particles, "threaded parts", through threaded pores in magnets.

Although magnets and magnetism were known much earlier, one of the first descriptions of the magnetic field was produced in 1269 by the French scholar Petrus Peregrinus[2] who mapped out the magnetic field on the surface of a spherical magnet using iron needles. Noting that the resulting field lines crossed at two points he named those points 'poles' in analogy to Earth's poles. Almost three centuries later, William Gilbert of Colchester replicated Petrus Peregrinus' work and was the first to state explicitly that Earth itself was a magnet. Gilbert's great work *De Magnete* was published in 1600 and helped to establish the study of magnetism as a science.

One of the first successful models of the magnetic field was developed in 1824 by Siméon-Denis Poisson (1781–1840). Poisson assumed that magnetism was due to 'magnetic charges' such that like 'magnetic charges' repulse while opposites attract. The model he created is exactly analogous to modern electrostatics with a magnetic ***H***-field being produced by 'magnetic charges' in the same way that an electric field ***E***-field is produced by electric charges. It predicts the correct ***H***-field for permanent magnets. It predicts the forces between magnets. And, it predicts the correct energy stored in the magnetic fields.[3]

Three remarkable discoveries though, would challenge Poisson's model. First, in 1819, Hans Christian Oersted discovered that an electric current generates a magnetic field encircling it. Then, André-Marie Ampère showed that parallel wires having currents in the same direction attract one another. Finally Jean-Baptiste Biot and Félix Savart discovered the Biot–Savart law which correctly predicts the magnetic field around any current-carrying wire.

Together, these discoveries suggested a model in which the magnetic ***B*** field of a material is produced by microscopic current loops. In this model, these current loops (called magnetic dipoles) would replace the dipoles of charge of the Poisson's model.[4] No magnetic charges are needed which has the additional benefit of explaining why magnetic charge can not be isolated; cutting a magnet in half does not result in two separate poles but in two separate magnets, each of which has both poles.

The next decade saw two developments that help lay the foundation for the full theory of electromagnetism. In 1825, Ampère published his Ampère's law which like the Biot–Savart law correctly described the magnetic field generated by a steady current but was more general. And, in 1831, Michael Faraday showed that a *changing* magnetic field generates an encircling electric field and thereby demonstrated that electricity and magnetism are even more tightly knitted.

Between 1861 and 1865, James Clerk Maxwell developed and published a set of Maxwell's equations which explained and united all of classical electricity and magnetism. The first set of these equations was published in a paper entitled *On Physical Lines of Force* in 1861. The mechanism that Maxwell proposed to underlie these equations in this paper was fundamentally incorrect, which is not surprising since it predated the modern understanding even of the atom. Yet, the equations were valid although incomplete. He completed the set of Maxwell's equations in his later 1865 paper *A Dynamical Theory of the Electromagnetic Field* and demonstrated the fact that light is an electromagnetic wave. Thus, he theoretically unified not only electricity and magnetism but light as well. This fact was then later confirmed experimentally by Heinrich Hertz in 1887.

Even though the classical theory of electrodynamics was essentially complete with Maxwell's equations, the twentieth century saw a number of improvements and extensions to the theory. Albert Einstein, in his great paper of 1905 that established relativity, showed that both the electric and magnetic fields are part of the same phenomena viewed from different reference frames. (See moving magnet and conductor problem for details about the thought experiment that eventually helped Albert Einstein to develop special relativity.) Finally, the emergent field of quantum mechanics was merged with electrodynamics to form quantum electrodynamics or QED.

Definitions, units, and measurement

Magnetic field can be defined in many equivalent ways based on the effects it has on its environment. For instance, a particle having an electric charge, q, and moving in a magnetic field with a velocity, v, experiences a force, F, called the Lorentz force. See Force on a charged particle below. Alternatively, the magnetic field can be defined in terms of the torque it produces on a magnetic dipole. See Torque on a magnet due to a B-field below.

Devices used to measure the local magnetic field are called magnetometers. Important classes of magnetometers include using a rotating coil, Hall effect magnetometers, NMR magnetometers, SQUID magnetometers, and fluxgate magnetometers. The magnetic fields of distant astronomical objects are measured through their effects on local charged particles. For instance, electrons spiraling around a field line produce synchrotron radiation which is detectable in radio waves.

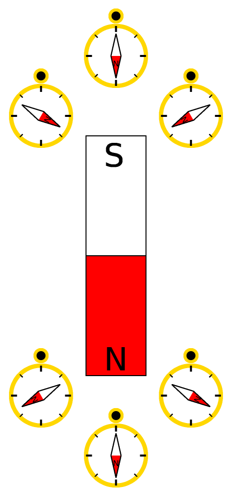
Alternative names for B [5]
• Magnetic flux density • Magnetic induction • Magnetic field
Alternative names for H [5] [6]
• Magnetic field intensity • Magnetic field strength • Magnetic field • Magnetizing field

There are two *magnetic fields*, **H** and **B**. In a vacuum they are indistinguishable, differing only by a multiplicative constant that depends on the physical units. Inside a material they are different (see H and B inside and outside of magnetic materials). The term *magnetic field* is historically reserved for **H** while using other terms for **B**. Informally, though, and formally for some recent textbooks mostly in physics, the term 'magnetic field' is used to describe **B** as well as or in place of **H**.[7] There are many alternative names for both (see sidebar to right).

The B-field is measured in teslas in SI units and in gauss in cgs units. (1 tesla = 10,000 gauss). The SI unit of tesla is equivalent to (newton·second)/(coulomb·metre).[8] The H-field is measured in ampere-turn per metre (A/m) in SI units, and in oersteds (Oe) in cgs units.[9]

The smallest precision level for a magnetic field measurement[10] is on the order of attoteslas (10^{-18} tesla); the largest magnetic field produced in a laboratory is 2,800 T (VNIIEF in Sarov, Russia, 1998).[11] The magnetic field of some astronomical objects such as magnetars are much higher; magnetars range from 0.1 to 100 GT (10^{8} to 10^{11} T).[12] See orders of magnitude (magnetic field).

Magnetic field lines

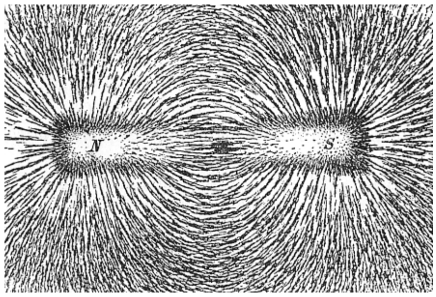


Compasses reveal the direction of the local magnetic field. As seen here, the magnetic field points towards a magnet's south pole and away from its north pole.

Mapping the magnetic field of an object is simple in principle. First, measure the strength and direction of the magnetic field at a large number of locations. Then, mark each location with an arrow (called a vector) pointing in the direction of the local magnetic field with a length proportional to the strength of the magnetic field.

A simpler way to visualize the magnetic field is to 'connect' the arrows to form "magnetic **field lines**". Magnetic field lines make it much easier to visualize and understand the complex mathematical relationships underlying magnetic field. If done carefully, a field line diagram contains the same information as the vector field it represents. The magnetic field can be estimated at *any* point on a magnetic field line diagram (whether on a field line or not) using the direction and density of nearby magnetic field lines.[13] A higher density of nearby field lines indicates a stronger magnetic field.

Various phenomena have the effect of "displaying" magnetic field lines as though the field lines are physical phenomena. For example, iron filings placed in a magnetic field line up to form lines that correspond to 'field lines'. Magnetic fields "lines" are also visually displayed in polar auroras, in which plasma particle dipole interactions create visible streaks of light that line up with the local direction of Earth's magnetic field. However, field lines are a visual and conceptual aid only and are no more real than (for example) the contour lines (constant altitude) on a topographic map. They do not exist in the actual field; a different choice of mapping scale could show twice as many "lines" or half as many.



The direction of magnetic field lines represented by the alignment of iron filings sprinkled on paper placed above a bar magnet. The mutual attraction of opposite poles of the iron filings results in the formation of elongated clusters of filings along "field lines". The field is not precisely the same as around the isolated magnet; the magnetization of the filings alters the field somewhat.

Field lines can be used as a qualitative tool to visualize magnetic forces. In ferromagnetic substances like iron and in plasmas, magnetic forces can be understood by imagining that the field lines exert a tension, (like a rubber band) along their length, and a pressure perpendicular to their length on neighboring field lines. 'Unlike' poles of magnets attract because they are linked by many field lines; 'like' poles repel because their field lines do not meet, but run parallel, pushing on each other.

Most physical phenomena that "display" magnetic field lines do not include which direction along the lines that the magnetic field is in. A compass, though, reveals that magnetic field lines *outside of a magnet* point from the north pole (compass points away from north pole) to the south (compass points toward the south pole). The magnetic field of a straight current-carrying wire encircles the wire with a direction that depends on the direction of the current and that can be measured with a compass as well.

The magnetic field and permanent magnets

Permanent magnets are objects that produce their own persistent magnetic fields. They are made of ferromagnetic materials, such as iron and nickel, that have been magnetized, and they have both a north and a south pole.

Magnetic field of permanent magnets

The magnetic field of permanent magnets can be quite complicated, especially near the magnet. The magnetic field of a small[14] straight magnet is proportional to the magnet's *strength* (called its magnetic dipole moment m). The equations are non-trivial and also depend on the distance from the magnet and the orientation of the magnet. For simple magnets, **m** points in the direction of a line drawn from the south to the north pole of the magnet. Flipping a bar magnet is equivalent to rotating its **m** by 180 degrees.

The magnetic field of larger magnets can be obtain by modelling them as a collection of a large number of small magnets called dipoles each having their own **m**. The magnetic field produced by the magnet then is the net magnetic field of these dipoles. And, any net force on the magnet is a result of adding up the forces on the individual dipoles.

There are two competing models for the nature of these dipoles. These two models produce two different magnetic fields, **H** and **B**. Outside a material, though, the two are identical (to a multiplicative constant) so that in many cases the distinction can be ignored. This is particularly true for magnetic fields, such as those due to electric currents, that are not generated by magnetic materials.

Magnetic pole model and the *H*-field

It is sometimes useful to model the force and torques between two magnets as due to magnetic poles repelling or attracting each other in the same manner as the Coulomb force between electric charges. In this model, a magnetic *H-field* is produced by *magnetic charges* that are 'smeared' around each pole. The H-field, therefore, is analogous to the electric field **E** which starts at a positive electric charge and ends at a negative electric charge. Near the north pole, therefore, all H-field lines point away from the north pole (whether inside the magnet or out) while near the south pole (whether inside the magnet or out) all H-field lines point toward the south pole. A north pole, then, feels a force in the direction of the H-field while the force on the south pole is opposite to the H-field.

In the magnetic pole model, the elementary magnetic dipole **m** is formed by two opposite magnetic charges (poles) of pole strength q_m separated by a very small distance d, such that m = q_m d.
Unfortunately, magnetic poles cannot exist apart from each other; all magnets have north/south pairs which cannot be separated without creating two magnets each having a north/south pair. Further, magnetic charge does not account for magnetism that is produced by electric currents nor the force that a magnetic field applies to moving electric charges.

Amperian loop model and the *B*-field

After Oersted discovered that electric currents produce a magnetic field and Ampere discovered that electric currents attracted and repelled each other similar to magnets, it was natural to hypothesize that all magnetic fields are due to electric current loops. In this model developed by Ampere, the elementary magnetic dipole that makes up all magnets is an Amperian loop of current I and extremely small area A with a dipole moment **m** = I**A** where the direction of A is perpendicular to the area and determined by the direction of the current around the area.

These magnetic dipoles produce a magnetic **B** field. One important property of the B-field produced this way is that magnetic **B** field lines neither start nor end (mathematically, **B** is a solenoidal vector field); a field line either extends to infinity or wraps around to form a closed curve.[15] To date no exception to this rule has been found. (See magnetic monopole below.) Magnetic field lines exit a magnet near its north pole and enter near its south pole, but inside the magnet B-field lines continue through the magnet from the south pole back to the north.[16] If a B-field line enters a magnet somewhere it has to leave somewhere else; it is not allowed to have an end point. Magnetic poles, therefore, always come in N and S pairs. Cutting a magnet in half results in two separate magnets each with both a north and a south pole.

More formally, since all the magnetic field lines that enter any given region must also leave that region, subtracting the 'number'[17] of field lines that enter the region from the number that exit gives identically zero. Mathematically this is equivalent to:

$$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0,$$

where the integral is a surface integral over the closed surface *S* (a closed surface is one that completely surrounds a region with no holes to let any field lines escape). Since d**A** points outward, the dot product in the integral is positive for B-field pointing out and negative for B-field pointing in.

There is also a corresponding differential form of this equation covered in Maxwell's equations below.

Force Between magnets

The force between two small magnets is quite complicated and depends on the strength and orientation of both magnets and the distance and direction of the magnets relative to each other. The force is particularly sensitive to rotations of the magnets due to magnetic torque. The force on each magnet depends on its magnetic moment and the magnetic field **B**[18] of the other.

To understand the force between magnets and to generalize it to other cases, it is useful to examine the *magnetic charge model* given above (with the caveats given above as well). In this model, the *H-field* of the first magnet pushes and pulls on the magnetic charges near *both* poles of the second magnet. If the H-field due to the first magnet is the same at both poles of the second magnet then there is no net force on that magnet since the force is opposite for opposite poles. The magnetic field is not the same, though; the magnetic field is significantly stronger near the poles of a magnet. In this *nonuniform* magnetic field, each pole sees a different field and is subject to a different force. This difference in the two forces moves the magnet in the direction of increasing magnetic field and may also cause a net torque.

This is a specific example of a general rule that magnets are attracted (or repulsed depending on the orientation of the magnet) into regions of higher magnetic field. Any non-uniform magnetic field whether caused by permanent magnets or by electric currents will exert a force on a small magnet in this way.

Mathematically, the force on a small magnet having a magnetic moment **m** due to a magnetic field **B** is:[19]

$$\mathbf{F} = \nabla\left(\mathbf{m}\cdot\mathbf{B}\right),$$

where the gradient ∇ is the change of the quantity $\mathbf{m}\cdot\mathbf{B}$ per unit distance and the direction is that of maximum increase of $\mathbf{m}\cdot\mathbf{B}$. To understand this equation, note that the dot product $\mathbf{m}\cdot\mathbf{B} = mB\cos(\theta)$, where m and B represent the magnitude of the **m** and **B** vectors and θ is the angle between them. If **m** is in the same direction as **B** then the dot product is positive and the gradient points 'uphill' pulling the magnet into regions of higher B-field (more strictly larger $\mathbf{m}\cdot\mathbf{B}$). This equation is strictly only valid for magnets of zero size, but is often a good approximation for not too large magnets. The magnetic force on larger magnets is determined by dividing them into smaller regions having their own **m** then summing up the forces on each of these regions.

Torque on a magnet due to a B-field

Magnetic torque on a magnet due to an external magnetic field can be observed by placing two magnets near each other while allowing one to rotate. Magnetic torque is used to drive simple electric motors. In one simple motor design, a magnet is fixed to a freely rotating shaft and subjected to a magnetic field from an array of electromagnets. By continuously switching the electric current through each of the electromagnets, thereby flipping the polarity of their magnetic fields, like poles are kept next to the rotor; the resultant torque is transferred to the shaft. See Rotating magnetic fields below.

Magnetic torque $\boldsymbol{\tau}$ tends to align a magnet's poles with the B-field lines (since **m** is in the direction of the poles this is equivalent to saying that it tends to align **m** in the same direction as **B**). This is why the magnetic needle of a compass points toward earth's north pole. By definition, the direction of the Earth's local magnetic field is the direction in which the north pole of a compass (or of any magnet) tends to point.

Mathematically, the torque $\boldsymbol{\tau}$ on a small magnet is proportional both to the applied B-field and to the magnetic moment **m** of the magnet:

$$\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B},$$

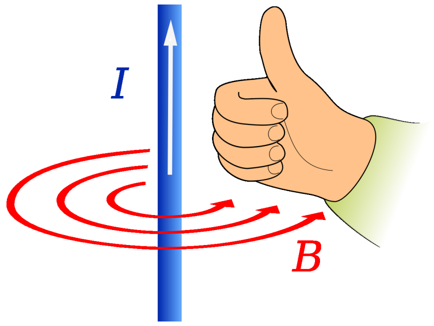
where × represents the vector cross product. Note that this equation includes all of the qualitative information included above. There is no torque on a magnet if **m** is in the same direction as **B**. (The cross product is zero for two vectors that are in the same direction.) Further, all other orientations feel a torque that twists them toward the direction of **B**.

The magnetic field and electric currents

Currents of electric charges both generate a magnetic field and feel a force due to magnetic B-fields.

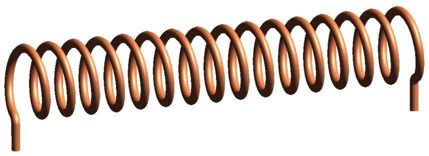
Magnetic field due to moving charges and electric currents

All moving charged particles produce magnetic fields. Moving point charges, such as electrons, produce complicated but well known magnetic fields that depend on the charge, velocity, and acceleration of the particles.[20]



Right hand grip rule: current (*I*) flowing through a conductor in the direction indicated by the white arrow produces a magnetic field (*B*) around the conductor as shown by the red arrows.

Magnetic field lines form in concentric circles around a cylindrical current-carrying conductor, such as a length of wire. The direction of such a magnetic field can be determined by using the "right hand grip rule" (see figure at right). The strength of the magnetic field decreases with distance from the wire. (For an infinite length wire the strength decreases inversely proportional to the distance.)



Solenoid

Bending a current-carrying wire into a loop concentrates the magnetic field inside the loop while weakening it outside. Bending a wire into multiple closely spaced loops to form a coil or "solenoid" enhances this effect. A device so formed around an iron core may act as an **electromagnet**, generating a strong, well-controlled magnetic field. An infinitely long cylindrical electromagnet has a uniform magnetic field inside, and no magnetic field outside. A finite length electromagnet produces a magnetic field that looks similar to that produced by a uniform permanent magnet, with its strength and polarity determined by the current flowing through the coil.

The magnetic field generated by a steady current I (a constant flow of electric charges in which charge is neither accumulating nor depleting at any point)[21] is described by the **Biot–Savart law**:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2},$$

where the integral sums over the wire length where vector $d\boldsymbol{\ell}$ is the direction of the current, μ_0 is the magnetic constant, r is the distance between the location of $d\boldsymbol{\ell}$ and the location at which the magnetic field is being calculated, and $\hat{\mathbf{r}}$ is a unit vector in the direction of $\mathbf{r}$.

A slightly more general[22] [23] way of relating the current I to the B-field is through **Ampère's law**:

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{\mathrm{enc}},$$

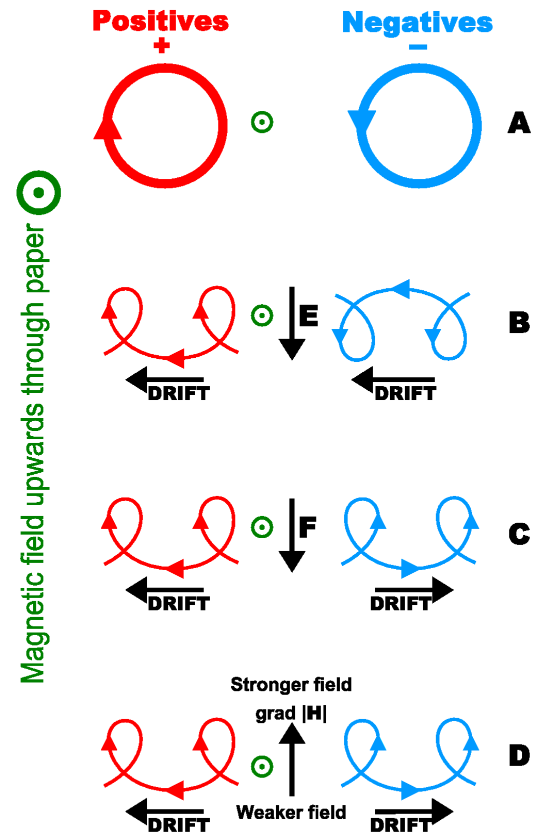
where the line integral is over any arbitrary loop and I_{enc} is the current enclosed by that loop. Ampère's law is always valid for steady currents and can be used to calculate the B-field for certain highly symmetric situations such as an infinite wire or an infinite solenoid.

In a modified form that accounts for time varying electric fields, Ampère's law is one of four Maxwell's equations that describe electricity and magnetism.

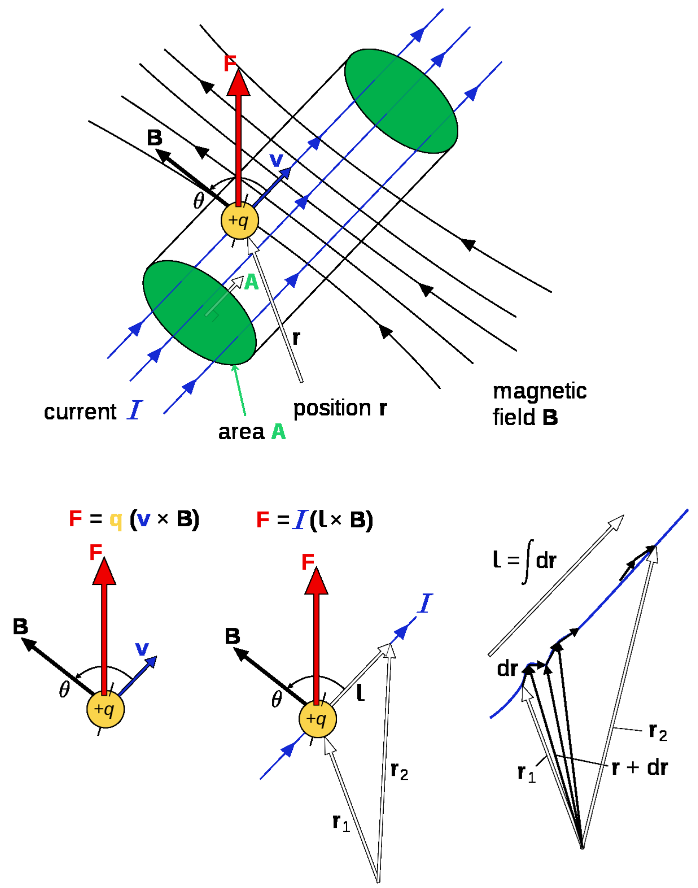
Force on moving charges and current

Force on a charged particle

A charged particle moving in a B-field experiences a *sideways* force that is proportional to the strength of the magnetic field, the component of the velocity that is perpendicular to the magnetic field and the charge of the particle. This force is known as the **Lorentz force**, and is given by



Charged particle drifts in a magnetic field with (A) no net force, (B) an electric field, **E**, (C) a charge independent force, **F** (e.g. gravity), and (D) an inhomogeneous magnetic field, grad **H**.



Magnetic force on charged particles and currents, shown in 3d. The electric current shown here is *conventional*, the real current would be a charge of $-q$ flowing in exactly the opposite direction. Only one charge carrier is shown to prevent cluttering the diagram. $\mathbf{r}_1$ is the position of entry into the field **B**, $\mathbf{r}_2$ is the exit. The vector **l** is the integral (sum) of all all infinitesimal vectors d**r** from $\mathbf{r}_1$ to $\mathbf{r}_2$.

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B},$$

where **F** is the force, q is the electric charge of the particle, **v** is the instantaneous velocity of the particle, and **B** is the magnetic field (in teslas).

The Lorentz force is always perpendicular to both the velocity of the particle and the magnetic field that created it. When a charged particle moves in a static magnetic field it will trace out a helical path in which the helix axis is parallel to the magnetic field and in which the speed of the particle will remain constant. Because the magnetic force is always perpendicular to the motion, the magnetic field can do no work on an isolated charge. It can only do work indirectly, via the electric field generated by a changing magnetic field. It is often claimed that the magnetic force can do work to a non-elementary magnetic dipole, or to charged particles whose motion is constrained by other forces, but this is incorrect[24] because the work in those cases is performed by the electric forces of the charges deflected by the magnetic field.

Force on current-carrying wire

The force on a current carrying wire is similar to that of a moving charge as expected since a charge carrying wire is a collection of moving charges. A current carrying wire feels a sideways force in the presence of a magnetic field. The Lorentz force on a macroscopic current is often referred to as the **Laplace force**. Consider a conductor of length l and area of cross section A and has charge q which is due to electric current i .If a conductor is placed in a magnetic field of induction B which makes an angle θ (theta) with the velocity of charges in the conductor which has i current flowing in it. then force exerted due to small particle q is

$$\mathbf{F} = q\mathbf{v}\sin(\theta),$$

then for *n* number of charges it has

$$N = nlA$$

then force exerted on the body is

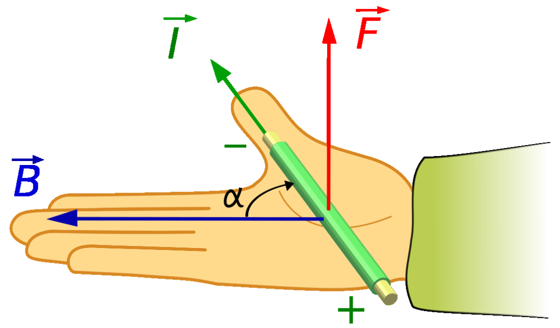
$$f = \mathbf{F}N = q\mathbf{v}\sin(\theta)(nlA)$$

but

$$nqvA = i$$

that is

$$f = \mathbf{B}il\sin(\theta)$$



The right-hand rule: Pointing the thumb of the right hand in the direction of the conventional current and the fingers in the direction of the B-field the force on the current points out of the palm. The force is reversed for a negative charge.

Direction of force

The direction of force on a charge or a current can be determined by a mnemonic known as the **right-hand rule**. See the figure on the left. Using the right hand and pointing the thumb in the direction of the moving positive charge or positive current and the fingers in the direction of the magnetic field the resulting force on the charge points outwards from the palm. The force on a negatively charged particle is in the opposite direction. If both the speed and the charge are reversed then the direction of the force remains the same. For that reason a magnetic field measurement (by itself) cannot distinguish whether there is a positive charge moving to the right or a negative charge moving to the left. (Both of these cases produce the same current.) On the other hand, a magnetic field combined with an electric field *can* distinguish between these, see Hall effect below.

An alternative mnemonic to the right hand rule is Fleming's left hand rule.

H and *B* inside and outside of magnetic materials

The formulas derived for the magnetic field above are correct when dealing with the entire current. A magnetic material placed inside a magnetic field, though, generates its own bound current which can be a challenge to calculate. (This bound current is due to the sum of atomic sized current loops and the spin of the subatomic particles such as electrons that make up the material.) The H-field as defined above helps factor out this bound current; but in order to see how, it helps to introduce the concept of *magnetization* first.

Magnetization

The **magnetization** field **M** represents how strongly a region of material is magnetized. For a uniform magnet, the magnetization is equal to its magnetic moment, **m**, divided by its volume. More generally, the magnetization of a region is defined as net magnetic dipole moment per unit volume of that region. Since the SI unit of magnetic moment is ampere-turn meter^2, the SI unit of magnetization **M** is ampere-turn per meter which is identical to that of the **H**-field.

The magnetization **M** field of a region points in the direction of the average magnetic dipole moment in the region and is in the same direction as the local **B**-field it produces. Therefore, **M** field lines move from near the south pole of a magnet to near its north. Unlike **B**, magnetization only exists inside a magnetic material. Therefore, magnetization field lines begin and end near magnetic poles.

The physically correct way to represent magnetization is to add all of the currents of the dipole moments that produce the magnetization. See Magnetic dipoles below and magnetic poles vs. atomic currents for more information. The resultant current is called *bound current* and is the source of the magnetic field due to the magnet. Given the definition of the magnetic dipole, the magnetization field follows a similar law to that of Ampere's law: [25]

$$\oint \mathbf{M} \cdot d\boldsymbol{\ell} = I_{\mathrm{b}},$$

where the integral is a line integral over any closed loop and I_b is the 'bound current' enclosed by that closed loop.

It is also possible to model the magnetization in terms of magnetic charge in which magnetization begins at and ends at bound 'magnetic charges'. If a given region, therefore, has a net positive 'magnetic charge' then it will have more magnetic field lines entering it than leaving it. Mathematically this is equivalent to:

$$\oint_S \mu_0 \mathbf{M} \cdot \mathrm{d}\mathbf{A} = -q_M,$$

where the integral is a closed surface integral over the closed surface *S* and q_M is the 'magnetic charge' (in units of magnetic flux) enclosed by *S*. (A closed surface completely surrounds a region with no holes to let any field lines escape.) The negative sign occurs because, like **B** *inside* a magnet, the magnetization field moves from south to north.

H-field and magnetic materials

The **H**-field is defined as:

$$\mathbf{H} \equiv \frac{\mathbf{B}}{\mu_0} - \mathbf{M},$$ (definition of **H** in SI units)

With this definition, Ampere's law becomes:

$$\oint \mathbf{H} \cdot d\boldsymbol{\ell} = \oint \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}\right) \cdot d\boldsymbol{\ell} = I_{\mathrm{tot}} - I_{\mathrm{b}} = I_{\mathrm{f}},$$

where I_f represents the 'free current' enclosed by the loop so that the line integral of **H** does not depend at all on the bound currents.[26] For the differential equivalent of this equation see Maxwell's equations. Ampere's law leads to the boundary condition

$$H_{1,\|} - H_{2,\|} = \mathbf{K}_{\mathrm{f}},$$

where $\mathbf{K}_f$ is the surface free current density.[27]

Similarly, a surface integral of **H** over any closed surface is independent of the free currents and picks out the 'magnetic charges' within that closed surface:

$$\oint_S \mu_0 \mathbf{H} \cdot \mathrm{d}\mathbf{A} = \oint_S (\mathbf{B} - \mu_0 \mathbf{M}) \cdot \mathrm{d}\mathbf{A} = (0 - (-q_M)) = q_M,$$

which does not depend on the free currents.

The **H**-field, therefore, can be separated into two[28] independent parts:

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_d,$$

where $\mathbf{H}_0$ is the applied magnetic field due only to the free currents and $\mathbf{H}_d$ is the demagnetizing field due only to the bound currents.

The magnetic **H**-field, therefore, re-factors the bound current in terms of 'magnetic charges'. The **H** field lines loop only around 'free current' and, unlike the magnetic **B** field, begins and ends at near magnetic poles as well.

Magnetism

Most materials respond to an applied B-field by producing their own magnetization **M** and therefore their own B-field. Typically, the response is very weak and exists only when the magnetic field is applied. The term **'magnetism**' describes how materials respond on the microscopic level to an applied magnetic field and is used to categorize the magnetic phase of a material. Materials are divided into groups based upon their magnetic behavior:

- Diamagnetic materials[29] produce a magnetization that opposes the magnetic field.
- Paramagnetic materials[29] produce a magnetization in the same direction as the applied magnetic field.
- Ferromagnetic materials and the closely related ferrimagnetic materials and antiferromagnetic materials[30] [31] can have a magnetization independent of an applied B-field with a complex relationship between the two fields.
- Superconductors (and ferromagnetic superconductors)[32] [33] are materials that are characterized by perfect conductivity below a critical temperature and magnetic field. They also are highly magnetic and can be perfect diamagnets below a lower critical magnetic field. Superconductors often have a broad range of temperatures and magnetic fields (the so named mixed state) for which they exhibit a complex hysteretic dependence of **M** on **B**.

In the case of paramagnetism and diamagnetism, the magnetization **M** is often proportional to the applied magnetic field such that:

$$\mathbf{B} = \mu\mathbf{H},$$

where μ is a material dependent parameter called the permeability. In some cases the permeability may be a second rank tensor so that **H** may not point in the same direction as **B**. These relations between **B** and **H** are examples of constitutive equations. However, superconductors and ferromagnets have a more complex **B** to **H** relation, see magnetic hysteresis.

Energy stored in magnetic fields

Energy is needed to generate a magnetic field both to work against the electric field that a changing magnetic field creates and to change the magnetization of any material within the magnetic field. For non-dispersive materials this same energy is released when the magnetic field is destroyed so that this energy can be modeled as being stored in the magnetic field.

For linear, non-dispersive, materials (such that $\mathbf{B} = \mu\mathbf{H}$ where μ is frequency-independent), the energy density is:

$$u = \frac{\mathbf{B} \cdot \mathbf{B}}{2\mu} = \frac{\mu\mathbf{H} \cdot \mathbf{H}}{2}.$$

If there are no magnetic materials around then μ can be replaced by μ_0. The above equation cannot be used for nonlinear materials, though; a more general expression given below must be used.

In general, the incremental amount of work per unit volume δW needed to cause a small change of magnetic field $\delta\mathbf{B}$ is:

$$\delta W = \mathbf{H} \cdot \delta\mathbf{B}.$$

Once the relationship between **H** and **B** is known this equation is used to determine the work needed to reach a given magnetic state. For hysteretic materials such as ferromagnets and superconductors the work needed will also depend on how the magnetic field is created. For linear non-dispersive materials, though, the general equation leads directly to the simpler energy density equation given above.

Electromagnetism: the relationship between magnetic and electric fields

Faraday's Law: Electric force due to a changing *B*-field

A changing magnetic field, such as a magnet moving through a conducting coil, generates an electric field (and therefore tends to drive a current in the coil). This is known as **Faraday's law** and forms the basis of many electrical generators and electric motors.

Mathematically, Faraday's law is:

$$\mathcal{E} = -\frac{d\Phi_m}{dt},$$

where $\mathcal{E}$ is the electromotive force (or *EMF*, the voltage generated around a closed loop) and Φ_m is the **magnetic flux**—the product of the area times the magnetic field normal to that area. (This definition of magnetic flux is why **B** is often referred to as **magnetic flux density**.)

The negative sign is necessary and represents the fact that any current generated by a changing magnetic field in a coil produces a magnetic field that *opposes* the **change** in the magnetic field that induced it. This phenomenon is known as Lenz's Law.

This integral formulation of Faraday's law can be converted[34] into a differential form, which applies under slightly different conditions. This form is covered as one of Maxwell's equations below.

Maxwell's correction to Ampère's Law: The magnetic field due to a changing electric field

Similar to the way that a changing magnetic field generates an electric field, a changing electric field generates a magnetic field. This fact is known as '**Maxwell's correction to Ampère's law**'. Maxwell's correction to Ampère's Law bootstrap together with Faraday's law of induction to form electromagnetic waves, such as light. Thus, a changing electric field generates a changing magnetic field which generates a changing electric field again.

Maxwell's correction to Ampère law is applied as an additive term to Ampere's law given above. This additive term is proportional to the time rate of change of the electric flux and is similar to Faraday's law above but with a different and positive constant out front. (The electric flux through an area is proportional to the area times the perpendicular part of the electric field.)

This full Ampère law including the correction term is known as the Maxwell–Ampère equation. It is not commonly given in integral form because the effect is so small that it can typically be ignored in most cases where the integral form is used. The Maxwell term *is* critically important in the creation and propagation of electromagnetic waves. These, though, are usually described using the differential form of this equation given below.

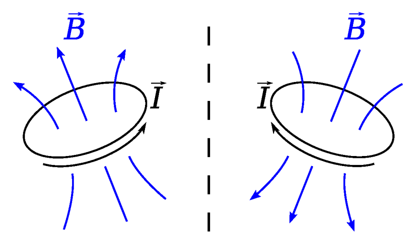
Maxwell's equations

Like all vector fields the B-field has two important mathematical properties that relates it to its *sources*. (For magnetic fields the *sources* are currents and changing electric fields.) These two properties, along with the two corresponding properties of the electric field, make up **Maxwell's Equations**. Maxwell's Equations together with the Lorentz force law form a complete description of classical electrodynamics including both electricity and magnetism.

The first property is the divergence of a vector field **A**, $\nabla \cdot \mathbf{A}$ which represents how **A** 'flows' outward from a given point. As discussed above, a B-field line never starts or ends at a point but instead forms a complete loop. This is mathematically equivalent to saying that the divergence of **B** is zero. (Such vector fields are called solenoidal vector fields.) This property is called Gauss's law for magnetism and is equivalent to the statement that there are no magnetic charges or magnetic monopoles. The electric field on the other hand begins and ends at electric charges so that its divergence is non-zero and proportional to the charge density (See Gauss's law).

The second mathematical property is called the curl, such that $\nabla \times \mathbf{A}$ represents how **A** curls or 'circulates' around a given point. The result of the curl is called a 'circulation source'. The equations for the curl of **B** and of **E** are called the Ampère–Maxwell equation and Faraday's law respectively. They represent the differential forms of the integral equations given above.

The complete set of Maxwell's equations then are:



Magnetic field, like all pseudovectors, changes sign when reflected in a mirror: When a current carrying loop (black) is reflected in a mirror (dotted line), its magnetic field (blue) is reflected *and* reversed.

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0},$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

where **J** = complete microscopic current density and ϱ is the charge density.

Technically, **B** is a pseudovector (also called an *axial vector*) due to being defined by a vector cross product. (See diagram to right.)

As discussed above, materials respond to an applied electric **E** field and an applied magnetic **B** field by producing their own internal 'bound' charge and current distributions that contribute to **E** and **B** but are difficult to calculate. To circumvent this problem the auxiliary **H** and **D** fields are defined so that Maxwell's equations can be re-factored in terms of the *free current density* $\mathbf{J}_f$ and *free charge density* ϱ_f:

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \cdot \mathbf{D} = \rho_f,$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t},$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

These equations are not any more general than the original equations (if the 'bound' charges and currents in the material are known'). They also need to be supplemented by the relationship between **B** and **H** as well as that between **E** and **D**. On the other hand, for simple relationships between these quantities this form of Maxwell's equations can circumvent the need to calculate the bound charges and currents.

Electric and magnetic fields: different aspects of the same phenomenon

According to the special theory of relativity, the partition of the electromagnetic force into separate electric and magnetic components is not fundamental, but varies with the observational frame of reference: An electric force perceived by one observer may be perceived by another (in a different frame of reference) as a magnetic force, or a mixture of electric and magnetic forces.

Formally, special relativity combines the electric and magnetic fields into a rank-2 tensor, called the **electromagnetic tensor**. Changing reference frames *mixes* these components. This is analogous to the way that special relativity *mixes* space and time into spacetime, and mass, momentum and energy into four-momentum.

Magnetic vector potential

In advanced topics such as quantum mechanics and relativity it is often easier to work with a potential formulation of electrodynamics rather than in terms of the electric and magnetic fields. In this representation, the **vector potential**, **A**, and the scalar potential, φ, are defined such that:

$$\mathbf{B} = \nabla \times \mathbf{A},$$
$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}.$$

The vector potential **A** may be interpreted as a *generalized potential momentum per unit charge*[35] just as φ is interpreted as a *generalized potential energy per unit charge*.

Maxwell's equations when expressed in terms of the potentials can be cast into a form that agrees with special relativity with little effort.[36] In relativity **A** together with φ forms the four-potential analogous to the four-momentum which combines the momentum and energy of a particle. Using the four potential instead of the electromagnetic tensor has the advantage of being much simpler; further it can be easily modified to work with quantum mechanics.

Quantum electrodynamics

In modern physics, the electromagnetic field is understood to be not a *classical* field, but rather a quantum field; it is represented not as a vector of three numbers at each point, but as a vector of three quantum operators at each point. The most accurate modern description of the electromagnetic interaction (and much else) is **Quantum electrodynamics** (QED),[37] which is incorporated into a more complete theory known as the "**Standard Model** of particle physics".

In QED, the magnitude of the electromagnetic interactions between charged particles (and their antiparticles) is computed using perturbation theory; these rather complex formulas have a remarkable pictorial representation as Feynman diagrams in which virtual photons are exchanged.

Predictions of QED agree with experiments to an extremely high degree of accuracy: currently about 10^{-12} (and limited by experimental errors); for details see precision tests of QED. This makes QED one of the most accurate physical theories constructed thus far.

All equations in this article are in the classical approximation, which is less accurate than the quantum description mentioned here. However, under most everyday circumstances, the difference between the two theories is negligible.

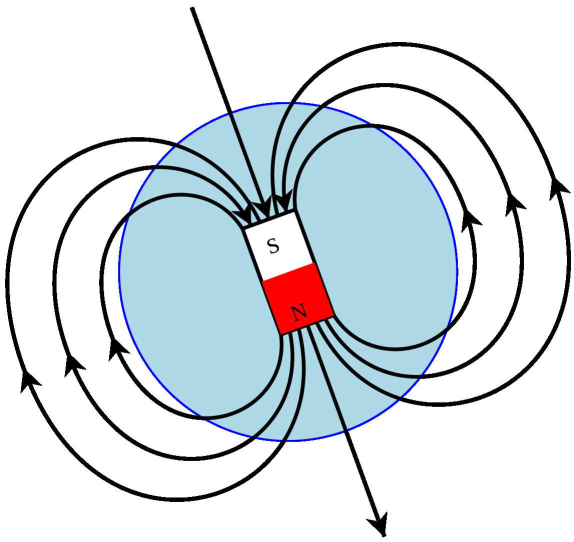
Important uses and examples of magnetic field

Earth's magnetic field

The **Earth's magnetic field** is thought to be produced by convection currents in the outer liquid of Earth's core. The Dynamo theory proposes that these movements produce electric currents which, in turn, produce the magnetic field.[38]

The presence of this field causes a compass, placed anywhere within it, to rotate so that the "north pole" of the magnet in the compass points roughly north, toward Earth's north magnetic pole. This is the traditional definition of the "north pole" of a magnet, although other equivalent definitions are also possible.

One confusion that arises from this definition is that, if Earth itself is considered as a magnet, the *south* pole of that magnet would be the one nearer the north magnetic pole, and vice-versa[39] (opposite poles attract, so the north pole of the compass magnet is attracted to the south pole of Earth's interior magnet).



A sketch of Earth's magnetic field representing the source of the field as a magnet. The geographic north pole of Earth is near the top of the diagram, the south pole near the bottom. The south pole of that magnet is deep in Earth's interior below Earth's North Magnetic Pole.

The north magnetic pole is so-named not because of the polarity of the field there but because of its geographical location. The north and south poles of a permanent magnet are so-called because they are "north-seeking" and "south-seeking", respectively.[40]

The figure to the right is a sketch of Earth's magnetic field represented by field lines. For most locations, the magnetic field has a significant up/down component in addition to the North/South component. (There is also an East/West component; Earth's magnetic poles do not coincide exactly with Earth's geological pole.) The magnetic field can be visualised as a bar magnet buried deep in Earth's interior.

Earth's magnetic field is not constant — the strength of the field and the location of its poles vary. Moreover, the poles periodically reverse their orientation in a process called geomagnetic reversal. The most recent reversal occurred 780,000 years ago.

Rotating magnetic fields

The **rotating magnetic field** is a key principle in the operation of alternating-current motors. A permanent magnet in such a field rotates so as to maintain its alignment with the external field. This effect was conceptualized by Nikola Tesla, and later utilized in his, and others', early AC (alternating-current) electric motors.

A rotating magnetic field can be constructed using two orthogonal coils with 90 degrees phase difference in their AC currents. However, in practice such a system would be supplied through a three-wire arrangement with unequal currents.

This inequality would cause serious problems in standardization of the conductor size and so, in order to overcome it, three-phase systems are used where the three currents are equal in magnitude and have 120 degrees phase difference. Three similar coils having mutual geometrical angles of 120 degrees create the rotating magnetic field in this case. The ability of the three-phase system to create a rotating field, utilized in electric motors, is one of the main reasons why three-phase systems dominate the world's electrical power supply systems.

Because magnets degrade with time, synchronous motors use DC voltage fed rotor windings which allows the excitation of the machine to be controlled and induction motors use short-circuited rotors (instead of a magnet) following the rotating magnetic field of a multicoiled stator. The short-circuited turns of the rotor develop eddy currents in the rotating field of the stator, and these currents in turn move the rotor by the Lorentz force.

In 1882, Nikola Tesla identified the concept of the rotating magnetic field. In 1885, Galileo Ferraris independently researched the concept. In 1888, Tesla gained U.S. Patent 381968 [41] for his work. Also in 1888, Ferraris published his research in a paper to the *Royal Academy of Sciences* in Turin.

Hall effect

The charge carriers of a current carrying conductor placed in a transverse magnetic field experience a sideways Lorentz force; this results in a charge separation in a direction perpendicular to the current and to the magnetic field. The resultant voltage in that direction is proportional to the applied magnetic field. This is known as the '**Hall effect**'.

The *Hall effect* is often used to measure the magnitude of a magnetic field. It is used as well to find the sign of the dominant charge carriers in materials such as semiconductors (negative electrons or positive holes).

Magnetic circuits

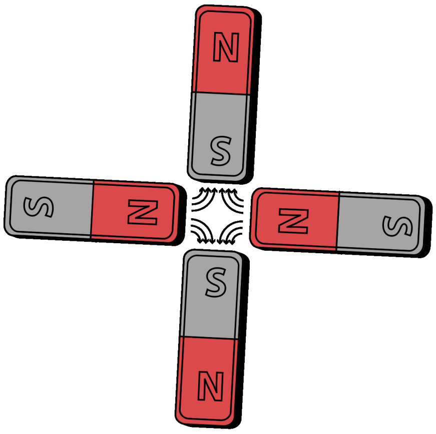
An important use of ***H*** is in **magnetic circuits** where inside a linear material ***B*** = μ ***H***. Here, μ is the magnetic permeability of the material. This result is similar in form to Ohm's law ***J*** = σ ***E***, where ***J*** is the current density, σ is the conductance and ***E*** is the electric field. Extending this analogy, the counterpart to the macroscopic Ohm's law ($I = V/R$) is:

$$\Phi = \frac{F}{R_m},$$

where $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$ is the magnetic flux in the circuit, $F = \int \mathbf{H} \cdot d\mathbf{l}$ is the magnetomotive force applied to the circuit, and R_m is the reluctance of the circuit. Here the reluctance R_m is a quantity similar in nature to resistance for the flux.

Using this analogy it is straight-forward to calculate the magnetic flux of complicated magnetic field geometries, by using all the available techniques of circuit theory.

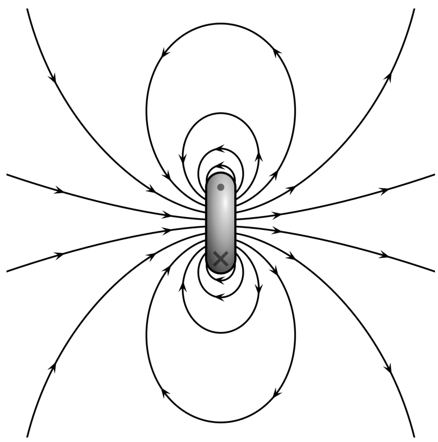
Magnetic field shape descriptions



Schematic quadrupole magnet ("*four-pole*") magnetic field. There are four steel pole tips, two opposing magnetic north poles and two opposing magnetic south poles.

- An **azimuthal** magnetic field is one that runs east-west.
- A **meridional** magnetic field is one that runs north-south. In the solar dynamo model of the Sun, differential rotation of the solar plasma causes the meridional magnetic field to stretch into an azimuthal magnetic field, a process called the *omega-effect*. The reverse process is called the *alpha-effect*.[42]
- A **dipole** magnetic field is one seen around a bar magnet or around a charged elementary particle with nonzero spin.
- A **quadrupole** magnetic field is one seen, for example, between the poles of four bar magnets. The field strength grows linearly with the radial distance from its longitudinal axis.
- A **solenoidal** magnetic field is similar to a dipole magnetic field, except that a solid bar magnet is replaced by a hollow electromagnetic coil magnet.
- A **toroidal** magnetic field occurs in a doughnut-shaped coil, the electric current spiraling around the tube-like surface, and is found, for example, in a tokamak.
- A **poloidal** magnetic field is generated by a current flowing in a ring, and is found, for example, in a tokamak.
- A **radial** magnetic field is one in which the field lines are directed from the center outwards, similar to the spokes in a bicycle wheel. An example can be found in a loudspeaker transducers (driver).[43]
- A **helical** magnetic field is corkscrew-shaped, and sometimes seen in space plasmas such as the Orion Molecular Cloud.[44]

Magnetic dipoles



Magnetic field lines around a "magnetostatic dipole" pointing to the right.

The magnetic field of a **magnetic dipole** is depicted on the right. From outside, the ideal magnetic dipole is identical to that of an ideal electric dipole of the same strength. Unlike the electric dipole, a magnetic dipole is properly modeled as a current loop having a current I and an area a. Such a current loop has a magnetic moment of:

$$m = Ia,$$

where the direction of $\boldsymbol{m}$ is perpendicular to the area of the loop and depends on the direction of the current using the right-hand rule. An ideal magnetic dipole is modeled as a real magnetic dipole whose area a has been reduced to zero and its current I increased to infinity such that the product $m = Ia$ is finite. In this model it is easy to see the connection between angular momentum and magnetic moment which is the basis of the Einstein-de Haas effect "rotation by magnetization" and its inverse, the Barnett effect or "magnetization by rotation".[45] Rotating the loop faster (in the same direction) increases the current and therefore the magnetic moment, for example.

It is sometimes useful to model the magnetic dipole similar to the electric dipole with two equal but opposite magnetic charges (one south the other north) separated by distance d. This model produces an $\boldsymbol{H}$-field not a $\boldsymbol{B}$-field. Such a model is deficient, though, both in that there are no magnetic charges and in that it obscures the link between electricity and magnetism. Further, as discussed above it fails to explain the inherent connection between angular momentum and magnetism.

Magnetic monopole (hypothetical)

A **magnetic monopole** is a hypothetical particle (or class of particles) that has, as its name suggests, only one magnetic pole (either a north pole or a south pole). In other words, it would possess a "magnetic charge" analogous to an electric charge. Magnetic field lines would start or end on magnetic monopoles, so if they exist, they would give exceptions to the rule that magnetic field lines neither start nor end.

Modern interest in this concept stems from particle theories, notably Grand Unified Theories and superstring theories, that predict either the existence, or the possibility, of magnetic monopoles. These theories and others have inspired extensive efforts to search for monopoles. Despite these efforts, no magnetic monopole has been observed to date.[46]

In recent research, materials known as spin ices can simulate monopoles, but do not contain actual monopoles.

Notes

[1] Technically, a magnetic field is a pseudo vector; pseudo-vectors, which also include torque and rotational velocity, are similar to vectors except that they remain unchanged when the coordinates are inverted.

[2] His *Epistola Petri Peregrini de Maricourt ad Sygerum de Foucaucourt Militem de Magnete*, which is often shortened to *Epistola de magnete*, is dated 1269 C.E.

[3] By the definition of magnetization, in this model, and in analogy to the physics of springs, the work done per unit volume, in stretching and twisting the *bonds* between *magnetic charge* to increment the magnetization by $\mu_0\delta\boldsymbol{M}$ is $W = \boldsymbol{H}\cdot\mu_0\delta\boldsymbol{M}$. In this model, $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ is an effective magnetization which includes the ***H***-field term to account for the energy of setting up the magnetic field in a vacuum. Therefore the total energy density increment needed to increment the magnetic field is $W = \boldsymbol{H}\cdot\delta\boldsymbol{B}$.

[4] It is a remarkable fact that from the 'outside' the field of a dipole of magnetic charge has the exact same form as that of an elementary current loop called a magnetic dipole. It is therefore only for the physics of magnetism 'inside' of magnetic material that the two models differ.

[5] Electromagnetics, by Rothwell and Cloud, p23 (http://books.google.com/books?id=jCqv1UygjA4C&pg=PA23)

[6] R.P. Feynman, R.B. Leighton, M. Sands (1963). *The Feynman Lectures on Physics, volume 2*.

[7] Edward Purcell, in Electricity and Magnetism, McGraw-Hill, 1963, writes, *Even some modern writers who treat **B** as the primary field feel obliged to call it the magnetic induction because the name magnetic field was historically preempted by **H**. This seems clumsy and pedantic. If you go into the laboratory and ask a physicist what causes the pion trajectories in his bubble chamber to curve, he'll probably answer "magnetic field", not "magnetic induction." You will seldom hear a geophysicist refer to the Earth's magnetic induction, or an astrophysicist talk about the magnetic induction of the galaxy. We propose to keep on calling **B** the magnetic field. As for **H**, although other names have been invented for it, we shall call it "the field **H**" or even "the magnetic field **H**."* In a similar vein, M Gerloch (1983). *Magnetism and Ligand-field Analysis* (http://books.google.com/?id=Ovo8AAAAIAAJ&pg=PA110). Cambridge University Press. p. 110. ISBN 0521249392. . says: "So we may think of both **B** and **H** as magnetic fields, but drop the word 'magnetic' from **H** so as to maintain the distinction ... As Purcell points out, 'it is only the names that give trouble, not the symbols'."

[8] This can be seen from the magnetic part of the Lorentz force law $\mathbf{F}_{mag} = (q\mathbf{vB})$.

[9] Magnetic Field Strength Converter (http://www.unitconversion.org/unit_converter/magnetic-field-strength.html), UnitConversion.org.

[10] "Gravity Probe B Executive Summary" (http://www.nasa.gov/pdf/168808main_gp-b_pfar_cvr-pref-execsum.pdf). pp. 10,21. .

[11] "With record magnetic fields to the 21st Century" (http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=823621). *IEEE Xplore*. .

[12] Kouveliotou, C.; Duncan, R. C.; Thompson, C. (February 2003). " Magnetars (http://solomon.as.utexas.edu/~duncan/sciam.pdf)". *Scientific American*; Page 36.

[13] The use of iron filings to display a field presents something of an exception to this picture; the filings alter the magnetic field so that it is much larger along the "lines" of iron, due to the large permeability of iron relative to air.

[14] Here 'small' means that the observer is sufficiently far away that it can be treated as being infinitesimally small. 'Larger' magnets need to include more complicated terms in the expression and depend on the entire geometry of the magnet not just **m**.

[15] Magnetic field lines may also wrap around and around without closing but also without ending. These more complicated non-closing non-ending magnetic field lines are moot, though, since the magnetic field of objects that produce them are calculated by adding the magnetic fields of 'elementary parts' having magnetic field lines that do form closed curves or extend to infinity.

[16] To see that this must be true imagine placing a compass inside a magnet. There, the north pole of the compass points toward the north pole of the magnet since magnets stacked on each other point in the same direction.

[17] As discussed above, magnetic field lines are primarily a conceptual tool used to represent the mathematics behind magnetic fields. The total 'number' of field lines is dependent on how the field lines are drawn. In practice, integral equations such as the one that follows in the main text are used instead.

[18] Either **B** or **H** may be used for the magnetic field outside of the magnet.

[19] See Eq. 11.42 in E. Richard Cohen, David R. Lide, George L. Trigg (2003). *AIP physics desk reference* (http://books.google.com/?id=JStYf6WlXpgC&pg=PA381) (3 ed.). Birkhäuser. p. 381. ISBN 0387989730. .

[20] Griffiths, David J. (1999). *Introduction to Electrodynamics* (3rd ed.). Prentice Hall. p. 438. ISBN 0-13-805326-X. OCLC 40251748.

[21] In practice, the Biot–Savart law and other laws of magnetostatics are often used even when the currents are changing in time as long as it is not changing too quickly. It is often used, for instance, for standard household currents which oscillate sixty times per second.

[22] Griffiths, David J. (1999). *Introduction to Electrodynamics* (3rd ed.). Prentice Hall. pp. 222–225. ISBN 0-13-805326-X. OCLC 40251748.

[23] The Biot–Savart law contains the additional restriction (boundary condition) that the B-field must go to zero fast enough at infinity. It also depends on the divergence of **B** being zero, which is always valid. (There are no magnetic charges.)

[24] Deissler, R.J. (2008). "Dipole in a magnetic field, work, and quantum spin" (http://academic.csuohio.edu/deissler/PhysRevE_77_036609.pdf). *Physical Review E* **77** (3, pt 2): 036609. Bibcode 2008PhRvE..77c6609D. doi:10.1103/PhysRevE.77.036609. PMID 18517545. .

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[26] John Clarke Slater, Nathaniel Herman Frank (1969). *Electromagnetism* (http://books.google.com/?id=GYsphnFwUuUC&pg=PA69) (first published in 1947 ed.). Courier Dover Publications. p. 69. ISBN 0486622630. .

[27] David Griffiths. *Introduction to Electrodynamics* (3rd 1999 ed.). p. 332.

[28] A third term is needed for changing electric fields and polarization currents; this displacement current term is covered in Maxwell's equations below.

[29] RJD Tilley (2004). *Understanding Solids* (http://books.google.com/?id=ZVgOLCXNoMoC&pg=PA368). Wiley. p. 368. ISBN 0470852755. .

[30] Sōshin Chikazumi, Chad D. Graham (1997). *Physics of ferromagnetism* (http://books.google.com/?id=AZVfuxXF2GsC&printsec=frontcover) (2 ed.). Oxford University Press. p. 118. ISBN 0198517769. .

[31] Amikam Aharoni (2000). *Introduction to the theory of ferromagnetism* (http://books.google.com/?id=9RvNuIDh0qMC&pg=PA27) (2 ed.). Oxford University Press. p. 27. ISBN 0198508085. .

[32] M Brian Maple *et al.* (2008). "Unconventional superconductivity in novel materials" (http://books.google.com/?id=PguAgEQTiQwC&pg=PA640). In K. H. Bennemann, John B. Ketterson. *Superconductivity*. Springer. p. 640. ISBN 3540732527. .

[33] Naoum Karchev (2003). "Itinerant ferromagnetism and superconductivity" (http://books.google.com/?id=3AFo_yxBkD0C&pg=PA169). In Paul S. Lewis, D. Di (CON) Castro. *Superconductivity research at the leading edge*. Nova Publishers. p. 169. ISBN 1590338618. .

[34] A complete expression for Faraday's law of induction in terms of the electric **E** and magnetic fields can be written as: $\mathcal{E} = -\frac{d\Phi_m}{dt} = \oint_{\partial\Sigma(t)} \left(\mathbf{E}(\mathbf{r},\ t) + \mathbf{v} \times \mathbf{B}(\mathbf{r},\ t)\right) \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \iint_{\Sigma(t)} d\boldsymbol{A} \cdot \mathbf{B}(\mathbf{r},\ t),$ where $\boldsymbol{\partial\Sigma}(t)$ is the moving closed path bounding the moving surface $\boldsymbol{\Sigma}(t)$, and d**A** is an element of surface area of $\boldsymbol{\Sigma}(t)$. The first integral calculates the work done moving a charge a distance d$\boldsymbol{\ell}$ based upon the Lorentz force law. In the case where the bounding surface is stationary, the Kelvin–Stokes theorem can be used to show this equation is equivalent to the Maxwell–Faraday equation.

[35] E. J. Konopinski (1978). "What the electromagnetic vector potential describes". *Am. J. Phys.* **46** (5): 499–502. Bibcode 1978AmJPh..46..499K. doi:10.1119/1.11298.

[36] Griffiths, David J. (1999). *Introduction to Electrodynamics (3rd ed.)*. Prentice Hall. p. 422. ISBN 0-13-805326-X. OCLC 40251748.

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[42] The Solar Dynamo (http://www.cora.nwra.com/~werne/eos/text/dynamo.html), retrieved Sep 15, 2007.

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[45] (See magnetic moment for further information.)

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Current density

Current density is a measure of the density of flow of a conserved charge. Usually the charge is the electric charge, in which case the associated current density is the electric current per unit area of cross section, but the term *current density* can also be applied to other conserved quantities. It is defined as a vector whose magnitude is the current per cross-sectional area.

In SI units, the electric current density is measured in amperes per square metre.

Definition

Electric current is a coarse, average quantity that tells what is happening in an entire wire. The *distribution* of flow of charge is described by the current density:

$$\mathbf{J}(\mathbf{r},t) = qn(\mathbf{r},t)\ \mathbf{v}_d(\mathbf{r},t) = \rho(\mathbf{r},t)\ \mathbf{v}_d(\mathbf{r},t)\,,$$

where

$\mathbf{J}(\mathbf{r}, t)$ is the current density vector at location $\mathbf{r}$ at time t (SI unit: amperes per square metre),

$n(\mathbf{r}, t)$ is the particle density in count per volume at location $\mathbf{r}$ at time t (SI unit: m^{-3}),

q is the charge of the individual particles with density n (SI unit: coulombs)

$\varrho(\mathbf{r}, t) = qn(\mathbf{r}, t)$ is the charge density (SI unit: coulombs per cubic metre), and

$\mathbf{v}_d(\mathbf{r}, t)$ is the particles' average drift velocity at position $\mathbf{r}$ at time t (SI unit: metres per second)

Importance

Current density is important to the design of electrical and electronic systems.

Circuit performance depends strongly upon the designed current level, and the current density then is determined by the dimensions of the conducting elements. For example, as integrated circuits are reduced in size, despite the lower current demanded by smaller devices, there is trend toward higher current densities to achieve higher device numbers in ever smaller chip areas. See Moore's law.

At high frequencies, current density can increase because the conducting region in a wire becomes confined near its surface, the so-called skin effect.

High current densities have undesirable consequences. Most electrical conductors have a finite, positive resistance, making them dissipate power in the form of heat. The current density must be kept sufficiently low to prevent the conductor from melting or burning up, or the insulating material failing. At high current densities the material forming the interconnections actually moves, a phenomenon called *electromigration*. In superconductors excessive current density may generate a strong enough magnetic field to cause spontaneous loss of the superconductive property.

The analysis and observation of current density also is used to probe the physics underlying the nature of solids, including not only metals, but also semiconductors and insulators. An elaborate theoretical formalism has developed to explain many fundamental observations.[1] [2]

The current density is an important parameter in Ampère's circuital law (one of Maxwell's equations), which relates current density to magnetic field.

In special relativity theory, charge and current are combined into a 4-vector.

Approximate calculation of current density

A common approximation to the current density assumes the current simply is proportional to the electric field, as expressed by:

$$\mathbf{J} = \sigma \mathbf{E}$$

where **E** is the electric field and σ is the electrical conductivity.

Conductivity σ is the reciprocal (inverse) of electrical resistivity and has the SI units of siemens per metre (S m^{-1}), and **E** has the SI units of newtons per coulomb (N C^{-1}) or, equivalently, volts per metre (V m^{-1}).

A more fundamental approach to calculation of current density is based upon:

$$\mathbf{J}(\mathbf{r}, t) = \int_{-\infty}^{t} \mathrm{d}t' \int \mathrm{d}^3\mathbf{r}' \, \sigma(\mathbf{r} - \mathbf{r}', t - t') \, \mathbf{E}(\mathbf{r}', \, t') \,,$$

indicating the lag in response by the time dependence of σ, and the non-local nature of response to the field by the spatial dependence of σ, both calculated in principle from an underlying microscopic analysis, for example, in the case of small enough fields, the linear response function for the conductive behavior in the material. See, for example, Giuliani or Rammer.[3] [4] The integral extends over the entire past history up to the present time.

As some reflection might indicate, the above conductivity and its associate current density reflect the fundamental mechanisms underlying charge transport in the medium, both in time and over distance.

A Fourier transform in space and time then results in:

$$\mathbf{J}(\mathbf{k}, \omega) = \sigma(\mathbf{k}, \omega) \, \mathbf{E}(\mathbf{k}, \omega) \,,$$

where $\sigma(\mathbf{k}, \omega)$ is now a complex function.

In many materials, for example, in crystalline materials, the conductivity is a tensor, and the current is not necessarily in the same direction as the applied field. Aside from the material properties themselves, the application of magnetic fields can alter conductive behavior.

Current through a surface

The current through a surface area S perpendicular to the flow can be calculated using a surface integral:

$$I = \int_S \mathbf{J} \cdot \mathrm{d}\mathbf{A}$$

where the current is in fact the integral of the dot product of the current density vector and the differential of the directed surface element d**A**, in other words, the net flux of the current density vector field flowing through the surface *S*.

Continuity equation

Because charge is conserved, the net flow out of a chosen volume must equal the net change in charge held inside the volume:

$$\int_S \mathbf{J} \cdot \mathrm{d}\mathbf{A} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho \, \mathrm{d}V = - \int_V \left(\frac{\partial \rho}{\partial t} \right) \mathrm{d}V \ ,$$

where ϱ is the charge density per unit volume, and dA is a surface element of the surface *S* enclosing the volume *V*. The surface integral on the left expresses the current *outflow* from the volume, and the negatively signed volume integral on the right expresses the *decrease* in the total charge inside the volume. From the divergence theorem,

$$\int_S \mathbf{J} \cdot \mathrm{d}\mathbf{A} = \int_V (\nabla \cdot \mathbf{J}) \mathrm{d}V \ .$$

Hence:

$$\int_V (\nabla \cdot \mathbf{J}) \mathrm{d}V \ = -\int_V \left(\frac{\partial \rho}{\partial t}\right) \mathrm{d}V \ .$$

Because this relation is valid for any volume, independent of size or location,

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \ .$$

This relation is called the continuity equation.[5] [6]

In practice

- In the domain of electrical wiring (isolated copper), maximum current density can vary from 4A/mm^2 for a wire isolated from free air to 6A/mm^2 for a wire in free air. However, for compact designs (e.g. windings of SMPS transformers) the value might be as low as 2A/mm^2.[7] If the wire is carrying high frequency currents, depending on its diameter, the skin effect may affect the distribution of the current across the section by concentrating the current on the surface of the conductor. This skin effect plays an important role in Switched-mode power supply transformers where the wires carries high currents and high frequencies (between 10 kHz and 1 MHz). Often in those transformers, the windings are made of multiple isolated wires in parallel with a diameter twice the skin depth and which are twisted together to increase the total skin area and to reduce the impact of the skin effect.
- In the domain of printed circuit boards, for TOP and BOTTOM layers, maximum current density can be as high as 35A/mm^2 with a copper thickness of 35 µm. Inner layers cannot dissipate as much power as outer layers; thus it is not a good idea to put high power lines in inner layers.
- In the domain of semiconductors, the maximum current density is given by the manufacturer. A common average is 1mA/µm^2 at 25°C for 180 nm technology. Above the maximum current density, apart from the joule effect, some other effects like electromigration appear in the micrometer scale.
- In biological systems, ion channels regulate the flow of ions (for example, sodium, calcium, potassium) across the membrane in all cells. Current density is measured in pA/pF (picoamperes per picofarad), that is, current divided by capacitance, a de facto measure of membrane area.
- In gas discharge lamps, such as flashlamps, current density plays an important role in the output spectrum produced. Low current densities produce spectral line emission and tend to favor longer wavelengths. High current densities produce continuum emission and tend to favor shorter wavelengths.[8] Low current densities for flash lamps are generally around 1000A/cm^2. High current densities can be more than 4000A/cm^2.

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[8] Xenon lamp photocathodes (https://kb.osu.edu/dspace/bitstream/1811/5654/1/V71N06_343.pdf)

External links

- A short explanation of the current density (http://maxwell.byu.edu/~spencerr/websumm122/node46.html)

Displacement current

In electromagnetism, **displacement current** is a quantity that is defined in terms of the rate of change of electric displacement field. Displacement current has the units of electric current density, and it has an associated magnetic field just as actual currents do. However it is not an electric current of moving charges, but a time-varying electric field. In materials, there is also a contribution from the slight motion of charges bound in atoms, dielectric polarization.

The idea was conceived by James Clerk Maxwell in his 1861 paper On Physical Lines of Force in connection with the displacement of electric particles in a dielectric medium. Maxwell added displacement current to the electric current term in Ampère's Circuital Law. In his 1865 paper A Dynamical Theory of the Electromagnetic Field Maxwell used this amended version of Ampère's Circuital Law to derive the electromagnetic wave equation. This derivation is now generally accepted as an historical landmark in physics by virtue of uniting electricity, magnetism and optics into one single unified theory. The displacement current term is now seen as a crucial addition that completed Maxwell's equations and is necessary to explain many phenomena, most particularly the existence of electromagnetic waves.

Explanation

The electric displacement field is defined as:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} \ .$$

where:

ε_0 is the permittivity of free space

E is the electric field intensity

P is the polarization of the medium

Differentiating this equation with respect to time defines the *displacement current*, which therefore has two components in a dielectric:[1]

$$\boldsymbol{J_D} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \frac{\partial \boldsymbol{P}}{\partial t} \ .$$

The first term on the right hand side is present in material media and in free space. It doesn't necessarily involve any actual movement of charge, but it does have an associated magnetic field, just as does a current due to charge motion. Some authors apply the name *displacement current* to only this contribution.[2]

The second term on the right hand side is associated with the polarization of the individual molecules of the dielectric material. Polarization results when the charges in molecules move a little under the influence of an applied electric field. The positive and negative charges in molecules separate, causing an increase in the state of polarization ***P***. A changing state of polarization corresponds to charge movement and so is equivalent to a current.

This polarization is the displacement current as it was originally conceived by Maxwell. Maxwell made no special treatment of the vacuum, treating it as a material medium. For Maxwell, the effect of ***P*** was simply to change the relative permittivity ε_r in the relation $\boldsymbol{D} = \varepsilon_r \varepsilon_0 \boldsymbol{E}$.

The modern justification of displacement current is explained below.

Isotropic dielectric case

In the case of a very simple dielectric material the constitutive relation holds:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \ ,$$

where the permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$,

- ε_r is the relative permittivity of the dielectric and
- ε_0 is the electric constant.

In this equation the use of ε, accounts for the polarization of the dielectric.

The scalar value of displacement current may also be expressed in terms of electric flux:

$$I_{\mathrm{D}} = \varepsilon \frac{\partial \Phi_E}{\partial t}.$$

The forms in terms of ε are correct only for linear isotropic materials. More generally ε may be replaced by a tensor, may depend upon the electric field itself, and may exhibit time dependence (dispersion).

For a linear isotropic dielectric, the polarization $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \varepsilon_0 \chi_e \boldsymbol{E} = \varepsilon_0 (\varepsilon_r - 1) \boldsymbol{E}$$

where χ_e is known as the electric susceptibility of the dielectric. Note that:

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi_e) \varepsilon_0.$$

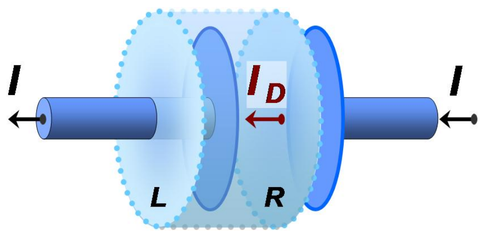
Necessity

Some implications of the displacement current follow, which agree with experimental observation, and with the requirements of logical consistency for the theory of electromagnetism.

Generalizing Ampère's circuital law

Current in capacitors

An example illustrating the need for the displacement current arises in connection with capacitors with no medium between the plates. Consider the charging capacitor in the figure. The capacitor is in a circuit that transfers charge (on a wire external to the capacitor) from the left plate to the right plate, charging the capacitor and increasing the electric field between its plates. The same current enters the right plate (say I) as leaves the left plate. Although current is flowing through the capacitor, no actual charge is transported through the vacuum between its plates. Nonetheless, a magnetic field exists between the plates as though a current were present there as well. The explanation is that a *displacement current* I_D flows in the vacuum, and this current produces the magnetic field in the region between the plates according to Ampère's law:[3] [4]



An electrically charging capacitor with an imaginary cylindrical surface surrounding the left-hand plate. Right-hand surface *R* lies in the space between the plates and left-hand surface *L* lies to the left of the left plate. No conduction current enters cylinder surface *R*, while current *I* leaves through surface *L*. Consistency of Ampère's law requires a displacement current $I_D = I$ to flow across surface *R*.

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I_D \ .$$

where

- $\oint_C$ is the closed line integral around some closed curve *C*.
- $\mathbf{B}$ is the magnetic field in tesla.
- $\cdot$ is the vector dot product.
- $\mathrm{d}\boldsymbol{\ell}$ is an infinitesimal element (differential) of the curve *C* (that is, a vector with magnitude equal to the length of the infinitesimal line element, and direction given by the tangent to the curve *C*).
- μ_0 is the magnetic constant also called the permeability of free space.
- I_D is the net displacement current that links the curve *C*.

The magnetic field between the plates is the same as that outside the plates, so the displacement current must be the same as the conduction current in the wires, that is,

$$I_D = I \ ,$$

which extends the notion of current beyond a mere transport of charge.

Next, this displacement current is related to the charging of the capacitor. Consider the current in the imaginary cylindrical surface shown surrounding the left plate. A current, say *I*, passes outward through the left surface *L* of the cylinder, but no conduction current (no transport of real charges) enters the right surface *R*. Notice that the electric field between the plates *E* increases as the capacitor charges. That is, in a manner described by Gauss's law, assuming no dielectric between the plates:

$$Q(t) = \varepsilon_0 \oint_S dS \cdot \boldsymbol{E}(t) \ ,$$

where *S* refers to the imaginary cylindrical surface. Assuming a parallel plate capacitor with uniform electric field, and neglecting fringing effects around the edges of the plates, differentiation provides:[3]

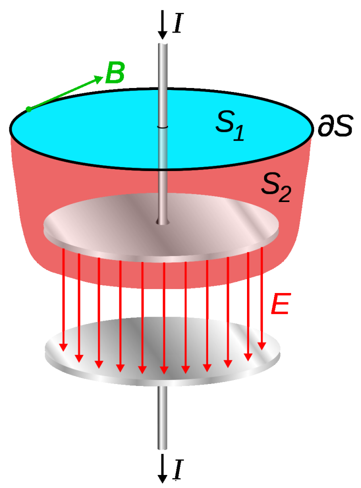
$$\frac{dQ}{dt} = I = \varepsilon_0 \oint_S dS \cdot \frac{\partial \boldsymbol{E}}{\partial t} \approx -S \ \varepsilon_0 \frac{\partial E}{\partial t} \ ,$$

where the sign is negative because charge leaves this plate (the charge is decreasing), and where *S* is the area of the face *R*. The electric field at face *L* is zero because the field due to charge on the right-hand plate is terminated by the equal but opposite charge on the left-hand plate. Under the assumption of a uniform electric field distribution inside the capacitor, the displacement current density J_D is found by dividing by the area of the surface:

$$J_D = \frac{I_D}{S} = -\frac{I}{S} = \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\partial D}{\partial t} \ ,$$

where I is the current leaving the cylindrical surface (which must equal $-I_D$ as the two currents sum to zero) and J_D is the flow of charge per unit area into the cylindrical surface through the face R.

Combining these results, the magnetic field is found using the integral form of Ampère's law with an arbitrary choice of contour provided the displacement current density term is added to the conduction current density (the Ampère-Maxwell equation):[5]



Example showing two surfaces S_1 and S_2 that share the same bounding contour ∂S. However, S_1 is pierced by conduction current, while S_2 is pierced by displacement current.

$$\oint_{\partial S} \boldsymbol{B} \cdot d\boldsymbol{\ell} = \mu_0 \int_S (\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}) \cdot d\boldsymbol{S}$$

This equation says that the integral of the magnetic field $\boldsymbol{B}$ around a loop ∂S is equal to the integrated current $\boldsymbol{J}$ through any surface spanning the loop, plus the displacement current term $\varepsilon_0\, \partial \boldsymbol{E} / \partial t$ through the surface. Applying the Ampère-Maxwell equation to surface S_1 we find:

$$B = \frac{\mu_0 I}{2\pi r}$$

However, applying this law to surface S_2, which is bounded by exactly the same curve ∂S, but lies between the plates, provides:

$$B = \frac{\mu_0 I_D}{2\pi r}$$

Any surface that intersects the wire has current I passing through it so Ampère's law gives the correct magnetic field. Also, any surface bounded by the same loop but passing between the capacitor's plates has no charge transport flowing through it, but the $\varepsilon_0\, \partial \boldsymbol{E} / \partial t$ term provides a second source for the magnetic field besides charge conduction current. Because the current is increasing the charge on the capacitor's plates, the electric field between the plates is increasing, and the rate of change of electric field gives the correct value for the field $\boldsymbol{B}$ found above.

Mathematical formulation

In a more mathematical vein, the same results can be obtained from the underlying differential equations. Consider for simplicity a non-magnetic medium where the relative magnetic permeability is unity, and the complication of magnetization current is absent. The current leaving a volume must equal the rate of decrease of charge in a volume. In differential form this continuity equation becomes:

$$\nabla\cdot\boldsymbol{J_f} = -\frac{\partial\rho_f}{\partial t} \ ,$$

where the left side is the divergence of the free current density and the right side is the rate of decrease of the free charge density. However, Ampère's law in its original form states:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J}_f \ ,$$

which implies that the divergence of the current term vanishes, contradicting the continuity equation. (Vanishing of the *divergence* is a result of the mathematical identity that states the divergence of a *curl* is always zero.) This conflict is removed by addition of the displacement current, as then:[6] [7]

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right) = \mu_0 \left(\boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t} \right) \ ,$$

and

$$\boldsymbol{\nabla}\cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) = 0 = \mu_0 \left(\nabla \cdot \boldsymbol{J}_f + \frac{\partial}{\partial t} \boldsymbol{\nabla} \cdot \boldsymbol{D} \right) \ ,$$

which is in agreement with the continuity equation because of Gauss's law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_f \ .$$

Wave propagation

The added displacement current also leads to wave propagation by taking the curl of the equation for magnetic field.[8]

$$\boldsymbol{J_D} = \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

Substituting this form for $\boldsymbol{J}$ into Ampère's law, and assuming there is no bound or free current density contributing to $\boldsymbol{J}$:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J_D} \ ,$$

with the result:

$$\boldsymbol{\nabla}\times (\boldsymbol{\nabla} \times \boldsymbol{B}) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \boldsymbol{\nabla} \times \boldsymbol{E} \ .$$

However,

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial}{\partial t} \boldsymbol{B} \ ,$$

leading to the wave equation:[9]

$$-\boldsymbol{\nabla}\times (\boldsymbol{\nabla} \times \boldsymbol{B}) = \nabla^2 \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \boldsymbol{B} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \boldsymbol{B} \ ,$$

where use is made of the vector identity that holds for any vector field $\boldsymbol{V}(\boldsymbol{r}, t)$:

$$\boldsymbol{\nabla}\times (\boldsymbol{\nabla} \times \boldsymbol{V}) = \boldsymbol{\nabla} (\boldsymbol{\nabla} \cdot \boldsymbol{V}) - \nabla^2 \boldsymbol{V} \ ,$$

and the fact that the divergence of the magnetic field is zero. An identical wave equation can be found for the electric field by taking the *curl*:

$$\boldsymbol{\nabla}\times (\boldsymbol{\nabla} \times \boldsymbol{E}) = -\frac{\partial}{\partial t} \boldsymbol{\nabla}\times \boldsymbol{B} = -\mu_0 \frac{\partial}{\partial t} \left(\boldsymbol{J} + \epsilon_0 \frac{\partial}{\partial t} \boldsymbol{E} \right) \ .$$

If ***J***, ***P*** and ϱ are zero, the result is:

$$\nabla^2 \boldsymbol{E} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \boldsymbol{E} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \boldsymbol{E} \ .$$

The electric field can be expressed in the general form:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi - \frac{\partial \boldsymbol{A}}{\partial t} \ ,$$

where φ is the electric potential (which can be chosen to satisfy Poisson's equation) and $\boldsymbol{A}$ is a vector potential. The $\boldsymbol{\nabla}\varphi$ component on the right hand side is the Gauss's law component, and this is the component that is relevant to the conservation of charge argument above. The second term on the right-hand side is the one relevant to the electromagnetic wave equation, because it is the term that contributes to the *curl* of $\boldsymbol{E}$. Because of the vector identity that says the *curl* of a *gradient* is zero, $\boldsymbol{\nabla}\varphi$ does not contribute to $\boldsymbol{\nabla}\times\boldsymbol{E}$.

History and interpretation

Maxwell's displacement current was postulated in part III of his 1861 paper 'On Physical Lines of Force'. Few topics in modern physics have caused as much confusion and misunderstanding as that of displacement current.[10] This is in part due to the fact that Maxwell used a sea of molecular vortices in his derivation, while modern textbooks operate on the basis that displacement current can exist in free space. Maxwell's derivation is unrelated to the modern day derivation for displacement current in the vacuum, which is based on consistency between Ampère's law for the magnetic field and the continuity equation for electric charge.

Maxwell's purpose is stated by him at (Part I, p. 161):

> I propose now to examine magnetic phenomena from a mechanical point of view, and to determine what tensions in, or motions of, a medium are capable of producing the mechanical phenomena observed.

He is careful to point out the treatment is one of analogy:

> The author of this method of representation does not attempt to explain the origin of the observed forces by the effects due to these strains in the elastic solid, but makes use of the mathematical analogies of the two problems to assist the imagination in the study of both.

In part III, in relation to displacement current, he says

> I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another.

Clearly Maxwell was driving at magnetization even though the same introduction clearly talks about dielectric polarization.

Maxwell concluded, using Newton's equation for the speed of sound (*Lines of Force*, Part III, equation (132)), that “light consists of transverse undulations in the same medium that is the cause of electric and magnetic phenomena.”

But although the above quotations point towards a magnetic explanation for displacement current, for example, based upon the divergence of the above *curl* equation, Maxwell's explanation ultimately stressed linear polarization of dielectrics:

> This displacement...is the commencement of a current...The amount of displacement depends on the nature of the body, and on the electromotive force so that if *h* is the displacement, *R* the electromotive force, and *E* a coefficient depending on the nature of the dielectric:
>
> $$R = -4\pi \mathrm{E}^2 h \ ;$$
>
> and if *r* is the value of the electric current due to displacement

$$r = \frac{dh}{dt} \ ,$$

These relations are independent of any theory about the mechanism of dielectrics; but when we find electromotive force producing electric displacement in a dielectric, and when we find the dielectric recovering from its state of electric displacement...we cannot help regarding the phenomena as those of an elastic body, yielding to a pressure and recovering its form when the pressure is removed.—Part III – *The theory of molecular vortices applied to statical electricity* , pp. 14–15

With some change of symbols (and units): $r \rightarrow J$, $R \rightarrow -E$ and the material constant $\mathrm{E}^{-2} \rightarrow 4\pi\ \varepsilon_r \varepsilon_0$ these equations take the familiar form:

$$J = \frac{d}{dt}\frac{1}{4\pi \mathrm{E}^2}E = \frac{d}{dt}\varepsilon_r\varepsilon_0 E = \frac{d}{dt}D \ .$$

When it came to deriving the electromagnetic wave equation from displacement current in his 1865 paper A Dynamical Theory of the Electromagnetic Field, he got around the problem of the non-zero divergence associated with Gauss's law and dielectric displacement by eliminating the Gauss term and deriving the wave equation exclusively for the solenoidal magnetic field vector.

Maxwell's emphasis on polarization diverted attention towards the electric capacitor circuit, and led to the common belief that Maxwell conceived of displacement current so as to maintain conservation of charge in an electric capacitor circuit. There are a variety of debatable notions about Maxwell's thinking, ranging from his supposed desire to perfect the symmetry of the field equations to the desire to achieve compatibility with the continuity equation.[11] [12]

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Maxwell's papers

- *On Faraday's Lines of Force* (http://blazelabs.com/On Faraday's Lines of Force.pdf) Maxwell's paper of 1855
- *On Physical Lines of Force* Maxwell's paper of 1861
- *A Dynamical Theory of the Electromagnetic Field* Maxwell's paper of 1864

Further reading

- AM Bork (http://dx.doi.org/10.1119/1.1969140) *Maxwell, Displacement Current, and Symmetry* (1963)
- AM Bork (http://dx.doi.org/10.1119/1.1974263) *Maxwell and the Electromagnetic Wave Equation* (1967)

Electric charge

electric charge	
SI symbol:	*Q*
SI quantity dimension:	**Q**
SI unit:	coulomb
other units:	e
Derivations from other quantities:	$Q = I \cdot t$

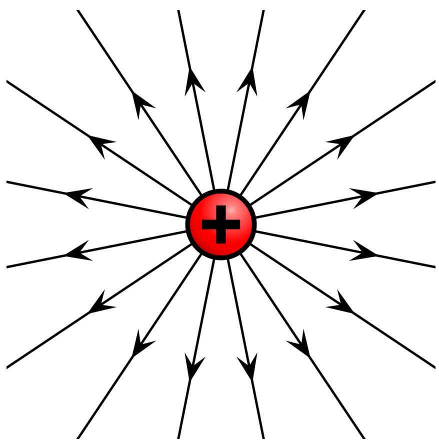
Electric charge is a physical property of matter that causes it to experience a force when near other electrically charged matter. Electric charge comes in two types, called positive and negative. Two positively charged substances, or objects, experience a mutual repulsive force, as do two negatively charged objects. Positively charged objects and negatively charged objects experience an attractive force. The SI unit of electric charge is the coulomb (C), although in electrical engineering it is also common to use the ampere-hour (Ah). The study of how charged substances interact is classical electrodynamics, which is accurate insofar as quantum effects can be ignored.

The *electric charge* is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction. Electrically charged matter is influenced by, and produces, electromagnetic fields. The interaction between a moving charge and an electromagnetic field is the source of the electromagnetic force, which is one of the four fundamental forces (See also: magnetic field).

Twentieth-century experiments demonstrated that electric charge is *quantized*; that is, it comes in multiples of individual small units called the elementary charge, e, approximately equal to 1.602×10^{-19} coulombs (except for particles called quarks, which have charges that are multiples of ⅓e). The proton has a charge of e, and the electron has a charge of $-e$. The study of charged particles, and how their interactions are mediated by photons, is quantum electrodynamics.

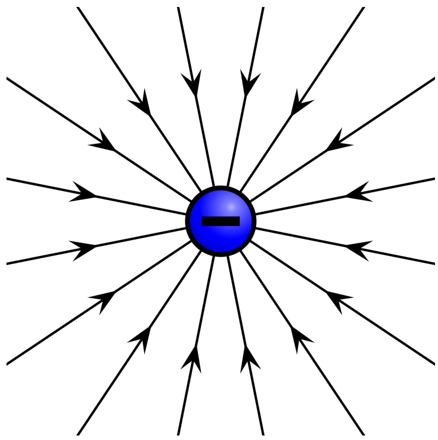
Overview

Charge is the fundamental property of forms of matter that exhibit electrostatic attraction or repulsion in the presence of other matter. Electric charge is a characteristic property of many subatomic particles. The charges of free-standing particles are integer multiples of the elementary charge e; we say that electric charge is *quantized*. Michael Faraday, in his electrolysis experiments, was the first to note the discrete nature of electric charge. Robert Millikan's oil-drop experiment demonstrated this fact directly, and measured the elementary charge.



Electric field induced by a positive electric charge

By convention, the charge of an electron is −1, while that of a proton is +1. Charged particles whose charges have the same sign repel one another, and particles whose charges have different signs attract. Coulomb's law quantifies the electrostatic force between two particles by asserting that the force is proportional to the product of their charges, and inversely proportional to the square of the distance between them.


Electric field induced by a negative electric charge

The charge of an antiparticle equals that of the corresponding particle, but with opposite sign. Quarks have fractional charges of either $-^1/_3$ or $+^2/_3$, but free-standing quarks have never been observed (the theoretical reason for this fact is asymptotic freedom).

The electric charge of a macroscopic object is the sum of the electric charges of the particles that make it up. This charge is often small, because matter is made of atoms, and atoms typically have equal numbers of protons and electrons, in which case their charges cancel out, yielding a net charge of zero, thus making the atom neutral.

An *ion* is an atom (or group of atoms) that has lost one or more electrons, giving it a net positive charge (cation), or that has gained one or more electrons, giving it a net negative charge (anion). *Monatomic ions* are formed from single atoms, while *polyatomic ions* are formed from two or more atoms that have been bonded together, in each case yielding an ion with a positive or negative net charge.

During the formation of macroscopic objects, usually the constituent atoms and ions will combine in such a manner that they form structures composed of neutral *ionic compounds* electrically bound to neutral atoms. Thus macroscopic objects tend toward being neutral overall, but macroscopic objects are rarely perfectly net neutral.

There are times when macroscopic objects contain ions distributed throughout the material, rigidly bound in place, giving an overall net positive or negative charge to the object. Also, macroscopic objects made of conductive elements, can more or less easily (depending on the element) take on or give off electrons, and then maintain a net negative or positive charge indefinitely. When the net electric charge of an object is non-zero and motionless, the phenomenon is known as static electricity. This can easily be produced by rubbing two dissimilar materials together, such as rubbing amber with fur or glass with silk. In this way non-conductive materials can be charged to a significant degree, either positively or negatively. Of course, charge taken from one material is simply moved to the other material, leaving an opposite charge of the same magnitude behind. The law of *conservation of charge* always applies, giving the object from which a negative charge has been taken a positive charge of the same magnitude, and vice-versa.

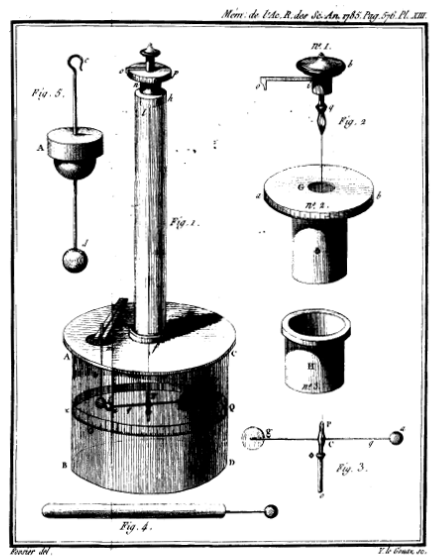
Even when an object's net charge is zero, charge can be distributed non-uniformly in the object (e.g., due to an external electromagnetic field, or bound polar molecules). In such cases the object is said to be polarized. The charge due to polarization is known as bound charge, while charge on an object produced by electrons gained or lost from outside the object is called *free charge*. The motion of electrons in conductive metals in a specific direction is known as electric current.

Units

The SI unit of quantity of electric charge is the coulomb, which is equivalent to about 6.242×10^{18} *e* (*e* is the charge of a proton). Hence, the charge of an electron is approximately -1.602×10^{-19} C. The coulomb is defined as the quantity of charge that has passed through the cross section of an electrical conductor carrying one ampere within one second. The symbol *Q* is often used to denote a quantity of electricity or charge. The quantity of electric charge can be directly measured with an electrometer, or indirectly measured with a ballistic galvanometer.

After finding the quantized character of charge, in 1891 George Stoney proposed the unit 'electron' for this fundamental unit of electrical charge. This was before the discovery of the particle by J.J. Thomson in 1897. The unit is today treated as nameless, referred to as "elementary charge", "fundamental unit of charge", or simply as "e". A measure of charge should be a multiple of the elementary charge *e*, even if at large scales, charge seems to behave as a real quantity. In some contexts it is meaningful to speak of fractions of a charge; for example in the charging of a capacitor, or in the fractional quantum Hall effect.

History



Coulomb's torsion balance

As reported by the ancient Greek philosopher Thales of Miletus around 600 BC, charge (or *electricity*) could be accumulated by rubbing fur on various substances, such as amber. The Greeks noted that the charged amber buttons could attract light objects such as hair. They also noted that if they rubbed the amber for long enough, they could even get an electric spark to jump. This property derives from the triboelectric effect.

In 1600, the English scientist William Gilbert returned to the subject in *De Magnete*, and coined the New Latin word *electricus* from *ηλεκτϱον* (*elektron*), the Greek word for "amber", which soon gave rise to the English words "electric" and "electricity." He was followed in 1660 by Otto von Guericke, who invented what was probably the first electrostatic generator. Other European pioneers were Robert Boyle, who in 1675 stated that electric attraction and repulsion can act across a vacuum; Stephen Gray, who in 1729 classified materials as conductors and insulators; and C. F. du Fay, who proposed in 1733[1] that electricity comes in two varieties that cancel each other, and expressed this in terms of a two-fluid theory. When glass was rubbed with silk, du Fay said that the glass was charged with *vitreous electricity*, and, when amber was rubbed with fur, the amber was said to be charged with *resinous electricity*. In 1839, Michael Faraday showed that the apparent division between static electricity, current electricity, and bioelectricity was incorrect, and all were a consequence of the behavior of a single kind of electricity appearing in opposite polarities. It is arbitrary which polarity is called positive and which is called negative. Positive charge can be defined as the charge left on a glass rod after being rubbed with silk.[2]

One of the foremost experts on electricity in the 18th century was Benjamin Franklin, who argued in favour of a one-fluid theory of electricity. Franklin imagined electricity as being a type of invisible fluid present in all matter; for example, he believed that it was the glass in a Leyden jar that held the accumulated charge. He posited that rubbing insulating surfaces together caused this fluid to change location, and that a flow of this fluid constitutes an electric current. He also posited that when matter contained too little of the fluid it was "negatively" charged, and when it had an excess it was "positively" charged. For a reason that was not recorded, he identified the term "positive" with vitreous electricity and "negative" with resinous electricity. William Watson arrived at the same explanation at about the same time.

Static electricity and electric current

Static electricity and electric current are two separate phenomena, both involving electric charge, and may occur simultaneously in the same object. Static electricity is a reference to the electric charge of an object and the related electrostatic discharge when two objects are brought together that are not at equilibrium. An electrostatic discharge creates a change in the charge of each of the two objects. In contrast, electric current is the flow of electric charge through an object, which produces no net loss or gain of electric charge. Although charge flows between two objects during an electrostatic discharge, time is too short for current to be maintained.

Electrification by friction

Further information: triboelectric effect

Experiment I

Let a piece of glass and a piece of resin, neither of which exhibiting any electrical properties, be rubbed together and left with the rubbed surfaces in contact. They will still exhibit no electrical properties. Let them be separated. They will now attract each other.

If a second piece of glass be rubbed with a second piece of resin, and if the piece be then separated and suspended in the neighbourhood of the former pieces of glass and resin, it may be observed:

1. that the two pieces of glass repel each other.
2. that each piece of glass attracts each piece of resin.
3. that the two pieces of resin repel each other.

These phenomena of attraction and repulsion are called electrical phenomena, and the bodies that exhibit them are said to be 'electrified', or to be 'charged with electricity'.

Bodies may be electrified in many other ways, as well as by friction.

The electrical properties of the two pieces of glass are similar to each other but opposite to those of the two pieces of resin: The glass attracts what the resin repels and repels what the resin attracts.

If a body electrified in any manner whatever behaves as the glass does, that is, if it repels the glass and attracts the resin, the body is said to be 'vitreously' electrified, and if it attracts the glass and repels the resin it is said to be 'resinously' electrified. All electrified bodies are found to be either vitreously or resinously electrified.

It is the established convention of the scientific community to define the vitreous electrification as positive, and the resinous electrification as negative. The exactly opposite properties of the two kinds of electrification justify our indicating them by opposite signs, but the application of the positive sign to one rather than to the other kind must be considered as a matter of arbitrary convention, just as it is a matter of convention in mathematical diagram to reckon positive distances towards the right hand.

No force, either of attraction or of repulsion, can be observed between an electrified body and a body not electrified.[3]

We now know that the Franklin/Watson model was fundamentally correct. There is only one kind of electrical charge, and only one variable is required to keep track of the amount of charge.[4] On the other hand, just knowing the charge is not a complete description of the situation. Matter is composed of several kinds of electrically charged particles, and these particles have many properties, not just charge.

The most common charge carriers are the positively charged proton and the negatively charged electron. The movement of any of these charged particles constitutes an electric current. In many situations, it suffices to speak of the *conventional current* without regard to whether it is carried by positive charges moving in the direction of the conventional current and/or by negative charges moving in the opposite direction. This macroscopic viewpoint is an approximation that simplifies electromagnetic concepts and calculations.

At the opposite extreme, if one looks at the microscopic situation, one sees there are many ways of carrying an electric current, including: a flow of electrons; a flow of electron "holes" that act like positive particles; and both negative and positive particles (ions or other charged particles) flowing in opposite directions in an electrolytic solution or a plasma).

Beware that, in the common and important case of metallic wires, the direction of the conventional current is opposite to the drift velocity of the actual charge carriers, i.e., the electrons. This is a source of confusion for beginners.

Properties

Aside from the properties described in articles about electromagnetism, charge is a relativistic invariant. This means that any particle that has charge Q, no matter how fast it goes, always has charge Q. This property has been experimentally verified by showing that the charge of *one* helium nucleus (two protons and two neutrons bound together in a nucleus and moving around at high speeds) is the same as *two* deuterium nuclei (one proton and one neutron bound together, but moving much more slowly than they would if they were in a helium nucleus).

Conservation of electric charge

The total electric charge of an isolated system remains constant regardless of changes within the system itself. This law is inherent to all processes known to physics and can be derived in a local form from gauge invariance of the wave function. The conservation of charge results in the charge-current continuity equation. More generally, the net change in charge density ϱ within a volume of integration V is equal to the area integral over the current density $\mathbf{J}$ through the closed surface $S = \partial V$, which is in turn equal to the net current I:

$$-\frac{d}{dt}\int_V \rho \, \mathrm{d}V = \oiint_{\partial V} \mathbf{J} \cdot \mathrm{d}\mathbf{S} = \int J dS \cos\theta = I.$$

Thus, the conservation of electric charge, as expressed by the continuity equation, gives the result:

$$I = \frac{dQ}{dt}.$$

The charge transferred between times t_i and t_f is obtained by integrating both sides:

$$Q = \int_{t_\mathrm{i}}^{t_\mathrm{f}} I \, \mathrm{d}t$$

where I is the net outward current through a closed surface and Q is the electric charge contained within the volume defined by the surface.

References

[1] Two Kinds of Electrical Fluid: Vitreous and Resinous - 1733 (http://www.sparkmuseum.com/BOOK_DUFAY.HTM)

[2] Electromagnetic Fields (2nd Edition), Roald K. Wangsness, Wiley, 1986. ISBN 0-471-81186-6 (intermediate level textbook)

[3] James Clerk Maxwell *A Treatise on Electricity and Magnetism*, pp. 32-33, Dover Publications Inc., 1954 ASIN: B000HFDK0K, 3rd ed. of 1891

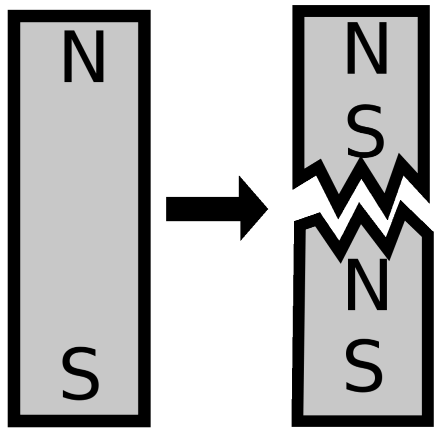
[4] One Kind of Charge (http://www.av8n.com/physics/one-kind-of-charge.htm)

External links

- How fast does a charge decay? (http://www.ce-mag.com/archive/2000/marapril/mrstatic.html)
- Science Aid: Electrostatic charge (http://www.scienceaid.co.uk/physics/electricity/charge.html) Easy-to-understand page on electrostatic charge.

Magnetic charge

A **magnetic monopole** is a hypothetical particle in particle physics that is a magnet with only one magnetic pole (a north pole without a south pole or vice-versa).[1] [2] In more technical terms, a magnetic monopole would have a net "magnetic charge". Modern interest in the concept stems from particle theories, notably the grand unified and superstring theories, which predict their existence.[3] [4] Magnetism in bar magnets and electromagnets does not arise from magnetic monopoles, and in fact there is no conclusive experimental evidence that magnetic monopoles exist at all in the universe.



It is impossible to make **magnetic monopoles** from a bar magnet. If a bar magnet is cut in half, it is *not* the case that one half has the north pole and the other half has the south pole. Instead, each piece has its own north and south poles. A magnetic monopole cannot be created from normal matter such as atoms and electrons, but would instead be a new elementary particle.

Many early scientists attributed the magnetism of lodestones to two different "magnetic fluids" ("effluvia"), a north-pole fluid at one end and a south-pole fluid at the other, which attracted and repelled each other in analogy to positive and negative electric charge.[5] [6] However, an improved understanding of electromagnetism in the nineteenth century showed that the magnetism of lodestones was caused by something else, not magnetic monopole fluids. It was concluded that magnetic monopoles did not exist: One of Maxwell's equations, now called Gauss's law for magnetism, is the mathematical statement that there are no magnetic monopoles. Nevertheless, it was pointed out by Pierre Curie in 1894[7] that magnetic monopoles *could* conceivably exist, despite not having been seen so far.

The *quantum* theory of magnetic charge started with a paper by the physicist Paul A.M. Dirac in 1931.[8] In this paper, Dirac showed that if *any* magnetic monopoles exist in the universe, then all electric charge in the universe must be quantized.[9] The electric charge *is*, in fact, quantized, which suggests (but does not necessarily prove) that monopoles exist.[9]

Since Dirac's paper, several systematic monopole searches have been performed. Experiments in 1975[10] and 1982[11] produced candidate events that were initially interpreted as monopoles, but are now regarded as inconclusive.[12] Therefore, it remains an open question whether or not monopoles exist.

Further advances in theoretical particle physics, particularly developments in grand unified theories and quantum gravity, have led to more compelling arguments that monopoles do exist. Joseph Polchinski, a prominent string-theorist, described the existence of monopoles as "one of the safest bets that one can make about physics not yet seen".[13] These theories are not necessarily inconsistent with the experimental evidence. In some theoretical models, magnetic monopoles are unlikely to be observed, because they are too massive to be created in particle accelerators, and also too rare in the Universe to enter a particle detector with much probability.[13]

Some condensed matter systems propose a structure superficially similar to a magnetic monopole, known as a flux tube. The ends of a flux tube form a magnetic dipole, but since they move independently, they can be treated for many purposes as independent magnetic monopole quasiparticles. Since 2009, numerous news reports from the popular media have incorrectly described these systems as the long-awaited discovery of the magnetic monopoles, but the two phenomena are only superficially related to one another.[14] These condensed-matter systems continue to be an area of active research. (See "Monopoles" in condensed-matter systems below.)

Background

Magnets exert forces on one another, similar to the force associated with electric charges. *Like* poles will repel each other, and *unlike* poles will attract. When any magnet (an object conventionally described as having magnetic north and south poles) is cut in half across the axis joining those "poles", the resulting pieces are two normal (albeit smaller) magnets. Each has its own north pole and south pole.

Even atoms and subatomic particles have tiny magnetic fields. In the Bohr model of an atom, electrons orbit the nucleus. Their constant motion gives rise to a magnetic field. Permanent magnets have measurable magnetic fields because the atoms and molecules in them are arranged in such a way that their individual magnetic fields align, combining to form large aggregate fields. In this model, the lack of a single pole makes intuitive sense: cutting a bar magnet in half does nothing to the arrangement of the molecules within. The end result is two bar magnetics whose atoms have the same orientation as before, and therefore generate a magnetic field with the same orientation as the original larger magnet.

Maxwell's equations

Maxwell's equations of electromagnetism relate the electric and magnetic fields to each other and to the motions of electric charges. The standard equations provide for electric charges, but they posit no magnetic charges. Except for this difference, the equations are symmetric under the interchange of the electric and magnetic fields.[15] In fact, symmetric Maxwell's equations can be written when all charges (and hence electric currents) are zero, and this is how the electromagnetic wave equation is derived.

Fully symmetric Maxwell's equations can also be written if one allows for the possibility of "magnetic charges" analogous to electric charges.[16] With the inclusion of a variable for the density of these magnetic charges, say ϱ_m, there will also be a "magnetic current density" variable in the equations, $\mathbf{j}_m$.

If magnetic charges do not exist - or if they do exist but are not present in a region of space - then the new terms in Maxwell's equations are all zero, and the extended equations reduce to the conventional equations of electromagnetism such as $\nabla\cdot\mathbf{B} = 0$ (where ∇ is divergence and $\mathbf{B}$ is the magnetic $\mathbf{B}$ field).

For a long time, the open question has been "*Why does the magnetic charge always seem to be zero?*"

In cgs units

The extended Maxwell's equations are as follows, in cgs units:[17]

Maxwell's equations in cgs

Name	Without magnetic monopoles	With magnetic monopoles
Gauss's law:	$\nabla \cdot \mathbf{E} = 4\pi\rho_e$	$\nabla \cdot \mathbf{E} = 4\pi\rho_e$
Gauss's law for magnetism:	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 4\pi\rho_m$
Faraday's law of induction:	$-\nabla \times \mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$-\nabla \times \mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_m$
Ampère's law (with Maxwell's extension):	$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_e$	$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_e$
ρ_m and j_m are defined above. For all other definitions and details, see Maxwell's equations article.		
Note: For the equations in nondimensionalized form, remove the factors of *c*.		

The equally-important Lorentz force equation becomes[17] [18]

$$\mathbf{F} = q_e\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) + q_m\left(\mathbf{B} - \frac{\mathbf{v}}{c} \times \mathbf{E}\right).$$

In these equations ϱ_m is the magnetic charge density, $\mathbf{j}_m$ is the magnetic current density, and q_m is the magnetic charge of a test particle, all defined analogously to the related quantities of electric charge and current; **v** is the particle's velocity and *c* is the speed of light.

In SI units

In SI units, there are two conflicting conventions in use for magnetic charge. In one, magnetic charge has units of webers, while in the other, magnetic charge has units of ampere-meters. Maxwell's equations then take the following forms:[19]

Maxwell's equations and Lorentz force equation with magnetic monopoles: SI units

Name	Without magnetic monopoles	Weber convention	Ampere·meter convention
Gauss's Law	$\nabla \cdot \mathbf{E} = \rho_e/\epsilon_0$	$\nabla \cdot \mathbf{E} = \rho_e/\epsilon_0$	$\nabla \cdot \mathbf{E} = \rho_e/\epsilon_0$
Gauss's Law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = \rho_m$	$\nabla \cdot \mathbf{B} = \mu_0\rho_m$
Faraday's Law of induction	$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$	$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{j}_m$	$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mu_0\mathbf{j}_m$
Ampère's Law	$\nabla \times \mathbf{B} = \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t} + \mu_0\mathbf{j}_e$	$\nabla \times \mathbf{B} = \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t} + \mu_0\mathbf{j}_e$	$\nabla \times \mathbf{B} = \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t} + \mu_0\mathbf{j}_e$
Lorentz force equation	$\mathbf{F} = q_e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$	$\mathbf{F} = q_e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \frac{q_m}{\mu_0}\left(\mathbf{B} - \mathbf{v} \times (\mathbf{E}/c^2)\right)$	$\mathbf{F} = q_e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + q_m\left(\mathbf{B} - \mathbf{v} \times (\mathbf{E}/c^2)\right)$

In these equations ϱ_m is the magnetic charge density, $\mathbf{j}_m$ is the magnetic current density, and q_m is the magnetic charge of a test particle, all defined analogously to the related quantities of electric charge and current.

Dirac's quantization

One of the defining advances in quantum theory was Paul Dirac's work on developing a relativistic quantum electromagnetism. Before his formulation, the presence of electric charge was simply "inserted" into the equations of quantum mechanics (QM), but in 1931 Dirac showed that a discrete charge naturally "falls out" of QM. That is to say, we can maintain the form of Maxwell's equations and still have magnetic charges.

Consider a system consisting of a single stationary electric monopole (an electron, say) and a single stationary magnetic monopole. Classically, the electromagnetic field surrounding them has a momentum density given by the Poynting vector, and it also has a total angular momentum, which is proportional to the product $q_e q_m$, and independent of the distance between them.

Quantum mechanics dictates, however, that angular momentum is quantized in units of $\hbar$, so therefore. the product $q_e q_m$ must also be quantized. This means that if even a single magnetic monopole existed in the universe, and the form of Maxwell's equations is valid, all electric charges would then be quantized.

What are the units in which magnetic charge would be quantized? Although it would be possible simply to integrate over all space to find the total angular momentum in the above example, Dirac took a different approach. This led him to new ideas. He considered a point-like magnetic charge whose magnetic field behaves as q_m / r^2 and is directed in the radial direction. Because the divergence of B is equal to zero almost everywhere, except for the locus of the magnetic monopole at $r = 0$, one can locally define the vector potential such that the curl of the vector potential A equals the magnetic field B.

However, the vector potential cannot be defined globally precisely because the divergence of the magnetic field is proportional to the Dirac delta function at the origin. We must define one set of functions for the vector potential on the northern hemisphere, and another set of functions for the southern hemispheres. These two vector potentials are matched at the equator, and they differ by a gauge transformation. The wave function of an electrically-charged particle (a "probe charge") that orbits the equator generally changes by a phase, much like in the Aharonov–Bohm effect. This phase is proportional to the electric charge q_e of the probe, as well as to the magnetic charge q_m of the source. Dirac was originally considering an electron whose wave function is described by the Dirac equation.

Because the electron returns to the same point after the full trip around the equator, the phase **exp**($i\varphi$) of its wave function must be unchanged, which implies that the phase φ added to the wave function must be a multiple of 2π:

$$2\frac{q_e q_m}{\hbar c} \in \mathbb{Z} \text{ (cgs units)}$$

$$\frac{q_e q_m}{2\pi\hbar} \in \mathbb{Z} \text{ (SI units, weber convention)}^{[20]}$$

$$\frac{q_e q_m}{2\pi\epsilon_0\hbar c^2} \in \mathbb{Z} \text{ (SI units, ampere·meter convention)}$$

where

- ϵ_0 is vacuum permittivity
- $\hbar$ is reduced Planck's constant
- c is speed of light
- $\mathbb{Z}$ is the set of integers.

This is known as the **Dirac quantization condition**. The hypothetical existence of a magnetic monopole would imply that the electric charge must be quantized in certain units; also, the existence of the electric charges implies that the magnetic charges of the hypothetical magnetic monopoles, if they exist, must be quantized in units inversely proportional to the elementary electric charge.

At the time it was not clear if such a thing existed, or even had to. After all, another theory could come along that would explain charge quantization without need for the monopole. The concept remained something of a curiosity. However, in the time since the publication of this seminal work, no other widely accepted explanation of charge quantization has appeared. (The concept of local gauge invariance—see gauge theory below—provides a natural explanation of charge quantization, without invoking the need for magnetic monopoles; but only if the U(1) gauge group is compact, in which case we will have magnetic monopoles anyway.)

If we maximally extend the definition of the vector potential for the southern hemisphere, it will be defined everywhere except for a semi-infinite line stretched from the origin in the direction towards the northern pole. This semi-infinite line is called the Dirac string and its effect on the wave function is analogous to the effect of the solenoid in the Aharonov–Bohm effect. The quantization condition comes from the requirement that the phases around the Dirac string are trivial, which means that the Dirac string must be unphysical. The Dirac string is merely an artifact of the coordinate chart used and should not be taken seriously.

The Dirac monopole is a singular solution of Maxwell's equation (because it requires removing the worldline from spacetime); in more complicated theories, it is superseded by a smooth solution such as the 't Hooft–Polyakov monopole.

Topological interpretation

Dirac string

A gauge theory like electromagnetism is defined by a gauge field, which associates a group element to each path in space time. For infinitesimal paths, the group element is close to the identity, while for longer paths the group element is the successive product of the infinitesimal group elements along the way.

In electrodynamics, the group is U(1), unit complex numbers under multiplication. For infinitesimal paths, the group element is $1+iA_\mu dx^\mu$ which implies that for finite paths parametrized by s, the group element is:

$$\prod_s \left(1 + ieA_\mu \frac{dx^\mu}{ds} ds\right) = \exp\left(ie \int A \cdot dx\right).$$

The map from paths to group elements is called the Wilson loop or the holonomy, and for a U(1) gauge group it is the phase factor which the wavefunction of a charged particle acquires as it traverses the path. For a loop:

$$e\oint_{\partial D} A \cdot dx = e\int_D (\nabla \times A)dS = e\int_D BdS.$$

So that the phase a charged particle gets when going in a loop is the magnetic flux through the loop. When a small solenoid has a magnetic flux, there are interference fringes for charged particles which go around the solenoid, or around different sides of the solenoid, which reveal its presence.

But if all particle charges are integer multiples of e, solenoids with a flux of $2\pi/e$ have no interference fringes, because the phase factor for any charged particle is $e^{2\pi i} = 1$. Such a solenoid, if thin enough, is quantum-mechanically invisible. If such a solenoid were to carry a flux of $2\pi/e$, when the flux leaked out from one of its ends it would be indistinguishable from a monopole.

Dirac's monopole solution in fact describes an infinitesimal line solenoid ending at a point, and the location of the solenoid is the singular part of the solution, the Dirac string. Dirac strings link monopoles and antimonopoles of opposite magnetic charge, although in Dirac's version, the string just goes off to infinity. The string is unobservable, so you can put it anywhere, and by using two coordinate patches, the field in each patch can be made nonsingular by sliding the string to where it cannot be seen.

Grand unified theories

In a U(1) gauge group with quantized charge, the group is a circle of radius $2\pi/e$. Such a U(1) gauge group is called compact. Any U(1) which comes from a Grand Unified Theory is compact - because only compact higher gauge groups make sense. The size of the gauge group is a measure of the inverse coupling constant, so that in the limit of a large-volume gauge group, the interaction of any fixed representation goes to zero.

The case of the U(1) gauge group is a special case because all its irreducible representations are of the same size — the charge is bigger by an integer amount, but the field is still just a complex number — so that in U(1) gauge field theory it is possible to take the decompactified limit with no contradiction. The quantum of charge becomes small, but each charged particle has a huge number of charge quanta so its charge stays finite. In a non-compact U(1) gauge group theory, the charges of particles are generically not integer multiples of a single unit. Since charge quantization is an experimental certainty, it is clear that the U(1) gauge group of electromagnetism is compact.

GUTs lead to compact U(1) gauge groups, so they explain charge quantization in a way that seems to be logically independent from magnetic monopoles. However, the explanation is essentially the same, because in any GUT which breaks down into a U(1) gauge group at long distances, there are magnetic monopoles.

The argument is topological:

1. The holonomy of a gauge field maps loops to elements of the gauge group. Infinitesimal loops are mapped to group elements infinitesimally close to the identity.
2. If you imagine a big sphere in space, you can deform an infinitesimal loop which starts and ends at the north pole as follows: stretch out the loop over the western hemisphere until it becomes a great circle (which still starts and ends at the north pole) then let it shrink back to a little loop while going over the eastern hemisphere. This is called *lassoing the sphere*.
3. Lassoing is a sequence of loops, so the holonomy maps it to a sequence of group elements, a continuous path in the gauge group. Since the loop at the beginning of the lassoing is the same as the loop at the end, the path in the group is closed.
4. If the group path associated to the lassoing procedure winds around the U(1), the sphere contains magnetic charge. During the lassoing, the holonomy changes by the amount of magnetic flux through the sphere.
5. Since the holonomy at the beginning and at the end is the identity, the total magnetic flux is quantized. The magnetic charge is proportional to the number of windings N, the magnetic flux through the sphere is equal to $2\pi N/e$. This is the Dirac quantization condition, and it is a topological condition which demands that the long

distance U(1) gauge field configurations be consistent.

6. When the U(1) gauge group comes from breaking a compact Lie group, the path which winds around the U(1) group enough times is topologically trivial in the big group. In a non-U(1) compact Lie group, the covering space is a Lie group with the same Lie algebra, but where all closed loops are contractible. Lie groups are homogenous, so that any cycle in the group can be moved around so that it starts at the identity, then its lift to the covering group ends at P, which is a lift of the identity. Going around the loop twice gets you to P^2, three times to P^3, all lifts of the identity. But there are only finitely many lifts of the identity, because the lifts can't accumulate. This number of times one has to traverse the loop to make it contractible is small, for example if the GUT group is SO(3), the covering group is SU(2), and going around any loop twice is enough.
7. This means that there is a continuous gauge-field configuration in the GUT group allows the U(1) monopole configuration to unwind itself at short distances, at the cost of not staying in the U(1). In order to do this with as little energy as possible, you should leave only the U(1) gauge group in the neighborhood of one point, which is called the **core** of the monopole. Outside the core, the monopole has only magnetic field energy.

Hence, the Dirac monopole is a topological defect in a compact U(1) gauge theory. When there is no GUT, the defect is a singularity — the core shrinks to a point. But when there is some sort of short-distance regulator on space time, the monopoles have a finite mass. Monopoles occur in lattice U(1), and there the core size is the lattice size. In general, they are expected to occur whenever there is a short-distance regulator.

String theory

In our universe, quantum gravity provides the regulator. When gravity is included, the monopole singularity can be a black hole, and for large magnetic charge and mass, the black hole mass is equal to the black hole charge, so that the mass of the magnetic black hole is not infinite. If the black hole can decay completely by Hawking radiation, the lightest charged particles cannot be too heavy. The lightest monopole should have a mass less than or comparable to its charge in natural units.

So in a consistent holographic theory, of which string theory is the only known example, there are always finite-mass monopoles. For ordinary electromagnetism, the mass bound is not very useful because it is about same size as the Planck mass.

Mathematical formulation

In mathematics, a gauge field is defined as a connection over a principal G-bundle over spacetime. G is the gauge group, and it acts on each fiber of the bundle separately.

A *connection* on a G bundle tells you how to glue F's together at nearby points of M. It starts with a continuous symmetry group G which acts on F, and then it associates a group element with each infinitesimal path. Group multiplication along any path tells you how to move from one point on the bundle to another, by acting the G element of a path on the fiber F.

In mathematics, the definition of bundle is designed to emphasize topology, so the notion of connection is added on as an afterthought. In physics, the connection is the fundamental physical object. One of the fundamental observations in the theory of characteristic classes in algebraic topology is that many homotopical structures of nontrivial principal bundles may be expressed as an integral of some polynomial over **any** connection over it. Note that any connection over a trivial bundle can never give us a nontrivial principal bundle.

If space time has no topology, if it is $\mathbf{R}^4$ the space of all possible connections of the G-bundle is connected. But consider what happens when we remove a timelike worldline from spacetime. The resulting spacetime is homotopically equivalent to the topological sphere S^2.

A principal G-bundle over S^2 is defined by covering S^2 by two charts, each homeomorphic to the open 2-ball such that their intersection is homeomorphic to the strip $S^1 \times I$. 2-balls are homotopically trivial and the strip is homotopically equivalent to the circle S^1. So a topological classification of the possible connections is reduced to classifying the transition functions. The transition function maps the strip to G, and the different ways of mapping a strip into G are given by the first homotopy group of *G*.

So in the G-bundle formulation, a gauge theory admits Dirac monopoles provided *G* is not simply connected, whenever there are paths that go around the group that cannot be deformed to nothing. *U*(1), which has quantized charges, is not simply connected and can have Dirac monopoles while **R**, its universal covering group, is simply connected, doesn't have quantized charges and does not admit Dirac monopoles. The mathematical definition is equivalent to the physics definition provided that, following Dirac, gauge fields are allowed which are defined only patch-wise and the gauge field on different patches are glued after a gauge transformation.

The total magnetic flux is none other than the first Chern number of the principal bundle, and depends only upon the choice of the principal bundle, and not the specific connection over it. In other words, it's a topological invariant.

This argument for monopoles is a restatement of the lasso argument for a pure U(1) theory. It generalizes to $d + 1$ dimensions with $d \geq 2$ in several ways. One way is to extend everything into the extra dimensions, so that U(1) monopoles become sheets of dimension d-3. Another way is to examine the type of topological singularity at a point with the homotopy group $\pi_{d-2}(G)$.

Grand unified theories

In more recent years, a new class of theories has also suggested the existence of magnetic monopoles.

During the early 1970s, the successes of quantum field theory and gauge theory in the development of electroweak theory and the mathematics of the strong nuclear force led many theorists to move on to attempt to combine them in a single theory known as a Grand Unified Theory (GUT). Several GUTs were proposed, most of which had the curious feature of implying the presence of a real magnetic monopole particle. More accurately, GUTs predicted a range of particles known as dyons, of which the most basic state was a monopole. The charge on magnetic monopoles predicted by GUTs is either 1 or 2 *gD*, depending on the theory.

The majority of particles appearing in any quantum field theory are unstable, and they decay into other particles in a variety of reactions that must satisfy various conservation laws. Stable particles are stable because there are no lighter particles into which they can decay and still satisfy the conservation laws. For instance, the electron has a lepton number of one and an electric charge of one, and there are no lighter particles that conserve these values. On the other hand, the muon, essentially a heavy electron, can decay into the electron plus two quanta of energy, and hence it is not stable.

The dyons in these GUTs are also stable, but for an entirely different reason. The dyons are expected to exist as a side effect of the "freezing out" of the conditions of the early universe, or a symmetry breaking. In this scenario, the dyons arise due to the configuration of the vacuum in a particular area of the universe, according to the original Dirac theory. They remain stable not because of a conservation condition, but because there is no simpler *topological* state into which they can decay.

The length scale over which this special vacuum configuration exists is called the *correlation length* of the system. A correlation length cannot be larger than causality would allow, therefore the correlation length for making magnetic monopoles must be at least as big as the horizon size determined by the metric of the expanding universe. According to that logic, there should be at least one magnetic monopole per horizon volume as it was when the symmetry breaking took place. Other arguments based on the critical density of the universe indicate that monopoles should be fairly common; the apparent problem of the observed scarcity of monopoles is resolved by cosmic inflation in the early universe, which greatly reduces the expected abundance of magnetic monopoles. For these reasons, monopoles became a major interest in the 1970s and 80s, along with the other "approachable" predictions of GUTs such as proton decay.

Many of the other particles predicted by these GUTs were beyond the abilities of current experiments to detect. For instance, a wide class of particles known as the X and Y bosons are predicted to mediate the coupling of the electroweak and strong forces, but these particles are extremely heavy and well beyond the capabilities of any reasonable particle accelerator to create.

Searches for magnetic monopoles

A number of attempts have been made to detect magnetic monopoles. One of the simpler ones is to use a loop of superconducting wire to look for even tiny magnetic sources, a so-called "superconducting quantum interference device", or SQUID. Given the predicted density, loops small enough to fit on a lab bench would expect to see about one monopole event per year. Although there have been tantalizing events recorded, in particular the event recorded by Blas Cabrera on the night of February 14, 1982 (thus, sometimes referred to as the "Valentine's Day Monopole"[21]), there has never been reproducible evidence for the existence of magnetic monopoles.[11] The lack of such events places a limit on the number of monopoles of about one monopole per 10^{29} nucleons.

Another experiment in 1975 resulted in the announcement of the detection of a moving magnetic monopole in cosmic rays by the team led by P. Buford Price.[10] Price later retracted his claim, and a possible alternative explanation was offered by Alvarez.[22] In his paper it was demonstrated that the path of the cosmic ray event that was claimed to have been be due to a magnetic monopole could be reproduced by the path followed by a platinum nucleus decaying first to osmium, and then to tantalum.

Other experiments rely on the strong coupling of monopoles with photons, as is the case for any electrically-charged particle as well. In experiments involving photon exchange in particle accelerators, monopoles should be produced in reasonable numbers, and detected due to their effect on the scattering of the photons. The probability of a particle being created in such experiments is related to their mass — with heavier particles being less likely to be created — so by examining the results of such experiments, limits on the mass of a magnetic monopole can be calculated. The most recent such experiments suggest that monopoles with masses below 600 GeV/c^2 do not exist, while upper limits on their mass due to the very existence of the universe - which would have collapsed by now if they were too heavy - are about 10^{17} GeV/c^2.

The MoEDAL experiment, installed at the Large Hadron Collider, is currently searching for magnetic monopoles and large supersymmetric particles using layers of special plastic sheets attached to the walls around LHCb's VELO detector. The particles it is looking for will damage the sheets along their path, with various identifying features.

"Monopoles" in condensed-matter systems

While a magnetic monopole particle has never been conclusively observed, there are a number of phenomena in condensed-matter physics where a material, due to the collective behavior of its electrons and ions, can show emergent phenomena that resemble magnetic monopoles in some respect.[23] [24] [25] [26] [27] These should not be confused with actual monopole particles: since all known particles have zero magnetic charge (including the protons, neutrons, and electrons that make up the whole periodic table), it is fundamentally impossible to find a *true* magnetic monopole in ordinary matter made from atoms; only quasiparticles are possible. In particular, the law $\nabla\cdot\mathbf{B}=0$ is true everywhere in these systems, which it would not be in the presence of a true magnetic monopole particle. The Dirac string model describes such a system in the theoretical limit of infinitely thin connecting fluxtubes.

One example of the work on magnetic monopole quasiparticles is a paper published in the journal *Science* in September 2009, in which researchers Jonathan Morris and Alan Tennant from the Helmholtz-Zentrum Berlin für Materialien und Energie (HZB) along with Santiago Grigera from Instituto de Física de Líquidos y Sistemas Biológicos (IFLYSIB, CONICET) and other colleagues from Dresden University of Technology, University of St. Andrews and Oxford University described the observation of quasiparticles resembling magnetic monopoles. A single crystal of dysprosium titanate in a highly frustrated pyrochlore lattice (F d -3 m) was cooled to a temperature between 0.6 kelvin and 2.0 kelvin. Using observations of neutron scattering, the magnetic moments were shown to align in the spin ice into interwoven tubelike bundles resembling Dirac strings. At the defect formed by the end of each tube, the magnetic field looks like that of a monopole. Using an applied magnetic field to break the symmetry of the system, the researchers were able to control the density and orientation of these strings. A contribution to the heat capacity of the system from an effective gas of these quasiparticles was also described.[28] [29]

Another example of the work on magnetic monopole quasiparticles is a paper in the February 11, 2011 issue of *Nature Physics* which describes creation and measurement of long-lived magnetic monopole quasiparticle currents in spin ice. By applying a magnetic-field pulse to a dysprosium titanate ($Dy_2Ti_2O_7$) spin-ice crystal at 0.36 K, the authors created a relaxing magnetic current that lasted for several minutes. They measured the current by means of the electromotive force it induced in a solenoid coupled to a sensitive amplifier, and quantitatively described it using a chemical kinetic model of point-like charges obeying the Onsager–Wien mechanism of carrier dissociation and recombination. They thus derived the microscopic parameters of monopole motion in spin ice and identified the distinct roles of free and bound magnetic charges.[30]

Notes

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[14] Magnetic monopoles spotted in spin ices (http://physicsworld.com/cws/article/news/40302), 3 September 2009. "Oleg Tchernyshyov at Johns Hopkins University [a researcher in this field] cautions that the theory and experiments are specific to spin ices, and are not likely to shed light on magnetic monopoles as predicted by Dirac."

[15] The fact that the electric and magnetic fields can be written in a symmetric way is specific to the fact that space is three-dimensional. When the equations of electromagnetism are extrapolated to other dimensions, the magnetic field is described as being a rank-two antisymmetric tensor, whereas the electric field remains a true vector. In dimensions other than three, these two mathematical objects do not have the same number of components.

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External links

- Magnetic Monopole Searches (lecture notes) (http://arxiv.org/abs/hep-ex/0302011)
- Particle Data Group summary of magnetic monopole search (http://pdg.lbl.gov/2004/listings/s028.pdf)
- 'Race for the Pole' Dr David Milstead (http://www.vega.org.uk/video/programme/56) Freeview 'Snapshot' video by the Vega Science Trust and the BBC/OU.
- Interview with Jonathan Morris (http://www.drillingsraum.com/magnetic_monopole/magnetic_monopole.html) about magnetic monopoles and magnetic monopole quasiparticles. Drillingsraum, 16 April 2010

Electric flux

In electromagnetism, **electric flux** is the flux of the electric field. Electric flux is proportional to the number of electric field lines going through a virtual surface. The electric flux $d\Phi_E$ through a small area $d\mathbf{A}$ is given by

$$d\Phi_E = \mathbf{E} \cdot d\mathbf{A}$$

(the electric field, **E**, multiplied by the component of area perpendicular to the field). The electric flux over a surface *S* is therefore given by the surface integral:

$$\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{A}$$

where **E** is the electric field and d**A** is a differential area on the closed surface S with an outward facing surface normal defining its direction.

For a closed Gaussian surface, electric flux is given by:

$$\Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_S}{\epsilon_0}$$

where Q_S is the net charge enclosed by the surface (including both free and bound charge), and ε_0 is the electric constant. This relation is known as Gauss' law for electric field in its integral form and it is one of the four Maxwell's equations.

It is important to note that while the electric flux is not affected by charges that are not within the closed surface, the net electric field, **E**, in the Gauss' Law equation, can be affected by charges that lie outside the closed surface. While Gauss' Law holds for all situations, it is only useful for "by hand" calculations when high degrees of symmetry exist in the electric field. Examples include spherical and cylindrical symmetry.

Electrical flux has SI units of volt metres (V m), or, equivalently, newton metres squared per coulomb (N m^2 C^{-1}). Thus, the SI base units of electric flux are kg•m^3•s^{-3}•A^{-1}.

External links

- Electric flux [1] — HyperPhysics

References

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Magnetic flux

Magnetic flux (most often denoted as Φ_m), is a measure of the amount of magnetic **B** field (also called "magnetic flux density") passing through a given surface (such as a conducting coil). The SI unit of magnetic flux is the weber (Wb) (in derived units: volt-seconds). The CGS unit is the maxwell.

Description

The magnetic flux through a given surface is proportional to the number of magnetic **B** field lines that pass through the surface. This is the *net* number, i.e. the number passing through in one direction, minus the number passing through in the other direction. (See below for how the positive sign is chosen.) For a uniform magnetic field **B** passing through a perpendicular area the magnetic flux is given by the product of the magnetic field and the area element. The magnetic flux for a uniform **B** at any angle to a surface is defined by a dot product of the magnetic field and the area element vector **a**.

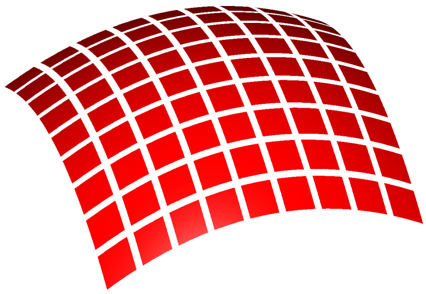


Figure 1: The definition of surface integral relies on splitting the surface into small surface elements. Each element is associated with a vector d**S** of magnitude equal to the area of the element and with direction normal to the element and pointing outward.

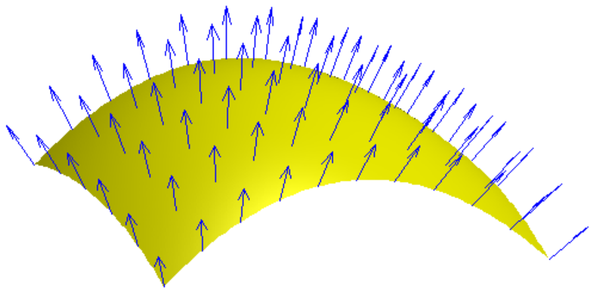


Figure 2: A vector field of normals to a surface

$$\Phi_m = \mathbf{B} \cdot \mathbf{a} = Ba\cos\theta \quad \text{(uniform } \mathbf{B} \text{ with flat area only)}$$

where θ is the angle between **B** and a vector **a** that is perpendicular (normal) to the surface.

In the general case, the magnetic flux through a surface *S* is defined as the integral of the magnetic field over the area of the surface (See Figures 1 and 2):

$$\Phi_m = \iint\limits_S \mathbf{B} \cdot d\mathbf{S},$$

where Φ_m is the magnetic flux, ***B*** is the magnetic field,

S is the surface (area), · denotes dot product, and d***S*** is an infinitesimal vector, whose magnitude is the area of a differential element of *S*, and whose direction is the surface normal. (See surface integral for more details.)

From the definition of the magnetic vector potential **A** and the fundamental theorem of the curl the magnetic flux may also be defined as:

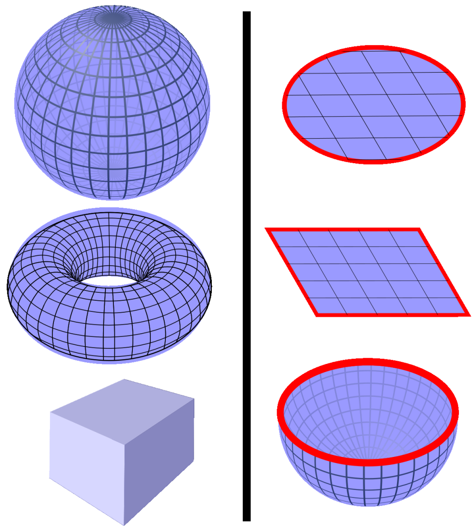
$$\Phi_m = \oint_{\Sigma} \mathbf{A} \cdot d\boldsymbol{\ell}$$

where the closed line integral is over the boundary of the surface and $d\boldsymbol{\ell}$ is an infinitesimal vector element of that contour $\mathbf{\Sigma}$.

The magnetic flux is usually measured with a fluxmeter. The fluxmeter contains measuring coils and electronics that evaluates the change of voltage in the measuring coils to calculate the magnetic flux.

Magnetic flux through a closed surface

Gauss's law for magnetism, which is one of the four Maxwell's equations, states that the total magnetic flux through a closed surface is equal to zero. (A "closed surface" is a surface that completely encloses a volume(s) with no holes.) This law is a consequence of the empirical observation that magnetic monopoles have never been found.



Some examples of closed surfaces (left) and open surfaces (right). Left: Surface of a sphere, surface of a torus, surface of a cube. Right: Disk surface, square surface, surface of a hemisphere. (The surface is blue, the boundary is red.)

In other words, Gauss's law for magnetism is the statement:

$$\Phi_m = \iint \mathbf{B} \cdot d\mathbf{S} = 0,$$

for any closed surface *S*.

Magnetic flux through an open surface

While the magnetic flux through a closed surface is always zero, the magnetic flux through an open surface need not be zero and is an important quantity in electromagnetism. For example, a change in the magnetic flux passing through a loop of conductive wire will cause an electromotive force, and therefore an electric current, in the loop. The relationship is given by Faraday's law:

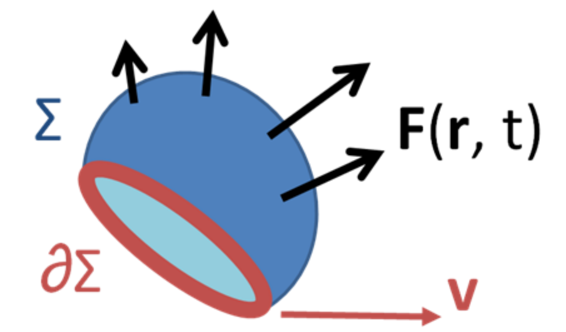


Figure 3: A vector field $\boldsymbol{F}(\boldsymbol{r}, t)$ defined throughout space, and a surface Σ bounded by curve ∂Σ moving with velocity $\boldsymbol{v}$ over which the field is integrated.

$$\mathcal{E} = \oint_{\partial\Sigma(t)} (\mathbf{E}(\mathbf{r},\ t) + \mathbf{v} \times \mathbf{B}(\mathbf{r},\ t)) \cdot d\boldsymbol{\ell} = -\frac{d\Phi_m}{dt},$$

where (see Figure 3):

$\mathcal{E}$ is the EMF,

Φ_m is the flux through a surface with an opening bounded by a curve $\boldsymbol{\partial\Sigma}(t)$,

$\boldsymbol{\partial\Sigma}(t)$ is a closed contour that can change with time; the EMF is found around this contour, and the contour is a boundary of the surface over which Φ_m is found,

d$\boldsymbol{\ell}$ is an infinitesimal vector element of the contour $\boldsymbol{\partial\Sigma}(t)$,

$\boldsymbol{v}$ is the velocity of the segment d$\boldsymbol{\ell}$,

$\boldsymbol{E}$ is the electric field,

$\boldsymbol{B}$ is the magnetic field.

The EMF is determined in this equation in two ways: first, as the work per unit charge done against the Lorentz force in moving a test charge around the (possibly moving) closed curve $\boldsymbol{\partial\Sigma}(t)$, and second, as the magnetic flux through the open surface $\boldsymbol{\Sigma}(t)$.

This equation is the principle behind an electrical generator.

Comparison with electric flux

By way of contrast, Gauss's law for electric fields, another of Maxwell's equations, is

$$\Phi_E = \iint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q}{\epsilon_0},$$

where

$\boldsymbol{E}$ is the electric field,

S is any closed surface,

Q is the total electric charge inside the surface S,

ϵ_0 is the electric constant (a universal constant, also called the "permittivity of free space").

Note that the flux of $\boldsymbol{E}$ through a closed surface is *not* always zero; this indicates the presence of electric "monopoles", that is, free positive or negative charges.

External articles

Patents

- Vicci, U.S. Patent 6720855 [1], *Magnetic-flux conduits*
- Magnetic Flux through a Loop of Wire [2] by Ernest Lee, Wolfram Demonstrations Project.
- Conversion Magnetic flux Φ in nWb per meter track width to flux level in dB - Tape Operating Levels and Tape Alignment Levels [3]

[1] http://www.google.com/patents?vid=6720855
[2] http://demonstrations.wolfram.com/MagneticFluxThroughALoopOfWire/
[3] http://www.sengpielaudio.com/calculator-magneticflux.htm

Electric potential

In classical electromagnetism, the **electric potential** (a scalar quantity denoted by φ, $\varphi_{\mathbf{E}}$ or V and also called the *electric field potential* or the *electrostatic potential*) at a point within a defined space is equal to the electric potential energy (measured in joules) at that location divided by the charge there (measured in coulombs). The electric potential at a specific location in the electric field is independent of q_t. That is to say, it is a characteristic only of the electric field that is present. The electric potential can be calculated at a point in either a static (time-invariant) electric field or in a dynamic (varying with time) electric field at a specific time, and has the units of joules per coulomb, or volts.

There is also a generalized electric scalar potential that is used in electrodynamics when time-varying electromagnetic fields are present. This generalized electric potential cannot be simply interpreted as the ratio of potential energy to charge, however.

Introduction

Objects may possess a property known as an electric charge. An electric field exerts a force on charged objects, accelerating them in the direction of the force, in either the same or the opposite direction of the electric field. If the charged object has a positive charge, the force and acceleration will be in the direction of the field. This force has the same direction as the electric field vector, and its magnitude is given by the size of the charge multiplied with the magnitude of the electric field. Classical mechanics explores the concepts such as force, energy, potential etc. in more detail.

Force and potential energy are directly related. As an object moves in the direction that the force accelerates it, its potential energy decreases. For example, the gravitational potential energy of a cannonball at the top of a hill is greater than at the base of the hill. As the object falls, that potential energy decreases and is translated to motion, or inertial (kinetic) energy.

For certain forces, it is possible to define the "potential" of a field such that the potential energy of an object due to a field is dependent only on the position of the object with respect to the field. Those forces must affect objects depending only on the intrinsic properties of the object and the position of the object, and obey certain other mathematical rules.

Two such forces are the gravitational force (gravity) and the electric force in the absence of time-varying magnetic fields. The potential of an electric field is called the electric potential. The synonymous term "electrostatic potential" is also in common use.

The electric potential and the magnetic vector potential together form a four vector, so that the two kinds of potential are mixed under Lorentz transformations.

In electrostatics

The electric potential at a point **r** in a static electric field **E** is given by the line integral

$$\Delta V_{\mathbf{E}} = -\int_C \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell}\,,$$

where C is an arbitrary path connecting the point with zero potential to **r**. When the curl $\nabla \times \mathbf{E}$ is zero, the line integral above does not depend on the specific path C chosen but only on its endpoints. In this case, the electric field is conservative and determined by the gradient of the potential:

$$\mathbf{E} = -\nabla V_{\mathbf{E}}.$$

Then, by Gauss's law, the potential satisfies Poisson's equation:

$$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla V_{\mathbf{E}}) = -\nabla^2 V_{\mathbf{E}} = \rho/\varepsilon_0,$$

where ϱ is the total charge density (including bound charge) and $\nabla\cdot$ denotes the divergence.

The concept of electric potential is closely linked with potential energy. A test charge q has an electric potential energy $U_{\mathbf{E}}$ given by

$$U_{\mathbf{E}} = q\,V.$$

The potential energy and hence also the electric potential is only defined up to an additive constant: one must arbitrarily choose a position where the potential energy and the electric potential are zero.

These equations cannot be used if the curl $\nabla \times \mathbf{E} \neq 0$, i.e., in the case of a *nonconservative electric field* (caused by a changing magnetic field; see Maxwell's equations). The generalization of electric potential to this case is described below.

Electric potential due to a point charge

The electric potential created by a point charge Q, at a distance r from the charge (relative to the potential at infinity), can be shown to be

$$V_{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where ε_0 is the electric constant (permittivity of free space). This is known as the Coulomb Potential.

The electric potential due to a system of point charges is equal to the sum of the point charges' individual potentials. This fact simplifies calculations significantly, since addition of potential (scalar) fields is much easier than addition of the electric (vector) fields.

The equation given above for the electric potential (and all the equations used here) are in the forms required by SI units. In some other (less common) systems of units, such as CGS-Gaussian, many of these equations would be altered.

Generalization to electrodynamics

When time-varying magnetic fields are present (which is true whenever there are time-varying electric fields and vice versa), it is not possible to describe the electric field simply in terms of a scalar potential V because the electric field is no longer conservative: $\int_C \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell}$ is path-dependent because $\nabla \times \mathbf{E} \neq 0$ (Faraday's law of induction).

Instead, one can still define a scalar potential by also including the magnetic vector potential **A**. In particular, **A** is defined to satisfy:

$$\mathbf{B} = \nabla \times \mathbf{A},$$

where **B** is the magnetic field. Because the divergence of the magnetic field is always zero due to the absence of magnetic monopoles, such an **A** can always be found. Given this, the quantity

$$\mathbf{F} = \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}$$

is a conservative field by Faraday's law and one can therefore write

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t},$$

where V is the scalar potential defined by the conservative field **F**.

The electrostatic potential is simply the special case of this definition where **A** is time-invariant. On the other hand, for time-varying fields, note that

$$\int_a^b \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} \neq V_{(b)} - V_{(a)},$$

unlike electrostatics.

Note that this definition of V depends on the gauge choice for the vector potential **A** (the gradient of any scalar field can be added to **A** without changing **B**). One choice is the Coulomb gauge, in which we choose $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$. In this case, we obtain

$$-\nabla^2 V = \rho/\varepsilon_0,$$

where ϱ is the charge density, just as for electrostatics. Another common choice is the Lorenz gauge, in which we choose **A** to satisfy

$$\nabla \cdot \mathbf{A} = -\frac{1}{c^2}\frac{\partial V}{\partial t}.$$

Units

The SI unit of electric potential is the volt (in honor of Alessandro Volta), which is why electric potential is also known as voltage. Older units are rarely used nowadays. Variants of the centimeter gram second system of units included a number of different units for electric potential, including the abvolt and the statvolt.

Galvani potential versus electrochemical potential

Inside metals (and other solids and liquids), the energy of an electron is affected not only by the electric potential, but also by the specific atomic environment that it is in. When a voltmeter is connected between two different types of metal, it measures not the electric potential difference, but instead the potential difference corrected for the different atomic environments.[1] The quantity measured by a voltmeter is called electrochemical potential or fermi level, while the pure unadjusted electric potential is sometimes called Galvani potential. The terms "voltage" and "electric potential" are a bit ambiguous in that, in practice, they can refer to *either* of these in different contexts.

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[1] Bagotskii, Vladimir Sergeevich (2006). *Fundamentals of electrochemistry* (http://books.google.com/books?id=09QI-assq1cC&pg=PA22). p. 22. ISBN 9780471700586. .

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Magnetic potential

The term **magnetic potential** can be used for either of two quantities in classical electromagnetism: the **magnetic vector potential**, **A**, (often simply called the **vector potential**) and the **magnetic scalar potential**, ψ. Both quantities can be used in certain circumstances to calculate the magnetic field.

The more frequently used magnetic vector potential **A** (often simply called the **vector potential**) is defined such that the curl of **A** is the magnetic **B** field. Together with the electric potential, the magnetic vector potential can be used to specify the electric field, **E** as well. Therefore, many equations of electromagnetism can be written either in terms of the **E** and **B**, *or* in terms of the magnetic vector potential and electric potential. In more advanced theories such as quantum mechanics, most equations use the potentials and not the **E** and **B** fields.

The magnetic scalar potential ψ is sometimes used to specify the magnetic **H**-field in cases when there are no free currents, in a manner analogous to using the electric potential to determine the electric field in electrostatics. One important use of ψ is to determine the magnetic field due to permanent magnets when their magnetization is known. With some care the scalar potential can be extended to include free currents as well.

Magnetic vector potential

The magnetic vector potential **A** is a vector field that together with the (scalar field) electric potential φ are defined as:

$$\mathbf{B} = \nabla \times \mathbf{A},$$ [1]

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t},$$ [1]

where **B** is the magnetic field and **E** is the electric field. In magnetostatics where there is no time varying charge distribution, only the first equation is needed. (In the context of electrodynamics, the terms "vector potential" and "scalar potential" are used for "magnetic vector potential" and "electric potential", respectively. In mathematics, vector potential and scalar potential have more general meanings.)

Defining the electric and magnetic fields from potentials automatically satisfies two of Maxwell's equations: Gauss's law for magnetism and Faraday's Law. For example, if **A** is continuous and well-defined everywhere, then it is guaranteed not to result in magnetic monopoles. (In the mathematical theory of magnetic monopoles, **A** is allowed to be either undefined or multiple-valued in some places; see magnetic monopole for details.)

Starting with the above definitions:

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times \mathbf{E} = \nabla \times \left(-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) = -\frac{\partial \mathbf{B}}{\partial t}.$$

Alternatively, the existence of **A** and φ is guaranteed from these two laws using the Helmholtz's theorem. For example, since the magnetic field is divergence-free (Gauss's law for magnetism), i.e. $\nabla \cdot \mathbf{B} = 0$, **A** always exists that satisfies the above definition.

The vector potential **A** is used when studying the Lagrangian in classical mechanics and in quantum mechanics (see Schrödinger equation for charged particles, Dirac equation, Aharonov-Bohm effect).

In the SI system, the units of **A** are volt-seconds per metre ($\mathrm{V{\cdot}s{\cdot}m^{-1}}$) and are the same as that of momentum per unit charge.

Although the magnetic field **B** is a pseudovector (also called axial vector), the vector potential **A** is not: **A** is a polar vector.[2] This means that if the right-hand rule for cross products were replaced with a left-hand rule, but without changing any other equations or definitions, then **B** would switch signs, but **A** would not change. This is an example of a general theorem: The curl of a polar vector is a pseudovector, and vice-versa.[2]

Gauge choices

The above definition does **not** define the magnetic vector potential uniquely because, by definition, we can arbitrarily add curl-free components to the magnetic potential without changing the observed magnetic field. Thus, there is a degree of freedom available when choosing **A**. This condition is known as gauge invariance.

Maxwell's equations in terms of vector potential

Using the above definition of the potentials and applying it to the other two Maxwell's equations (the ones that are not automatically satisfied) results in a complicated differential equation that can be simplified using the Lorenz gauge where **A** is chosen so as to satisfy:

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0.$$ [1]

Using the Lorenz gauge, Maxwell's equations can be written compactly in terms of the magnetic vector potential **A** and the electric scalar potential *Φ*.

$$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\rho/\epsilon_0$$ [1]

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$$ [1]

In other gauges, the equations are different. A different notation to write these same equations (using four-vectors) is shown below.

Calculation of potentials from source distributions

The solutions of Maxwell's equations (in the Lorenz gauge) Feynman [1] and Jackson[3] with the boundary condition that both potentials go to zero sufficiently fast as they approach infinity are called the *'retarded potentials* which are:

$$\mathbf{A}(\mathbf{p_1}, t) = \frac{\mu_0}{4\pi}\int_{V_2} \frac{\mathbf{j}(\mathbf{p_2}, t_r)}{r_{12}}\,\mathrm{d}V \text{ where } t_r = t - \frac{r_{12}}{c}$$

$$\phi(\mathbf{p_1}, t) = \frac{1}{4\pi\epsilon_0}\int_{V_2} \frac{\rho(\mathbf{p_2}, t_r)}{r_{12}}\,\mathrm{d}V$$

where

t is the time at which the value of **A** and ϕ are to be calculated.

$\mathbf{p_1}$ is the point at which the value of **A** and ϕ are to be calculated.

$\mathbf{p_2}$ is the integration variable.

r_{12} is the distance from point $\mathbf{p_1}$ to point $\mathbf{p_2}$.

t_r is a time earlier than t by $\frac{r_{12}}{c}$ which is the time it takes an effect generated at $\mathbf{p_2}$ to propagate to $\mathbf{p_1}$ at the speed of light. t_r is also called *retarded time*.

A ($\mathbf{p_1}$, t) is the magnetic vector potential at point $\mathbf{p_1}$ and time t.

ϕ ($\mathbf{p_1}$, t)is the electric scalar potential at point $\mathbf{p_1}$ and time t.

j ($\mathbf{p_1}$, t_r) is the current density at point $\mathbf{p_2}$ and time t_r

ρ ($\mathbf{p_1}$, t_r) is the charge density at point $\mathbf{p_2}$ and time t_r.

$\mathbf{V_2}$ is the volume of all points $\mathbf{p_2}$ where $\mathbf{j}$ or ρ is non-zero at least sometimes.

There are a few notable things about **A** and ϕ calculated in this way:

(The Lorenz gauge condition): $\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$ is satisfied.

The position of the source point $\mathbf{p_2}$ only enters the equation as a scalar distance from $\mathbf{p_1}$ to $\mathbf{p_2}$. The direction from $\mathbf{p_1}$ to $\mathbf{p_2}$ does not enter into the equation. The only thing that matters about a source point is how far away it is.

The integrand uses *retarded time*. This simply reflects the fact that changes in the sources propagate at the speed of light

The equation for **A** is a vector equation. In Cartesian coordinates, the equation separates into three equations thus[4] :

$$\mathbf{A_x}(\mathbf{p_1}, t) = \frac{\mu_0}{4\pi} \int_{V_2} \frac{\mathbf{j_x}(\mathbf{p_2}, t_r)}{r_{12}} \, \mathrm{d}V$$ where $\mathbf{A_x}$ and $\mathbf{j_x}$ are the components of **A** and **j** in the direction of the x axis.

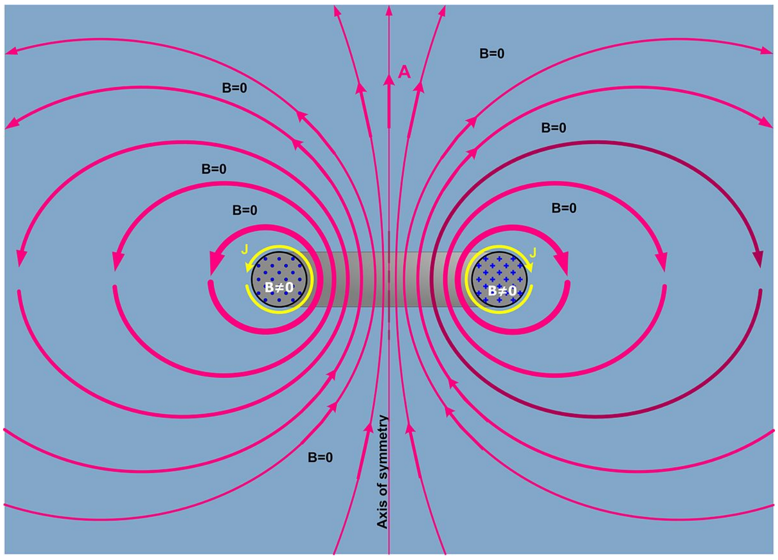
$$\mathbf{A_y}(\mathbf{p_1}, t) = \frac{\mu_0}{4\pi} \int_{V_2} \frac{\mathbf{j_y}(\mathbf{p_2}, t_r)}{r_{12}} \, \mathrm{d}V$$

$$\mathbf{A_z}(\mathbf{p_1}, t) = \frac{\mu_0}{4\pi} \int_{V_2} \frac{\mathbf{j_z}(\mathbf{p_2}, t_r)}{r_{12}} \, \mathrm{d}V$$

In this form it is easy to see that the component of **A** in a given direction depends only on the components of $\mathbf{j}$ that are in the same direction. If the current is carried in a long straight wire, the **A** points in the same direction as the wire.

In other gauges the formula for **A** and ϕ is different—for example, see Coulomb gauge for another possibility.

Depiction of the A field



Representing the Coulomb gauge magnetic vector potential (A), magnetic flux (B), and current density (j) fields around a toroidal inductor of circular cross section. Thicker lines indicate field lines of higher average intensity. Circles in cross section of the core represent B flux coming out of the picture. Plus signs on the other cross section of the core represent B flux going into the picture. Div **A** = 0 has been assumed.

See Feynman[5] for the depiction of the **A** field around a long thin solenoid.

Since $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (assuming quasi-static conditions, i.e. $\frac{\partial E}{\partial t} \to 0$) and $\nabla \times \mathbf{A} = \mathbf{B}$, the lines and contours of **A** relate to **B** like the lines and contours of **B** relate to **j**. Thus, a depiction of the **A** field around a loop of **B** flux (as would be produced in a toroidal inductor) is qualitatively the same as the **B** field around a loop of current.

The figure to the left is an artist's depiction of the **A** field. The thicker lines indicate paths of higher average intensity (shorter paths have higher intensity so that the path integral is the same). The lines are just drawn to look good and impart general look of the **A** field.

The drawing tacitly assumes $\nabla \cdot \mathbf{A} = 0$. This would be true under the following assumptions:

- the Coulomb gauge is assumed
- the Lorenz gauge is assumed and there is no distribution of charge, $\rho = 0$
- the Lorenz gauge is assumed and zero frequency is assumed
- the Lorenz gauge is assumed and a non-zero frequency that is low enough to neglect $\frac{1}{c^2}\frac{\partial \phi}{\partial t}$ is assumed

Electromagnetic four-potential

In the context of special relativity, it is natural to join the magnetic vector potential together with the (scalar) electric potential into the electromagnetic potential, also called "four-potential".

One motivation for doing so is that the four-potential is a mathematical four-vector. Thus, using standard four-vector transformation rules, if the electric and magnetic potentials are known in one inertial reference frame, they can be simply calculated in any other inertial reference frame.

Another, related motivation is that the content of classical electromagnetism can be written in a concise and convenient form using the electromagnetic four potential, especially when the Lorenz gauge is used. In particular, in abstract index notation, the set of Maxwell's equations (in the Lorenz gauge) may be written (in Gaussian units) as follows:

$$\partial^{\mu} A_{\mu} = 0$$

$$\Box A_{\mu} = \frac{4\pi}{c} J_{\mu}$$

where $\Box$ is the d'Alembertian and *J* is the four-current. The first equation is the Lorenz gauge condition while the second contains Maxwell's equations.

Yet another motivation for creating the electromagnetic four-potential is that it plays a very important role in quantum electrodynamics.

Magnetic scalar potential

The **magnetic scalar potential** is another useful tool in describing the magnetic field, especially for permanent magnets.

In a simply connected domain where there is no free current,

$$\nabla \times \mathbf{H} = 0,$$

hence we can define **magnetic scalar potential** ψ as[6]

$$\mathbf{H} = -\nabla \psi.$$

And since

$$\nabla \cdot \mathbf{B} = \mu_0 \nabla \cdot (\mathbf{H} + \mathbf{M}) = 0,$$

it follows that

$$\nabla^2 \psi = -\nabla \cdot \mathbf{H} = \nabla \cdot \mathbf{M}.$$

Here $\nabla \cdot \mathbf{M}$ acts as the source for magnetic field, much like $\nabla \cdot \mathbf{P}$ as the source for electric field. So analogously to bound electric charge, we can call

$$\rho_m = -\nabla \cdot \mathbf{M}$$

bound magnetic charge.

If there is free current, one may subtract the contribution of free current per Biot-Savart law from total magnetic field and solve the remainder with the scalar potential method.

Notes

[1] Feynman (1964, p. 15_15)
[2] Tensors and pseudo-tensors, lecture notes by Richard Fitzpatrick (http://farside.ph.utexas.edu/teaching/em/lectures/node120.html)
[3] Jackson (1999, p. 246)
[4] Kraus (1984, p. 189)
[5] Feynman (1964, p. 15_11)
[6] Vanderlinde (2005, pp. 194~199)

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Electric susceptibility

In electromagnetism, the **electric susceptibility** χ_e (latin: *susceptibilis* "receptiveness") is a dimensionless proportionality constant that indicates the degree of polarization of a dielectric material in response to an applied electric field. The greater the electric susceptibility, the greater the ability of a material to polarize in response to the field, and thereby reduce the total electric field inside the material (and store energy). It is in this way that the electric susceptibility influences the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.[1] [2]

Definition of Volume Susceptibility

Electric susceptibility is defined as the constant of proportionality (which may be a tensor) relating an electric field **E** to the induced dielectric polarization density **P** such that:

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E},$$

Where:

- $\mathbf{P}$ is the Polarization Density
- ε_0 is the Electric Permittivity of Free Space
- χ_e is the Electric Susceptibility
- $\mathbf{E}$ is the Electric Field

The susceptibility is also related to the polarizability of individual particles in the medium by the Clausius-Mossotti relation. The susceptibility is related to its relative permittivity ε_r by:

$$\chi_e = \varepsilon_r - 1$$

So in the case of a vacuum:

$$\chi_e \;= 0$$

At the same time, the electric displacement **D** is related to the polarization density **P** by:

$$\mathbf{D} \;=\; \varepsilon_0\mathbf{E} + \mathbf{P} \;=\; \varepsilon_0(1+\chi_e)\mathbf{E} \;=\; \varepsilon_r\varepsilon_0\mathbf{E}.$$

Molecular Polarizability

A similar parameter exists to relate the magnitude of the induced dipole moment **p** of an individual molecule to the local electric field **E** that induced the dipole. This parameter is the ***molecular polarizability*** and the dipole moment resulting from the local electric field E_{local} is given by:

$$\mathbf{p} = \varepsilon_0\alpha\mathbf{E}_{local}$$

This introduces a complication however, as locally the field can differ significantly from the overall applied field. We have:

$$\mathbf{P} = Np = N\varepsilon_0\alpha\mathbf{E}_{local},$$

where **P** is the polarization per unit volume, and **N** is the number of molecules per unit volume contributing to the polarization. Thus, if the local electric field is parallel to the ambient electric field, we have:

$$\frac{\chi_e}{N\alpha} = \frac{\mathbf{E}_{local}}{\mathbf{E}}$$

Thus only if the local field equals the ambient field can we write:

$$\chi_e \;= N\alpha$$

Dispersion and causality

In general, a material cannot polarize instantaneously in response to an applied field, and so the more general formulation as a function of time is

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{t} \chi_e(t-t')\mathbf{E}(t')\,dt'.$$

That is, the polarization is a convolution of the electric field at previous times with time-dependent susceptibility given by $\chi_e(\Delta t)$. The upper limit of this integral can be extended to infinity as well if one defines $\chi_e(\Delta t) = 0$ for $\Delta t < 0$. An instantaneous response corresponds to Dirac delta function susceptibility $\chi_e(\Delta t) = \chi_e\delta(\Delta t)$.

It is more convenient in a linear system to take the Fourier transform and write this relationship as a function of frequency. Due to the convolution theorem, the integral becomes a simple product,

$$\mathbf{P}(\omega) = \varepsilon_0\chi_e(\omega)\mathbf{E}(\omega).$$

This frequency dependence of the susceptibility leads to frequency dependence of the permittivity. The shape of the susceptibility with respect to frequency characterizes the dispersion properties of the material.

Moreover, the fact that the polarization can only depend on the electric field at previous times (i.e. $\chi_e(\Delta t) = 0$ for $\Delta t < 0$), a consequence of causality, imposes Kramers–Kronig constraints on the susceptibility $\chi_e(0)$.

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[1] "Electric susceptibility". *Encyclopedia Brittanica*.

[2] Cardarelli, François (2000, 2008). *Materials Handbook: A Concise Desktop Reference* (http://books.google.com/books?id=PvU-qbQJq7IC&pg=PA524&dq=Electric+susceptibility#v=onepage&q=Electric susceptibility&f=false) (2nd ed.). London: Springer-Verlag. pp. 524 (Section 8.1.16). doi:10.1007/978-1-84628-669-8. ISBN 9781846286681. .

Magnetic susceptibility

In electromagnetism, the **magnetic susceptibility** χ_m (latin: *susceptibilis* “receptiveness”) is a dimensionless proportionality constant that indicates the degree of magnetization of a material in response to an applied magnetic field. A related term is **magnetizability**, the proportion between magnetic moment and magnetic flux density.[1]

Definition of volume susceptibility

See also Relative permeability.

The **volume magnetic susceptibility**, represented by the symbol χ_v (often simply χ, sometimes χ_m — magnetic, to distinguish from the electric susceptibility), is defined by the relationship

$$\mathbf{M} = \chi_v \mathbf{H}$$

where, in SI units,

M is the magnetization of the material (the magnetic dipole moment per unit volume), measured in amperes per meter, and

H is the magnetic field strength, also measured in amperes per meter.

The magnetic induction **B** is related to **H** by the relationship

$$\mathbf{B}\ =\ \mu_0(\mathbf{H}+\mathbf{M})\ =\ \mu_0(1+\chi_v)\mathbf{H}\ =\ \mu\mathbf{H}$$

where μ_0 is the magnetic constant (see table of physical constants), and $(1+\chi_v)$ is the relative permeability of the material. Thus the **volume magnetic susceptibility** χ_v and the magnetic permeability μ are related by the following formula:

$$\mu = \mu_0(1+\chi_v).$$

Sometimes[2] an auxiliary quantity, called **intensity of magnetization** (also referred to as magnetic polarisation *J*) and measured in teslas, is defined as

$$\mathbf{I} = \mu_0 \mathbf{M}.$$

This allows an alternative description of all magnetization phenomena in terms of the quantities **I** and **B**, as opposed to the commonly used **M** and **H**.

Conversion between SI and CGS units

Note that these definitions are according to SI conventions. However, many tables of magnetic susceptibility give CGS values (more specifically emu-cgs, short for electromagnetic units, or Gaussian-cgs; both are the same in this context) that rely on a different definition of the permeability of free space:[3]

$$\mathbf{B}^{\rm cgs}\ =\ \mathbf{H}^{\rm cgs}+4\pi\mathbf{M}^{\rm cgs}\ =\ (1+4\pi\chi_v^{\rm cgs})\mathbf{H}^{\rm cgs}$$

The dimensionless CGS value of volume susceptibility is multiplied by 4π to give the dimensionless SI volume susceptibility value:[3]

$$\chi_v^{\rm SI} = 4\pi\chi_v^{\rm cgs}$$

For example, the CGS volume magnetic susceptibility of water at 20°C is -7.19×10^{-7} which is -9.04×10^{-6} using the SI convention.

Mass susceptibility and molar susceptibility

There are two other measures of susceptibility, the **mass magnetic susceptibility** (χ_{mass} or χ_g, sometimes χ_m), measured in $m^3 \cdot kg^{-1}$ in SI or in $cm^3 \cdot g^{-1}$ in CGS and the **molar magnetic susceptibility** (χ_{mol}) measured in $m^3 \cdot mol^{-1}$ (SI) or $cm^3 \cdot mol^{-1}$ (CGS) that are defined below, where ρ is the density in $kg \cdot m^{-3}$ (SI) or $g \cdot cm^{-3}$ (CGS) and M is molar mass in $kg \cdot mol^{-1}$ (SI) or $g \cdot mol^{-1}$ (CGS).

$$\chi_{\text{mass}} = \chi_v / \rho$$
$$\chi_{\text{mol}} = M \chi_{\text{mass}} = M \chi_v / \rho$$

Sign of susceptibility: diamagnetics and other types of magnetism

If χ is positive, the material can be paramagnetic . In this case, the magnetic field in the material is strengthened by the induced magnetization. Alternatively, if χ is negative, the material is diamagnetic. As a result, the magnetic field in the material is weakened by the induced magnetization. Generally, non-magnetic materials are said para- or diamagnetic because they do not possess permanent magnetization without external magnetic field. Ferromagnetic, ferrimagnetic, or antiferromagnetic materials, which have positive susceptibility, possess permanent magnetization even without external magnetic field.

Experimental methods to determine susceptibility

Volume magnetic susceptibility is measured by the force change felt upon the application of a magnetic field gradient.[4] Early measurements were made using the Gouy balance where a sample is hung between the poles of an electromagnet. The change in weight when the electromagnet is turned on is proportional to the susceptibility. Today, high-end measurement systems use a superconductive magnet. An alternative is to measure the force change on a strong compact magnet upon insertion of the sample. This system, widely used today, is called the Evans balance.[5] For liquid samples, the susceptibility can be measured from the dependence of the NMR frequency of the sample on its shape or orientation.[6] [7] [8] [9] [10]

Tensor susceptibility

The **magnetic susceptibility** of most crystals is not a scalar. Magnetic response **M** is dependent upon the orientation of the sample and can occur in directions other than that of the applied field **H**. In these cases, volume susceptibility is defined as a tensor

$$M_i = \chi_{ij} H_j$$

where *i* and *j* refer to the directions (e.g., *x*, *y* and *z* in Cartesian coordinates) of the applied field and magnetization, respectively. The tensor is thus rank 2, dimension (3,3) describing the component of magnetization in the *i*-th direction from the external field applied in the *j*-th direction.

Differential susceptibility

In ferromagnetic crystals, the relationship between **M** and **H** is not linear. To accommodate this, a more general definition of **differential susceptibility** is used

$$\chi_{ij}^{d} = \frac{\partial M_i}{\partial H_j}$$

where χ_{ij}^{d} is a tensor derived from partial derivatives of components of **M** with respect to components of **H**. When the coercivity of the material parallel to an applied field is the smaller of the two, the differential susceptibility is a function of the applied field and self interactions, such as the magnetic anisotropy. When the material is not saturated, the effect will be nonlinear and dependent upon the domain wall configuration of the material.

Susceptibility in the frequency domain

When the magnetic susceptibility is measured in response to an AC magnetic field (i.e. a magnetic field that varies sinusoidally), this is called **AC susceptibility**. AC susceptibility (and the closely related "AC permeability") are complex quantities, and various phenomena (such as resonances) can be seen in AC susceptibility that cannot in constant-field (DC) susceptibility. In particular, when an ac-field is applied perpendicular to the detection direction (called the "transverse susceptibility" regardless of the frequency), the effect has a peak at the ferromagnetic resonance frequency of the material with a given static applied field. Currently, this effect is called the **microwave permeability** or **network ferromagnetic resonance** in the literature. These results are sensitive to the domain wall configuration of the material and eddy currents.

In terms of ferromagnetic resonance, the effect of an ac-field applied along the direction of the magnetization is called **parallel pumping**.

For a tutorial with more information on AC susceptibility measurements, see here (external link) [11].

Examples

Magnetic susceptibility of some materials

Material	Temperature	Pressure	χ_{mol} (molar susc.)		χ_{mass} (mass susc.)		χ_v (volume susc.)		M (molar mass)	ρ (density)
Units	(°C)	(atm)	SI ($m^3 \cdot mol^{-1}$)	CGS ($cm^3 \cdot mol^{-1}$)	SI ($m^3 \cdot kg^{-1}$)	CGS ($cm^3 \cdot g^{-1}$)	SI	CGS (*emu*)	(10^{-3} kg/mol) or (g/mol)	(10^3 kg/m^3) or (g/cm^3)
vacuum	Any	0	0	0	0	0	0	0	–	0
water [12]	20	1	-1.631×10^{-10}	-1.298×10^{-5}	-9.051×10^{-9}	-7.203×10^{-7}	-9.035×10^{-6}	-7.190×10^{-7}	18.015	0.9982
bismuth [13]	20	1	-3.55×10^{-9}	-2.82×10^{-4}	-1.70×10^{-8}	-1.35×10^{-6}	-1.66×10^{-4}	-1.32×10^{-5}	208.98	9.78
Diamond [14]	R.T.	1	-7.4×10^{-11}	-5.9×10^{-6}	-6.2×10^{-9}	-4.9×10^{-7}	-2.2×10^{-5}	-1.7×10^{-6}	12.01	3.513
Graphite [15] $\chi_\perp$ (to c-axis)	R.T.	1	-7.5×10^{-11}	-6.0×10^{-6}	-6.3×10^{-9}	-5.0×10^{-7}	-1.4×10^{-5}	-1.1×10^{-6}	12.01	2.267
Graphite [15] $\chi_{\parallel}$	R.T.	1	-3.2×10^{-9}	-2.6×10^{-4}	-2.7×10^{-7}	-2.2×10^{-5}	-6.1×10^{-4}	-4.9×10^{-5}	12.01	2.267
Graphite [15] $\chi_{\parallel}$	-173	1	-4.4×10^{-9}	-3.5×10^{-4}	-3.6×10^{-7}	-2.9×10^{-5}	-8.3×10^{-4}	-6.6×10^{-5}	12.01	2.267
He [16]	20	1	-2.38×10^{-11}	-1.89×10^{-6}	-5.93×10^{-9}	-4.72×10^{-7}	-9.85×10^{-10}	-7.84×10^{-11}	4.0026	0.000166
Xe [16]	20	1	-5.71×10^{-10}	-4.54×10^{-5}	-4.35×10^{-9}	-3.46×10^{-7}	-2.37×10^{-8}	-1.89×10^{-9}	131.29	0.00546
O_2 [16]	20	0.209	4.3×10^{-8}	3.42×10^{-3}	1.34×10^{-6}	1.07×10^{-4}	3.73×10^{-7}	2.97×10^{-8}	31.99	0.000278

N_2 [16]	20	0.781	-1.56×10^{-10}	-1.24×10^{-5}	-5.56×10^{-9}	-4.43×10^{-7}	-5.06×10^{-9}	-4.03×10^{-10}	28.01	0.000910
Al		1	2.2×10^{-10}	1.7×10^{-5}	7.9×10^{-9}	6.3×10^{-7}	2.2×10^{-5}	1.75×10^{-6}	26.98	2.70
Ag [17]	961	1					-2.31×10^{-5}	-1.84×10^{-6}	107.87	

Sources of confusion in published data

There are tables of magnetic susceptibility values published on-line that seem to have been uploaded from a substandard source,[18] which itself has probably borrowed heavily from the CRC Handbook of Chemistry and Physics. Some of the data (e.g. for Al, Bi, and diamond) are apparently in cgs **Molar Susceptibility** units, whereas that for water is in **Mass Susceptibility** units (see discussion above). The susceptibility table in the CRC Handbook is known to suffer from similar errors, and even to contain sign errors. Effort should be made to trace the data in such tables to the original sources, and to double-check the proper usage of units.

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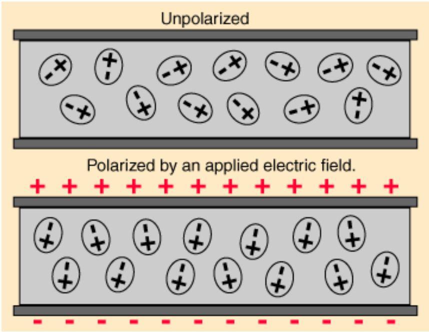
Permittivity

In electromagnetism, **absolute permittivity** is the measure of the resistance that is encountered when forming an electric field in a medium. In other words, permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium. The permittivity of a medium describes how much electric field (more correctly, flux) is 'generated' per unit charge. Less electric flux exists in a medium with a high permittivity (per unit charge) due to polarization effects. Permittivity is directly related to electric susceptibility, which is a measure of how easily a dielectric polarizes in response to an electric field. Thus, permittivity relates to a material's ability to transmit (or "permit") an electric field.



A dielectric medium showing orientation of charged particles creating polarization effects. Such a medium can have a higher ratio of electric flux to charge (permittivity) than empty space

In SI units, permittivity ε is measured in farads per meter (F/m); electric susceptibility χ is dimensionless. They are related to each other through

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1+\chi)\varepsilon_0$$

where ε_r is the relative permittivity of the material, and ϵ_0= 8.85… × 10^{-12} F/m is the vacuum permittivity.

Explanation

In electromagnetism, the electric displacement field **D** represents how an electric field **E** influences the organization of electrical charges in a given medium, including charge migration and electric dipole reorientation. Its relation to permittivity in the very simple case of *linear, homogeneous, isotropic* materials with *"instantaneous" response* to changes in electric field is

$$\mathbf{D} = \varepsilon \mathbf{E}$$

where the permittivity ε is a scalar. If the medium is anisotropic, the permittivity is a second rank tensor.

In general, permittivity is not a constant, as it can vary with the position in the medium, the frequency of the field applied, humidity, temperature, and other parameters. In a nonlinear medium, the permittivity can depend on the strength of the electric field. Permittivity as a function of frequency can take on real or complex values.

In SI units, permittivity is measured in farads per meter (F/m or $A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}$). The displacement field **D** is measured in units of coulombs per square meter (C/m^2), while the electric field **E** is measured in volts per meter (V/m). **D** and **E** describe the interaction between charged objects. **D** is related to the *charge densities* associated with this interaction, while **E** is related to the *forces* and *potential differences*.

Vacuum permittivity

The vacuum permittivity ε_0 (also called **permittivity of free space** or the **electric constant**) is the ratio **D/E** in free space. It also appears in the Coulomb force constant $1/4\pi\varepsilon_0$.

Its value is[1]

$$\varepsilon_0 \stackrel{\text{def}}{=} \frac{1}{c_0^2 \mu_0} = \frac{1}{35950207149.4727056\pi} \frac{\text{F}}{\text{m}} \approx 8.8541878176\ldots \times 10^{-12} \frac{\text{F}}{\text{m}}$$

where

c_0 is the speed of light in free space,[2]

μ_0 is the vacuum permeability.

Constants c_0 and μ_0 are defined in SI units to have exact numerical values, shifting responsibility of experiment to the determination of the meter and the ampere.[3] (The approximation in the second value of ε_0 above stems from π being an irrational number.)

Relative permittivity

The linear permittivity of a homogeneous material is usually given relative to that of free space, as a relative permittivity ε_r (also called dielectric constant, although this sometimes only refers to the static, zero-frequency relative permittivity). In an anisotropic material, the relative permittivity may be a tensor, causing birefringence. The actual permittivity is then calculated by multiplying the relative permittivity by ε_0:

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1+\chi)\varepsilon_0$$

where

χ (frequently written χ_e) is the electric susceptibility of the material.

The susceptibility is defined as the constant of proportionality (which may be a tensor) relating an electric field **E** to the induced dielectric polarization density **P** such that

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E},$$

where ε_0 is the electric permittivity of free space.

The susceptibility of a medium is related to its relative permittivity ε_r by

$$\chi \ = \varepsilon_r - 1.$$

So in the case of a vacuum,

$$\chi \ = 0.$$

The susceptibility is also related to the polarizability of individual particles in the medium by the Clausius-Mossotti relation.

The electric displacement **D** is related to the polarization density **P** by

$$\mathbf{D} \ = \ \varepsilon_0 \mathbf{E} + \mathbf{P} \ = \ \varepsilon_0 (1+\chi) \mathbf{E} \ = \ \varepsilon_r \varepsilon_0 \mathbf{E}.$$

The permittivity ε and permeability μ of a medium together determine the phase velocity $v = c/n$ of electromagnetic radiation through that medium:

$$\varepsilon \mu = \frac{1}{v^2}.$$

Dispersion and causality

In general, a material cannot polarize instantaneously in response to an applied field, and so the more general formulation as a function of time is

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{t} \chi(t-t') \mathbf{E}(t') \, dt'.$$

That is, the polarization is a convolution of the electric field at previous times with time-dependent susceptibility given by $\chi(\Delta t)$. The upper limit of this integral can be extended to infinity as well if one defines $\chi(\Delta t) = 0$ for $\Delta t < 0$. An instantaneous response corresponds to Dirac delta function susceptibility $\chi(\Delta t) = \chi \delta(\Delta t)$.

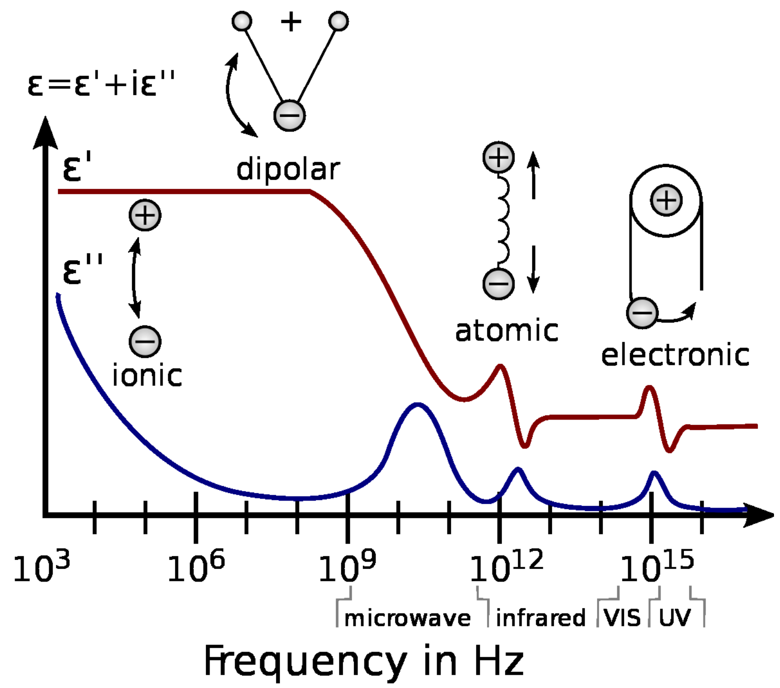
It is more convenient in a linear system to take the Fourier transform and write this relationship as a function of frequency. Due to the convolution theorem, the integral becomes a simple product,

$$\mathbf{P}(\omega) = \varepsilon_0 \chi(\omega) \mathbf{E}(\omega).$$

This frequency dependence of the susceptibility leads to frequency dependence of the permittivity. The shape of the susceptibility with respect to frequency characterizes the dispersion properties of the material.

Moreover, the fact that the polarization can only depend on the electric field at previous times (i.e. $\chi(\Delta t) = 0$ for $\Delta t < 0$), a consequence of causality, imposes Kramers–Kronig constraints on the susceptibility $\chi(0)$.

Complex permittivity



A dielectric permittivity spectrum over a wide range of frequencies. ε′ and ε″ denote the real and the imaginary part of the permittivity, respectively. Various processes are labeled on the image: ionic and dipolar relaxation, and atomic and electronic resonances at higher energies.[4]

As opposed to the response of a vacuum, the response of normal materials to external fields generally depends on the frequency of the field. This frequency dependence reflects the fact that a material's polarization does not respond instantaneously to an applied field. The response must always be *causal* (arising after the applied field) which can be represented by a phase difference. For this reason permittivity is often treated as a complex function (since complex numbers allow specification of magnitude and phase) of the (angular) frequency of the applied field ω, $\varepsilon \longrightarrow \hat{\varepsilon}(\omega)$. The definition of permittivity therefore becomes

$$D_0 e^{-i\omega t} = \hat{\varepsilon}(\omega) E_0 e^{-i\omega t},$$

where

D_0 and E_0 are the amplitudes of the displacement and electrical fields, respectively,
i is the imaginary unit, $i^2 = -1$.

The response of a medium to static electric fields is described by the low-frequency limit of permittivity, also called the static permittivity ε_s (also ε_{DC}):

$$\varepsilon_s = \lim_{\omega \to 0} \hat{\varepsilon}(\omega).$$

At the high-frequency limit, the complex permittivity is commonly referred to as ε_∞. At the plasma frequency and above, dielectrics behave as ideal metals, with electron gas behavior. The static permittivity is a good approximation for alternating fields of low frequencies, and as the frequency increases a measurable phase difference δ emerges between **D** and **E**. The frequency at which the phase shift becomes noticeable depends on temperature and the details of the medium. For moderate fields strength (E_0), **D** and **E** remain proportional, and

$$\hat{\varepsilon} = \frac{D_0}{E_0} = |\varepsilon| e^{i\delta}.$$

Since the response of materials to alternating fields is characterized by a complex permittivity, it is natural to separate its real and imaginary parts, which is done by convention in the following way:

$$\hat{\varepsilon}(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) = \frac{D_0}{E_0}\left(\cos\delta + i\sin\delta\right).$$

where

ε" is the imaginary part of the permittivity, which is related to the dissipation (or loss) of energy within the medium.

ε' is the real part of the permittivity, which is related to the stored energy within the medium.

It is important to realize that the choice of sign for time-dependence, $\exp(-i\omega t)$, dictates the sign convention for the imaginary part of permittivity. The signs used here correspond to those commonly used in physics, whereas for the engineering convention one should reverse all imaginary quantities.

The complex permittivity is usually a complicated function of frequency ω, since it is a superimposed description of dispersion phenomena occurring at multiple frequencies. The dielectric function ε(ω) must have poles only for frequencies with positive imaginary parts, and therefore satisfies the Kramers–Kronig relations. However, in the narrow frequency ranges that are often studied in practice, the permittivity can be approximated as frequency-independent or by model functions.

At a given frequency, the imaginary part of $\hat{\varepsilon}$ leads to absorption loss if it is positive (in the above sign convention) and gain if it is negative. More generally, the imaginary parts of the eigenvalues of the anisotropic dielectric tensor should be considered.

In the case of solids, the complex dielectric function is intimately connected to band structure. The primary quantity that characterizes the electronic structure of any crystalline material is the probability of photon absorption, which is directly related to the imaginary part of the optical dielectric function ε(ω). The optical dielectric function is given by the fundamental expression:[5]

$$\varepsilon(\omega) = 1 + \frac{8\pi^2 e^2}{m^2} \sum_{c,v} \int W_{cv}(E)\left[\varphi(\hbar\omega - E) - \varphi(\hbar\omega + E)\right]\, dx.$$

In this expression, $W_{cv}(E)$ represents the product of the Brillouin zone-averaged transition probability at the energy E with the joint density of states,[6] [7] $J_{cv}(E)$; φ is a broadening function, representing the role of scattering in smearing out the energy levels.[8] In general, the broadening is intermediate between Lorentzian and Gaussian;[9] [10] for an alloy it is somewhat closer to Gaussian because of strong scattering from statistical fluctuations in the local composition on a nanometer scale.

Classification of materials

Materials can be classified according to their permittivity and conductivity, σ. Materials with a large amount of loss inhibit the propagation of electromagnetic waves. In this case, generally when $\sigma/(\omega\varepsilon') \gg 1$, we consider the material to be a good conductor. Dielectrics are associated with lossless or low-loss materials, where $\sigma/(\omega\varepsilon') \ll 1$. Those that do not fall under either limit are considered to be general media. A *perfect dielectric* is a material that has no conductivity, thus exhibiting only a displacement current. Therefore it stores and returns electrical energy as if it were an ideal capacitor.

Lossy medium

In the case of lossy medium, i.e. when the conduction current is not negligible, the total current density flowing is:

$$J_{\text{tot}} = J_c + J_d = \sigma E - i\omega\varepsilon' E = -i\omega\hat{\varepsilon} E$$

where

σ is the conductivity of the medium;

ε' is the real part of the permittivity.

$\hat{\varepsilon}$ is the complex permittivity

The size of the displacement current is dependent on the frequency ω of the applied field *E*; there is no displacement current in a constant field.

In this formalism, the complex permittivity is defined as[11] :

$$\hat{\varepsilon} = \varepsilon' + i\frac{\sigma}{\omega}$$

In general, the absorption of electromagnetic energy by dielectrics is covered by a few different mechanisms that influence the shape of the permittivity as a function of frequency:

- First, are the relaxation effects associated with permanent and induced molecular dipoles. At low frequencies the field changes slowly enough to allow dipoles to reach equilibrium before the field has measurably changed. For frequencies at which dipole orientations cannot follow the applied field due to the viscosity of the medium, absorption of the field's energy leads to energy dissipation. The mechanism of dipoles relaxing is called dielectric relaxation and for ideal dipoles is described by classic Debye relaxation.
- Second are the resonance effects, which arise from the rotations or vibrations of atoms, ions, or electrons. These processes are observed in the neighborhood of their characteristic absorption frequencies.

The above effects often combine to cause non-linear effects within capacitors. For example, dielectric absorption refers to the inability of a capacitor that has been charged for a long time to completely discharge when briefly discharged. Although an ideal capacitor would remain at zero volts after being discharged, real capacitors will develop a small voltage, a phenomenon that is also called *soakage* or *battery action*. For some dielectrics, such as many polymer films, the resulting voltage may be less than 1-2% of the original voltage. However, it can be as much as 15 - 25% in the case of electrolytic capacitors or supercapacitors.

Quantum-mechanical interpretation

In terms of quantum mechanics, permittivity is explained by atomic and molecular interactions.

At low frequencies, molecules in polar dielectrics are polarized by an applied electric field, which induces periodic rotations. For example, at the microwave frequency, the microwave field causes the periodic rotation of water molecules, sufficient to break hydrogen bonds. The field does work against the bonds and the energy is absorbed by the material as heat. This is why microwave ovens work very well for materials containing water. There are two maxima of the imaginary component (the absorptive index) of water, one at the microwave frequency, and the other at far ultraviolet (UV) frequency. Both of these resonances are at higher frequencies than the operating frequency of microwave ovens.

At moderate frequencies, the energy is too high to cause rotation, yet too low to affect electrons directly, and is absorbed in the form of resonant molecular vibrations. In water, this is where the absorptive index starts to drop sharply, and the minimum of the imaginary permittivity is at the frequency of blue light (optical regime).

At high frequencies (such as UV and above), molecules cannot relax, and the energy is purely absorbed by atoms, exciting electron energy levels. Thus, these frequencies are classified as ionizing radiation.

While carrying out a complete *ab initio* (that is, first-principles) modelling is now computationally possible, it has not been widely applied yet. Thus, a phenomenological model is accepted as being an adequate method of capturing experimental behaviors. The Debye model and the Lorentz model use a 1st-order and 2nd-order (respectively) lumped system parameter linear representation (such as an RC and an LRC resonant circuit).

Measurement

The dielectric constant of a material can be found by a variety of static electrical measurements. The complex permittivity is evaluated over a wide range of frequencies by using different variants of dielectric spectroscopy, covering nearly 21 orders of magnitude from 10^{-6} to 10^{15} Hz. Also, by using cryostats and ovens, the dielectric properties of a medium can be characterized over an array of temperatures. In order to study systems for such diverse excitation fields, a number of measurement setups are used, each adequate for a special frequency range.

Various microwave measurement techniques are outlined in Chen *et al.*.[12] Typical errors for the Hakki-Coleman method employing a puck of material between conducting planes are about 0.3%.[13]

- Low-frequency time domain measurements (10^{-6}-10^{3} Hz)
- Low-frequency frequency domain measurements (10^{-5}-10^{6} Hz)
- Reflective coaxial methods (10^{6}-10^{10} Hz)
- Transmission coaxial method (10^{8}-10^{11} Hz)
- Quasi-optical methods (10^{9}-10^{10} Hz)
- Fourier-transform methods (10^{11}-10^{15} Hz)

At infrared and optical frequencies, a common technique is ellipsometry. Dual polarisation interferometry is also used to measure the complex refractive index for very thin films at optical frequencies.

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[2] Current practice of standards organizations such as NIST and BIPM is to use c_0, rather than *c*, to denote the speed of light in vacuum according to ISO 31. In the original Recommendation of 1983, the symbol *c* was used for this purpose. See NIST *Special Publication 330*, Appendix 2, p. 45 (http://physics.nist.gov/Pubs/SP330/sp330.pdf).

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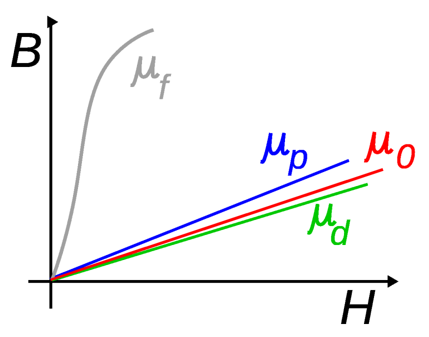
Further reading

- *Theory of Electric Polarization: Dielectric Polarization*, C.J.F. Böttcher, ISBN 0-444-41579-3
- *Dielectrics and Waves* edited by von Hippel, Arthur R., ISBN 0-89006-803-8
- *Dielectric Materials and Applications* edited by Arthur von Hippel, ISBN 0-89006-805-4.

External links

- Electromagnetism (http://lightandmatter.com/html_books/0sn/ch11/ch11.html), a chapter from an online textbook
- What's all this trapped charge stuff . . . (http://keith-snook.info/capacitor-soakage.html), A different approach to some capacitor problems

Permeability



Simplified comparison of permeabilities for: ferromagnets (μ_f), paramagnets(μ_p), free space(μ_0) and diamagnets (μ_d)

In electromagnetism, **permeability** is the measure of the ability of a material to support the formation of a magnetic field within itself. In other words, it is the degree of magnetization that a material obtains in response to an applied magnetic field. Magnetic permeability is typically represented by the Greek letter μ. The term was coined in September, 1885 by Oliver Heaviside. The reciprocal of magnetic permeability is **magnetic reluctivity**.

We can simplify it by saying, the more conductive a material is to a magnetic field, the higher its permeability.

In SI units, permeability is measured in the henry per metre ($H \cdot m^{-1}$), or newton per ampere squared ($N \cdot A^{-2}$). The permeability constant (μ_0), also known as the magnetic constant or the permeability of free space, is a measure of the amount of resistance encountered when forming a magnetic field in a classical vacuum. The magnetic constant has the exact (defined)[1] value $\mu_0 = 4\pi \times 10^{-7} \approx 1.2566370614\ldots \times 10^{-6}$ $H \cdot m^{-1}$ or $N \cdot A^{-2}$).

Explanation

In electromagnetism, the auxiliary magnetic field **H** represents how a magnetic field **B** influences the organization of magnetic dipoles in a given medium, including dipole migration and magnetic dipole reorientation. Its relation to permeability is

$$\mathbf{B} = \mu \mathbf{H}$$

where the **permeability μ** is a scalar if the medium is isotropic or a second rank tensor for an anisotropic medium.

In general, permeability is not a constant, as it can vary with the position in the medium, the frequency of the field applied, humidity, temperature, and other parameters. In a nonlinear medium, the permeability can depend on the strength of the magnetic field. Permeability as a function of frequency can take on real or complex values. In ferromagnetic materials, the relationship between **B** and **H** exhibits both non-linearity and hysteresis: **B** is not a single-valued function of **H**,[2] but depends also on the history of the material. For these materials it is sometimes useful to consider the *incremental permeability* defined as

$$\Delta \mathbf{B} = \mu_\Delta \Delta \mathbf{H}.$$

This definition is useful in local linearizations of non-linear material behavior, for example in a Newton-Raphson iterative solution scheme that computes the changing saturation of a magnetic circuit.

Permeability is the inductance per unit length. In SI units, permeability is measured in henries per metre ($H \cdot m^{-1} = J/(A^2 \cdot m) = N\ A^{-2}$). The auxiliary magnetic field **H** has dimensions current per unit length and is measured in units of amperes per metre ($A\ m^{-1}$). The product **μH** thus has dimensions inductance times current per unit area ($H \cdot A/m^2$). But inductance is magnetic flux per unit current, so the product has dimensions magnetic flux per unit area. This is just the magnetic field **B**, which is measured in webers (volt-seconds) per square-metre ($V \cdot s/m^2$), or teslas (T).

B is related to the Lorentz force on a moving charge q:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

The charge q is given in coulombs (C), the velocity v in m/s, so that the force F is in newtons (N):

$$q\mathbf{v} \times \mathbf{B} = \mathrm{C} \cdot \frac{\mathrm{m}}{\mathrm{s}} \cdot \frac{\mathrm{V} \cdot \mathrm{s}}{\mathrm{m}^2} = \frac{\mathrm{C} \cdot (\mathrm{J} \ / \ \mathrm{C})}{\mathrm{m}} = \frac{\mathrm{J}}{\mathrm{m}} = \mathrm{N}$$

H is related to the magnetic dipole density. A magnetic dipole is a closed circulation of electric current. The dipole moment has dimensions current times area, units ampere square-metre ($A \cdot m^2$), and magnitude equal to the current around the loop times the area of the loop.[3] The **H** field at a distance from a dipole has magnitude proportional to the dipole moment divided by distance cubed,[4] which has dimensions current per unit length.

Relative permeability

Relative permeability, sometimes denoted by the symbol μ_r, is the ratio of the permeability of a specific medium to the permeability of free space given by the magnetic constant $\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$:

$$\mu_r = \frac{\mu}{\mu_0}.$$

In terms of relative permeability, the magnetic susceptibility is:

$$\chi_m = \mu_r - 1.$$

χ_m, a dimensionless quantity, is sometimes called *volumetric* or *bulk* susceptibility, to distinguish it from χ_p (*magnetic mass* or *specific* susceptibility) and χ_M (*molar* or *molar mass* susceptibility).

Diamagnetism

Diamagnetism is the property of an object which causes it to create a magnetic field in opposition of an externally applied magnetic field, thus causing a repulsive effect. Specifically, an external magnetic field alters the orbital velocity of electrons around their nuclei, thus changing the magnetic dipole moment in the direction opposing the external field. Diamagnets are materials with a magnetic permeability less than μ_0 (a relative permeability less than 1).

Consequently, diamagnetism is a form of magnetism that a substance exhibits only in the presence of an externally applied magnetic field. It is generally a quite weak effect in most materials, although superconductors exhibit a strong effect.

Paramagnetism

Paramagnetism is a form of magnetism which occurs only in the presence of an externally applied magnetic field. Paramagnetic materials are attracted to magnetic fields, hence have a relative magnetic permeability greater than one (or, equivalently, a positive magnetic susceptibility). The magnetic moment induced by the applied field is *linear* in the field strength and rather *weak*. It typically requires a sensitive analytical balance to detect the effect. Unlike ferromagnets, paramagnets do not retain any magnetization in the absence of an externally applied magnetic field, because thermal motion causes the spins to become *randomly oriented* without it. Thus the total magnetization will drop to zero when the applied field is removed. Even in the presence of the field there is only a small *induced* magnetization because only a small fraction of the spins will be oriented by the field. This fraction is proportional to the field strength and this explains the linear dependency. The attraction experienced by ferromagnets is non-linear and much stronger, so that it is easily observed, for instance, in magnets on one's refrigerator.

Values for some common materials

The following table should be used with caution as the permeability of ferromagnetic materials varies greatly with field strength. For example 4% Si steel has an initial relative permeability (at or near 0T) of 2,000 and a maximum of 35,000[5] and, indeed, the relative permeability of any material at a sufficiently high field strength tends to 1.

Magnetic susceptibility and permeability data for selected materials

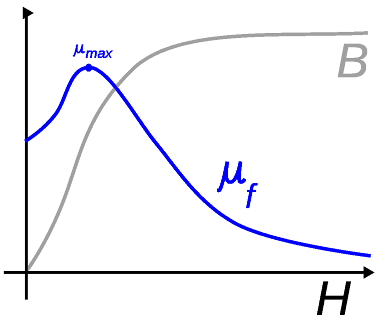
Medium	Susceptibility χ_m (volumetric SI)	Permeability μ [H/m]	Relative Permeability μ/μ_0	Magnetic field	Frequency max.
Metglas		1.25×10^{-1}	1000000[6]	at 0.5 T	100kHz
Nanoperm		10×10^{-2}	80000[7]	at 0.5 T	10kHz
Mu-metal		2.5×10^{-2}	20000[8]	at 0.002 T	
Mu-metal			50000[9]		
Permalloy		1.0×10^{-2}	8000[8]	at 0.002 T	
Electrical steel		5.0×10^{-3}	4000[8]	at 0.002 T	
Ferrite (nickel zinc)		$2.0 \times 10^{-5} – 8.0 \times 10^{-4}$	16–640		100 kHz ~ 1 MHz
Ferrite (manganese zinc)		$>8.0 \times 10^{-4}$	640 (or more)		100 kHz ~ 1 MHz
Steel		8.75×10^{-4}	100[8]	at 0.002 T	
Nickel		1.25×10^{-4}	100[8] – 600	at 0.002 T	
Neodymium magnet			1.05[10]		
Platinum		1.2569701×10^{-6}	1.000265		
Aluminum	2.22×10^{-5}[11]	1.2566650×10^{-6}	1.000022		
Wood			1.00000043[11]		
Air			1.00000037 [12]		
Concrete			1[13]		
Vacuum	0	1.2566371×10^{-6} (μ_0)	1[14]		
Hydrogen	-2.2×10^{-9}[11]	1.2566371×10^{-6}	1.0000000		
Teflon		1.2567×10^{-6}[8]	1.0000		
Sapphire	-2.1×10^{-7}	1.2566368×10^{-6}	0.99999976		
Copper	-6.4×10^{-6} or -9.2×10^{-6}[11]	1.2566290×10^{-6}	0.999994		
Water	-8.0×10^{-6}	1.2566270×10^{-6}	0.999992		
Bismuth	-1.66×10^{-4}		0.999834		

Superconductors	−1	0	0		

A good magnetic core material must have high permeability.

For magnetic levitation a relative permeability below 1 is needed (corresponding to a negative susceptibility).

Permeability varies with magnetic field. Values shown above are approximate and valid only at the magnetic fields shown. Moreover, they are given for a zero frequency; in practice, the permeability is generally a function of the frequency. When frequency is considered the permeability can be complex, corresponding to the in phase and out of phase response.



Magnetisation curve for ferromagnets (and ferrimagnets) and corresponding permeability

Note that the magnetic constant μ_0 has an exact value in SI units (that is, there is no uncertainty in its value), because the definition of the ampere fixes its value to $4\pi \times 10^{-7}$ H/m exactly.

Complex permeability

A useful tool for dealing with high frequency magnetic effects is the complex permeability. While at low frequencies in a linear material the magnetic field and the auxiliary magnetic field are simply proportional to each other through some scalar permeability, at high frequencies these quantities will react to each other with some lag time.[15] These fields can be written as phasors, such that

$$H = H_0 e^{j\omega t} \qquad B = B_0 e^{j(\omega t - \delta)}$$

where δ is the phase delay of B from H . Understanding permeability as the ratio of the magnetic field to the auxiliary magnetic field, the ratio of the phasors can be written and simplified as

$$\mu = \frac{B}{H} = \frac{B_0 e^{j(\omega t - \delta)}}{H_0 e^{j\omega t}} = \frac{B_0}{H_0} e^{-j\delta},$$

so that the permeability becomes a complex number. By Euler's formula, the complex permeability can be translated from polar to rectangular form,

$$\mu = \frac{B_0}{H_0} \cos\delta - j\frac{B_0}{H_0} \sin\delta = \mu' - j\mu''.$$

The ratio of the imaginary to the real part of the complex permeability is called the loss tangent,

$$\tan\delta = \frac{\mu''}{\mu'},$$

which provides a measure of how much power is lost in a material versus how much is stored.

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External links

- Electromagnetism (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html) - a chapter from an online textbook
- Relative Permeability (http://hyperphysics.phy-astr.gsu.edu/hbase/solids/ferro.html)
- Soil Permeability Test (http://www.denichsoiltest.com)
- Magnetic Properties of Materials (http://www.ee.surrey.ac.uk/Workshop/advice/coils/mu/)

Magnetization

In classical electromagnetism, **magnetization** [1] or **magnetic polarization** is the vector field that expresses the density of permanent or induced magnetic dipole moments in a magnetic material. The origin of the magnetic moments responsible for magnetization can be either microscopic electric currents resulting from the motion of electrons in atoms, or the spin of the electrons or the nuclei. Net magnetization results from the response of a material to an external magnetic field, together with any unbalanced magnetic dipole moments that may be inherent in the material itself; for example, in ferromagnets. Magnetization is not always homogeneous within a body, but rather varies between different points. Magnetization also describes how a material responds to an applied magnetic field as well as the way the material changes the magnetic field, and can be used to calculate the forces that result from those interactions. It can be compared to electric polarization, which is the measure of the corresponding response of a material to an electric field in electrostatics. Physicists and engineers define magnetization as the quantity of magnetic moment per unit volume. It is represented by a vector M.

Definition

Magnetization can be defined according to the following equation:

$$\mathbf{M} = \frac{N}{V}\mathbf{m} = n\mathbf{m}$$

Here, **M** represents magnetization; **m** is the vector that defines the magnetic moment; V represents volume; and *N* is the number of magnetic moments in the sample. The quantity *N/V* is usually written as *n*, the number density of magnetic moments. The M-field is measured in amperes per meter (A/m) in SI units.[2]

Magnetization in Maxwell's equations

The behavior of magnetic fields (***B***, ***H***), electric fields (***E***, ***D***), charge density (ϱ), and current density (***J***) is described by Maxwell's equations. The role of the magnetization is described below.

Relations between B, H, and M

The magnetization defines the auxiliary magnetic field ***H*** as

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \text{ (SI units)}$$

$$\mathbf{B} = (\mathbf{H} + 4\pi\mathbf{M}) \text{ (Gaussian units)}$$

which is convenient for various calculations. The vacuum permeability μ_0 is, by definition, 4×10^{-7} V·s/(A·m).

A relation between ***M*** and ***H*** exists in many materials. In diamagnets and paramagnets, the relation is usually linear:

$$\mathbf{M} = \chi_m \mathbf{H}$$

where χ_m is called the volume magnetic susceptibility.

In ferromagnets there is no one-to-one correspondence between ***M*** and ***H*** because of hysteresis.

Magnetization current

The magnetization ***M*** makes a contribution to the current density ***J***, known as the **magnetization current** or **bound current**:

$$\mathbf{J_m} = \nabla \times \mathbf{M}$$

so that the total current density that enters Maxwell's equations is given by

$$\mathbf{J} = \mathbf{J_f} + \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t}$$

where $\boldsymbol{J}_{\mathrm{f}}$ is the electric current density of free charges (also called the **free current**), the second term is the contribution from the magnetization, and the last term is related to the electric polarization ***P***.

Magnetostatics

In the absence of free electric currents and time-dependent effects, Maxwell's equations describing the magnetic quantities reduce to

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$$
$$\nabla \times \mathbf{H} = 0$$

These equations can be easily solved in analogy with electrostatic problems where

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \times \mathbf{E} = 0$$

In this sense $-\nabla \cdot \mathbf{M}$ plays the role of a "magnetic charge density" analogous to the electric charge density ρ (see also demagnetizing field).

Magnetization is volume density of magnetic moment. That is: if a certain volume has magnetization $\mathbf{M}$ then the volume element dV has a magnetic moment of $d\mathbf{m} = \mathbf{M}\,dV$

Magnetization dynamics

Main article: Magnetization dynamics

The time-dependent behavior of magnetization becomes important when considering nanoscale and nanosecond timescale magnetization. Rather than simply aligning with an applied field, the individual magnetic moments in a material begin to precess around the applied field and come into alignment through relaxation as energy is transferred into the lattice.

Demagnetization

In addition to magnetization, there is also demagnetization. Demagnetization is the process by which the magnetic field of an object is reduced or eliminated.[3] The process of demagnetizing can be accomplished in many ways. One technique used for demagnetization is to heat the object above its Curie Temperature. The reason for this is that when a magnetic material is heated to its Curie Temperature, the material's magnetivity is eliminated. One other way of achieving demagnetization is to use an electric coil. If the object is retracted out of a coil with aternating current running through it, the object's dipoles will become randomized and the object will be demagnetized.[4]

Applications of Demagnetization

One application of demagnetization is to eliminate unwanted magnetic fields. The reason for doing this is that magnetic fields can have unwanted effects on different devices. In particular magnetic fields can affect electronic devices such as cell phones or computers. If such a device is going to be coming into contact with other possibly magnetic objects, the magnetic fields might need to be reduced in order to protect the electronic device. Therefore demagnetization is sometimes used to keep magnetic fields from damaging electrical devices.[5]

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Polarization

In classical electromagnetism, **polarization density** (or **electric polarization**, or simply **polarization**) is the vector field that expresses the density of permanent or induced electric dipole moments in a dielectric material. When a dielectric is placed in an external electric field, its molecules gain electric dipole moment and the dielectric is said to be polarized. The electric dipole moment induced per unit volume of the dielectric material is called the electric polarization of the dielectric. Polarization density also describes how a material responds to an applied electric field as well as the way the material changes the electric field, and can be used to calculate the forces that result from those interactions. It can be compared to magnetization, which is the measure of the corresponding response of a material to an magnetic field in magnetism. The SI unit of measure is coulombs per square metre, and polarization density is represented by a vector **P**.

Polarization density in Maxwell's equations

The behavior of electric fields (**E** and **D**), magnetic fields (**B**, **H**), charge density (ρ) and current density (**J**) are described by Maxwell's equations. The role of the polarization density **P** is described below.

Relations between E, D and P

The polarization density **P** defines the electric displacement field **D** as [1]

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

which is convenient for various calculations. Here ε_0 is the electric permittivity.

A relation between **P** and **E** exists in many materials, as described later in the article.

Bound charge

Electric polarization corresponds to a rearrangement of the bound electrons in the material, which creates an additional charge density, known as the **bound charge density** ρ_b:

$$\rho_b = -\nabla \cdot \mathbf{P}$$

so that the total charge density that enters Maxwell's equation for the divergence of **E** is given by

$$\rho = \rho_f + \rho_b$$

where ρ_f is the **free charge density** given by the divergence of **D**.

At the surface of the polarized material, the bound charge appears as a surface charge density

$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}_{\text{out}}$$

where $\hat{\mathbf{n}}_{\text{out}}$ is the normal vector. If **P** is uniform inside the material, this surface charge is the only bound charge.

When the polarization density changes with time, the time-dependent bound-charge density creates a *polarization current density* of

$$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t}$$

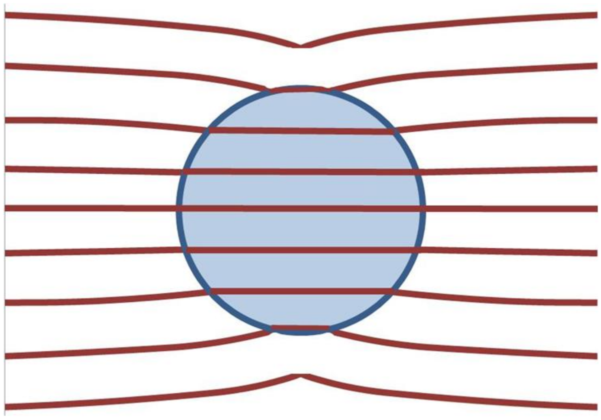
so that the total current density that enters Maxwell's equations is given by

$$\mathbf{J} = \mathbf{J}_f + \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t}$$

where $\mathbf{J}_f$ is the free-charge current density, and the second term is the magnetization current density (also called the *bound current density*), a contribution from atomic-scale magnetic dipoles (when they are present).

Relation between P and E in various materials

In a homogeneous linear and isotropic dielectric medium, the **polarization** is aligned with and proportional to the electric field **E**:



Field lines of the **D**-field in a dielectric sphere with greater susceptibility than its surroundings, placed in a previously-uniform field.[2] The field lines of the **E**-field are not shown: These point in the same directions, but many field lines start and end on the surface of the sphere, where there is bound charge. As a result, the density of E-field lines is lower inside the sphere than outside, which corresponds to the fact that the E-field is weaker inside the sphere than outside.

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E},$$

where ε_0 is the electric constant, and χ is the electric susceptibility of the medium.

In an *anisotropic* material, the polarization and the field are not necessarily in the same direction. Then, the i^{th} component of the polarization is related to the j^{th} component of the electric field according to:

$$P_i = \sum_j \epsilon_0 \chi_{ij} E_j,$$

where ε_0 is the electric constant, and $\chi_{ij,}$ is the electric susceptibility tensor of the medium. This relation shows, for example, that a material can polarize in the x direction by applying a field in the z direction, and so on. The case of an anisotropic dielectric medium is described by the field of crystal optics.

As in most electromagnetism, this relation deals with macroscopic averages of the fields and dipole density, so that one has a continuum approximation of the dielectric materials that neglects atomic-scale behaviors. The polarizability of individual particles in the medium can be related to the average susceptibility and polarization density by the Clausius-Mossotti relation.

In general, the susceptibility is a function of the frequency ω of the applied field. When the field is an arbitrary function of time *t*, the polarization is a convolution of the Fourier transform of χ(ω) with the **E**(*t*). This reflects the fact that the dipoles in the material cannot respond instantaneously to the applied field, and causality considerations lead to the Kramers–Kronig relations.

If the polarization **P** is not linearly proportional to the electric field **E**, the medium is termed *nonlinear* and is described by the field of nonlinear optics. To a good approximation (for sufficiently weak fields, assuming no permanent dipole moments are present), **P** is usually given by a Taylor series in **E** whose coefficients are the nonlinear susceptibilities:

$$P_i/\epsilon_0 = \sum_j \chi_{ij}^{(1)} E_j + \sum_{jk} \chi_{ijk}^{(2)} E_j E_k + \sum_{jk\ell} \chi_{ijk\ell}^{(3)} E_j E_k E_\ell + \cdots$$

where $\chi^{(1)}$ is the linear susceptibility, $\chi^{(2)}$ is the second-order susceptibility (describing phenomena such as the Pockels effect, optical rectification and second-harmonic generation), and $\chi^{(3)}$ is the third-order susceptibility (describing third-order effects such as the Kerr effect and electric field-induced optical rectification).
In ferroelectric materials, there is no one-to-one correspondence between **P** and **E** at all because of hysteresis.

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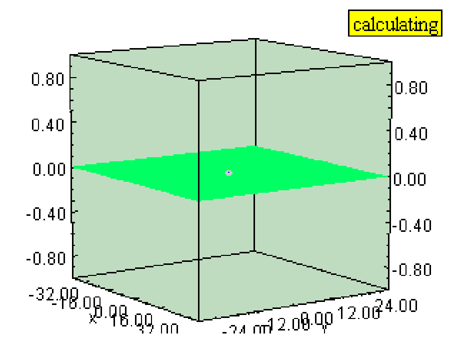
Scalar potential

A **scalar potential** is a fundamental concept in vector analysis and physics (the adjective *scalar* is frequently omitted if there is no danger of confusion with vector potential). The scalar potential is an example of a scalar field. Given a vector field **F**, the scalar potential *P* is defined such that:



gravitational potential well of a increasing mass where $\mathbf{F} = -\nabla P$

$$\mathbf{F} = -\nabla P = -\left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z}\right),$$ [1]

where **∇**P is the gradient of P and the second part of the equation is minus the gradient for a function of the Cartesian coordinates x,y,z.[2] In some cases, mathematicians may use a positive sign in front of the gradient to define the potential.[3] Because of this definition of P in terms of the gradient, the direction of **F** at any point is the direction of the steepest decrease of P at that point, its magnitude is the rate of that decrease per unit length.

In order for **F** to be described in terms of a scalar potential only, the following have to be true:

1. $-\int_a^b \mathbf{F} \cdot d\mathbf{l} = P(\mathbf{b}) - P(\mathbf{a})$, where the integration is over a Jordan arc passing from location **a** to location **b** and P(**b**) is P evaluated at location **b** .
2. $\oint \mathbf{F} \cdot d\mathbf{l} = 0$, where the integral is over any simple closed path, otherwise known as a Jordan curve.
3. $\nabla \times \mathbf{F} = 0$

The first of these conditions represents the fundamental theorem of the gradient and is true for any vector field that is a gradient of a differentiable single valued scalar field P. The second condition is a requirement of **F** so that it can be expressed as the gradient of a scalar function. The third condition re-expresses the second condition in terms of the curl of **F** using the fundamental theorem of the curl. A vector field **F** that satisfies these conditions is said to be irrotational (Conservative).

Scalar potentials play a prominent role in many areas of physics and engineering. The gravity potential is the scalar potential associated with the gravity per unit mass, i.e., the acceleration due to the field, as a function of position. The gravity potential is the gravitational potential energy per unit mass. In electrostatics the electric potential is the scalar potential associated with the electric field, i.e., with the electrostatic force per unit charge. The electric potential is in this case the electrostatic potential energy per unit charge. In fluid dynamics, irrotational lamellar fields have a scalar potential only in the special case when it is a Laplacian field. Certain aspects of the nuclear force can be described by a Yukawa potential. The potential play a prominent role in the Lagrangian and Hamiltonian formulations of classical mechanics. Further, the scalar potential is the fundamental quantity in quantum mechanics.

Not every vector field has a scalar potential. Those that do are called **conservative**, corresponding to the notion of conservative force in physics. Examples of non-conservative forces include frictional forces, magnetic forces, and in fluid mechanics a solenoidal field velocity field. By the Helmholtz decomposition theorem however, all vector fields can be describable in terms of a scalar potential and corresponding vector potential. In electrodynamics the electromagnetic scalar and vector potentials are known together as the electromagnetic four-potential.

Integrability conditions

If **F** is a conservative vector field (also called *irrotational*, *curl-free*, or *potential*), and its components have continuous partial derivatives, the **potential** of **F** with respect to a reference point $\mathbf{r}_0$ is defined in terms of the line integral:

$$V(\mathbf{r}) = -\int_C \mathbf{F}(\mathbf{r})\cdot\, d\mathbf{r} = -\int_a^b \mathbf{F}(\mathbf{r}(t))\cdot \mathbf{r}'(t)\, dt.$$

where *C* is a parametrized path from $\mathbf{r}_0$ to $\mathbf{r}$,

$$\mathbf{r}(t), a \leq t \leq b, \mathbf{r}(a) = \mathbf{r_0}, \mathbf{r}(b) = \mathbf{r}.$$

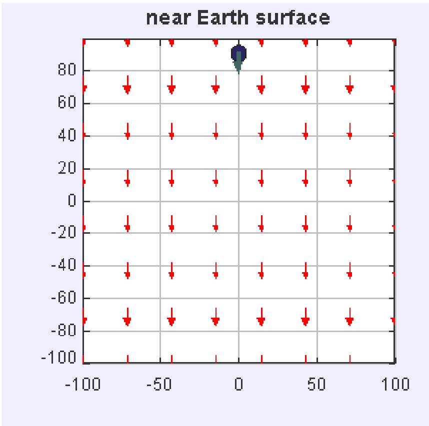
The fact that the line integral depends on the path *C* only through its terminal points $\mathbf{r}_0$ and $\mathbf{r}$ is, in essence, the **path independence property** of a conservative vector field. The fundamental theorem of calculus for line integrals implies that if *V* is defined in this way, then $\mathbf{F} = -\nabla V$, so that *V* is a scalar potential of the conservative vector field **F**. Scalar potential is not determined by the vector field alone: indeed, the gradient of a function is unaffected if a constant is added to it. If *V* is defined in terms of the line integral, the ambiguity of *V* reflects the freedom in the choice of the reference point $\mathbf{r}_0$.

Altitude as gravitational potential energy

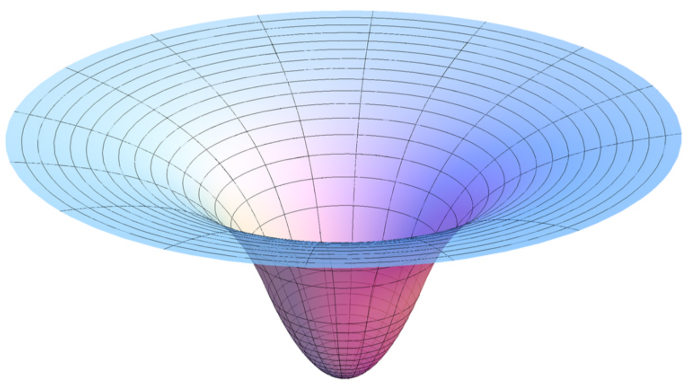
An example is the (nearly) uniform gravitational field near the Earth's surface. It has a potential energy

$$U = mgh$$

where *U* is the gravitational potential energy and *h* is the height above the surface. This means that gravitational potential energy on a contour map is proportional to altitude. On a contour map, the two-dimensional negative gradient of the altitude is a two-dimensional vector field, whose vectors are always perpendicular to the contours and also perpendicular to the direction of gravity. But on the hilly region represented by the contour map, the three-dimensional negative gradient of *U* always points straight downwards in the direction of gravity; **F**. However, a ball rolling down a hill cannot move directly downwards due to the normal force of the hill's surface, which cancels out the component of gravity perpendicular to the hill's surface. The component of gravity that remains to move the ball is parallel to the surface:



uniform gravitational field near the Earth's surface


Plot of a two-dimensional slice of the gravitational potential in and around a uniform spherical body. The inflection points of the cross-section are at the surface of the body.

$$F_S = -mg \ \sin\theta$$

where θ is the angle of inclination, and the component of F_S perpendicular to gravity is

$$F_P = -mg \ \sin\theta \ \cos\theta = -\frac{1}{2} mg \sin 2\theta.$$

This force F_P, parallel to the ground, is greatest when θ is 45 degrees.

Let Δh be the uniform interval of altitude between contours on the contour map, and let Δx be the distance between two contours. Then

$$\theta = \tan^{-1} \frac{\Delta h}{\Delta x}$$

so that

$$F_P = -mg \frac{\Delta x \, \Delta h}{\Delta x^2 + \Delta h^2}.$$

However, on a contour map, the gradient is inversely proportional to Δx, which is not similar to force F_P: altitude on a contour map is not exactly a two-dimensional potential field. The magnitudes of forces are different, but the directions of the forces are the same on a contour map as well as on the hilly region of the Earth's surface represented by the contour map.

Pressure as buoyant potential

In fluid mechanics, a fluid in equilibrium, but in the presence of a uniform gravitational field is permeated by a uniform buoyant force that cancels out the gravitational force: that is how the fluid maintains its equilibrium. This buoyant force is the negative gradient of pressure:

$$\mathbf{f_B} = -\nabla p.$$

Since buoyant force points upwards, in the direction opposite to gravity, then pressure in the fluid increases downwards. Pressure in a static body of water increases proportionally to the depth below the surface of the water. The surfaces of constant pressure are planes parallel to the ground. The surface of the water can be characterized as a plane with zero pressure.

If the liquid has a vertical vortex (whose axis of rotation is perpendicular to the ground), then the vortex causes a depression in the pressure field. The surfaces of constant pressure are parallel to the ground far away from the vortex, but near and inside the vortex the surfaces of constant pressure are pulled downwards, closer to the ground. This also happens to the surface of zero pressure. Therefore, inside the vortex, the top surface of the liquid is pulled downwards into a depression, or even into a tube (a solenoid).

The buoyant force due to a fluid on a solid object immersed and surrounded by that fluid can be obtained by integrating the negative pressure gradient along the surface of the object:

$$F_B = -\oint_S \nabla p \cdot \, d\mathbf{S}.$$

A moving airplane wing makes the air pressure above it decrease relative to the air pressure below it. This creates enough buoyant force to counteract gravity.

Calculating the scalar potential

Given a vector field **E**, its scalar potential Φ can be calculated to be

$$\phi(\mathbf{R_0}) = \frac{1}{4\pi}\int_\tau \frac{\nabla\cdot\mathbf{E}(\tau)}{\|\mathbf{R}(\tau)-\mathbf{R_0}\|}\,d\tau$$

where τ is volume. Then, if **E** is irrotational (Conservative),

$$\mathbf{E} = -\nabla\phi = -\frac{1}{4\pi}\nabla\int_\tau \frac{\nabla\cdot\mathbf{E}(\tau)}{\|\mathbf{R}(\tau)-\mathbf{R_0}\|}\,d\tau.$$

This formula is known to be correct if **E** is continuous and vanishes asymptotically to zero towards infinity, decaying faster than $1/r$ and if the divergence of **E** likewise vanishes towards infinity, decaying faster than $1/r^2$.

References

[1] Herbert Goldstein. *Classical Mechanics* (2 ed.). pp. 3–4. ISBN 9780201029185.

[2] The second part of this equation is ONLY valid for Cartesian coordinates, other coordinate systems such as cylindrical or spherical coordinates will have more complicated representations. derived from the fundamental theorem of the gradient.

[3] See (http://www.math.umn.edu/~nykamp/m2374/readings/findpot/) for an example where the potential is defined without a negative. Other references such as Louis Leithold, *The Calculus with Analytic Geometry* (5 ed.), p. 1199 avoid using the term *potential* when solving for a function from its gradient.

Vector potential

In vector calculus, a **vector potential** is a vector field whose curl is a given vector field. This is analogous to a *scalar potential*, which is a scalar field whose negative gradient is a given vector field.

Formally, given a vector field **v**, a *vector potential* is a vector field **A** such that

$$\mathbf{v} = \nabla\times\mathbf{A}.$$

If a vector field **v** admits a vector potential **A**, then from the equality

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

(divergence of the curl is zero) one obtains

$$\nabla\cdot\mathbf{v} = \nabla\cdot(\nabla\times\mathbf{A}) = 0,$$

which implies that **v** must be a solenoidal vector field.

An interesting question is then if any solenoidal vector field admits a vector potential. The answer is yes, if the vector field satisfies certain conditions.

Theorem

Let

$$\mathbf{v}:\mathbb{R}^3\to\mathbb{R}^3$$

be a solenoidal vector field which is twice continuously differentiable. Assume that **v**(**x**) decreases sufficiently fast as $\|\mathbf{x}\|\to\infty$. Define

$$\mathbf{A}(\mathbf{x}) = \frac{1}{4\pi}\nabla\times\int_{\mathbb{R}^3}\frac{\mathbf{v}(\mathbf{y})}{\|\mathbf{x}-\mathbf{y}\|}\,d\mathbf{y}.$$

Then, **A** is a vector potential for **v**, that is,

$$\nabla\times\mathbf{A} = \mathbf{v}.$$

A generalization of this theorem is the Helmholtz decomposition which states that any vector field can be decomposed as a sum of a solenoidal vector field and an irrotational vector field.

Nonuniqueness

The vector potential admitted by a solenoidal field is not unique. If **A** is a vector potential for **v**, then so is

$$\mathbf{A} + \nabla m$$

where *m* is any continuously differentiable scalar function. This follows from the fact that the curl of the gradient is zero.

This nonuniqueness leads to a degree of freedom in the formulation of electrodynamics, or gauge freedom, and requires choosing a gauge.

References

- *Fundamentals of Engineering Electromagnetics* by David K. Cheng, Addison-Wesley, 1993.

Vacuum permeability

Vacuum permeability, **permeability of free space**, or **magnetic constant** is an ideal, (baseline) physical constant, which is the value of magnetic permeability in a classical vacuum. *Vacuum permeability* is derived from production of a magnetic field by an electric current or by a moving electric charge and in all other formulas for magnetic-field production in a vacuum. When the permeability is that of the vacuum, denoted μ_0 has an exact defined value:[1] [2]

$$\mu_0 = 4 \times 10^{-7}\ \mathrm{V{\cdot}s/(A{\cdot}m)} \approx 1.2566370614 \times 10^{-6}\ \mathrm{H{\cdot}m^{-1}}\ \text{or}\ \mathrm{N{\cdot}A^{-2}}\ \text{or}\ \mathrm{T{\cdot}m/A}\ \text{or}\ \mathrm{Wb/(A{\cdot}m)}$$

in the SI system of units.

As a constant, it can also be defined as a fundamental invariant quantity, and is also one of three components that defines free space through Maxwell's equations. In classical physics, *free space* is a concept of electromagnetic theory, corresponding to a theoretically perfect vacuum and sometimes referred to as the **vacuum of free space**, or as **classical vacuum**, and is appropriately viewed as a *reference* medium.[1] [3]

The ampere defines vacuum permeability

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.

Adopted in 1948, the effect of this definition is to fix the magnetic constant (permeability of vacuum) at exactly $4 \times 10^{-7}\ \mathrm{H{\cdot}m^{-1}}$.[4] To further illustrate:

Two thin, straight, stationary, parallel wires, a distance *r* apart in free space, each carrying a current *I*, will exert a force on each other. Ampère's force law states that the force per unit length is given by[5]

$$F_{\mathrm{m}} = \frac{\mu_0 I^2}{2\pi r}.$$

The ampere is defined such that if the wires are 1 m apart and the current in each wire is 1 A, the force between the two wires is $2 \times 10^{-7}\ \mathrm{N{\cdot}m^{-1}}$. Hence the value of μ_0 is *defined* to be exactly

$$\mu_0 = 4 \times 10^{-7}\ \mathrm{N{\cdot}A^{-2}} \approx 1.2566370614 \times 10^{-6}\ \mathrm{N{\cdot}A^{-2}}.$$ [6] [1]

Terminology

Historically, the constant μ_0 has had different names. In the 1987 IUPAP Red book, for example, this constant was still called *permeability of vacuum*.[7] Another, now rather rare and obsolete, term is "*magnetic permittivity of vacuum*". See, for example, Servant *et al.*[8] The term "vacuum permeability" (and variations thereof, such as "permeability of free space") remains very widespread. However, Standards Organizations have recently moved to **magnetic constant** as the preferred name for μ_0, although the older name continues to be listed as a synonym.[1]

The name "magnetic constant" is used by standards organizations in order to avoid use of the terms "permeability" and "vacuum", which have physical meanings. This change of preferred name has been made because μ_0 is a defined value, and is not the result of experimental measurement (see below).

Systems of units and historical origin of value of μ_0

In principle, there are several equation systems that could be used to set up a system of electrical quantities and units.[9] Since the late 19th century, the fundamental definitions of current units have been related to the definitions of mass, length and time units, using Ampère's force law. However, the precise way in which this has "officially" been done has changed many times, as measurement techniques and thinking on the topic developed. The overall history of the unit of electric current, and of the related question of how to define a set of equations for describing electromagnetic phenomena, is very complicated. Briefly, the basic reason why μ_0 has the value it does is as follows.

Ampère's force law describes the experimentally-derived fact that, for two thin, straight, stationary, parallel wires, a distance r apart, in each of which a current I flows, the force per unit length, F_m, that one wire exerts upon the other in the vacuum of free space would be given by

$$F_{\mathrm{m}} \propto \frac{I^2}{r}.$$

Writing the constant of proportionality as k_m gives

$$F_{\mathrm{m}} = k_{\mathrm{m}} \frac{I^2}{r}.$$

The form of k_m needs to be chosen in order to set up a system of equations, and a value then needs to be allocated in order to define the unit of current.

In the old "electromagnetic (emu)" system of equations defined in the late 1800s, k_m was chosen to be a pure number, 2, distance was measured in centimetres, force was measured in the cgs unit dyne, and the currents defined by this equation were measured in the "electromagnetic unit (emu) of current" (also called the "abampere"). A practical unit to be used by electricians and engineers, the ampere, was then defined as equal to one tenth of the electromagnetic unit of current.

In another system, the "rationalized-metre-kilogram-second (rmks) system" (or alternatively the "metre-kilogram-second-ampere (mksa) system"), k_m is written as $\mu_0/2\pi$, where μ_0 is a measurement-system constant called the "magnetic constant".[10] The value of μ_0 was chosen such that the rmks unit of current is equal in size to the ampere in the emu system: μ_0 is *defined* to be $4\pi \times 10^{-7}$ N A^{-2}.[4]

Historically, several different systems (including the two described above) were in use simultaneously. In particular, physicists and engineers used different systems, and physicists used three different systems for different parts of physics theory and a fourth different system (the engineers' system) for laboratory experiments. In 1948, international decisions were made by standards organizations to adopt the rmks system, and its related set of electrical quantities and units, as the single main international system for describing electromagnetic phenomena in the International System of Units.

Ampère's law as stated above describes a physical property of the world. However, the choices about the form of k_m and the value of μ_0 are totally human decisions, taken by international bodies composed of representatives of the national standards organizations of all participating countries. The parameter μ_0 is a measurement-system constant, not a physical constant that can be measured. It does not, in any meaningful sense, describe a physical property of the vacuum.[11] This is why the relevant Standards Organizations prefer the name "magnetic constant", rather than any name that carries the hidden and misleading implication that μ_0 describes some physical property of the vacuum.

Significance in electromagnetism

The magnetic constant μ_0 appears in Maxwell's equations, which describe the properties of electric and magnetic fields and electromagnetic radiation, and relate them to their sources. In particular, it appears in relationship to quantities such as permeability and magnetization density, such as the relationship that defines the magnetic **H**-field in terms of the magnetic **B**-field. In real media, this relationship has the form:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M},$$

where **M** is the magnetization density. In free space, **M** = 0.

Maxwell's laws show that speed of light in a vacuum, c_0 is related to the magnetic constant and the electric constant (vacuum permittivity), ε_0, by the formula

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

References and notes

[1] CODATA. "Magnetic constants" (http://physics.nist.gov/cgi-bin/cuu/Value?mu0) (2006 CODATA recommended values: Source of the CODATA internationally recommended values (http://physics.nist.gov/cuu/Constants/bibliography.html)). *Fundamental Physical Constants*. NIST. . Retrieved 2010-02-04.

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[3] Werner S. Weiglhofer and Akhlesh Lakhtakia (2003). "§ 4.1 The classical vacuum as reference medium" (http://books.google.com/?id=QtIP_Lr3gngC&pg=PA34). *Introduction to complex mediums for optics and electromagnetics*. SPIE Press. p. 34 *ff*. ISBN 9780819449474. .

[4] This choice defines the SI unit of current, the ampere: "Unit of electric current (ampere)" (http://physics.nist.gov/cuu/Units/ampere.html). *Historical context of the SI*. NIST. . Retrieved 2007-08-11.

[5] See for example Tipler, Paul A. (1992). *Physics for Scientists and Engineers, Third Edition, Extended Version*. New York, NY: Worth Publishers. p. 826. ISBN 0-87901-434-2.Equation 25-14

[6] "Magnetic constant" (http://physics.nist.gov/cgi-bin/cuu/Value?mu0). *2006 CODATA recommended values*. NIST. . Retrieved 2007-08-08.

[7] SUNAMCO Commission (1987). "Recommended values of the fundamental physical constants" (http://www-v2.sp.se/metrology/IUPAP_SUNAMCO/IUPAP SUNAMCO Commission_files/IUPAP_Red_book_1987/SUNAMCO Red book 1987/6_Recommended_fundamental_constants_iupap_sunamco_red_book_1987.pdf). *Symbols, Units, Nomenclature and Fundamental Constants in Physics* (http://www-v2.sp.se/metrology/IUPAP_SUNAMCO/IUPAP SUNAMCO Commission_files/IUPAP_Red_book_1987/SUNAMCO Red book 1987/index_red_book_iupap_sunamco_1987.htm). pp. 54. ; (the IUPAP "Red book").

[8] J R Lalanne, F Carmona & L Servant (1999). *Optical spectroscopies of electronic absorption.* (http://books.google.com/?id=7rWD-TdxKkMC&pg=PA10&lpg=PA10&dq=+"magnetic+permittivity") (World Scientific series in contemporary chemical physics, vol. 17. ed.). Singapore;London: World Scientific. p. 10. ISBN 9810238614. .

[9] For an introduction to the subject of choices for independent units, see John David Jackson (1998). *Classical electrodynamics* (http://worldcat.org/isbn/047130932X) (Third ed.). New York: Wiley. p. 154. ISBN 047130932X. .

[10] The decision to explicitly include the factor of 2π in k_m stems from the "rationalization" of the equations used to describe physical electromagnetic phenomena.

[11] The intrinsic properties of realizable vacuum, such as quantum vacuum, QCD vacuum, outer space or ultra-high vacuum are entirely separate from the definition of μ_0.

Vacuum permittivity

The physical constant ε_0, commonly called the **vacuum permittivity**, **permittivity of free space** or **electric constant**, relates the units for electric charge to mechanical quantities such as length and force.[1] For example, the force between two separated electric charges (in vacuum) is given by Coulomb's law:

$$F_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where q_1 and q_2 are the charges, and r is the distance between them. Likewise, ε_0 appears in Maxwell's equations, which describe the properties of electric and magnetic fields and electromagnetic radiation, and relate them to their sources.

Value

The value of ε_0 is *defined* by the formula

$$\varepsilon_0 = \frac{1}{\mu_0 c_0^2}$$

where c_0 is the speed of light in vacuum,[2] and μ_0 is the parameter that international Standards Organizations call the "magnetic constant" (commonly called vacuum permeability). Since μ_0 has the *defined* value $4\pi \times 10^{-7}$ H m^{-1},[3] and c_0 has the *defined* value 299792458 m·s^{-1},[4] it follows that ε_0 has a *defined* value given approximately by

$\varepsilon_0 \approx 8.854187817620... \times 10^{-12}$ F·m^{-1} (or $A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}$ in SI base units, or $C^2 \cdot N^{-1} \cdot m^{-2}$ or $C \cdot V^{-1} \cdot m^{-1}$ using other SI coherent units).[5] [6]

The ellipsis (...) does not indicate experimental uncertainty, but the arbitrary termination of a nonrecurring decimal. The historical origins of the electric constant ε_0, and its value, are explained in more detail below.

Under the proposals to redefine the ampere as a fixed number of elementary charges per second,[7] the electric constant would no longer have an exact fixed value. Instead, it would be defined by the equation

$$\varepsilon_0 = \frac{e^2}{2\alpha h c_0}$$

where e is the elementary charge, α is the fine structure constant and h is the Planck constant. The relative uncertainty in the value would be the same as that of the fine structure constant, currently 6.8×10^{-10}.[5]

Terminology

Historically, the parameter ε_0 has been known by many different names. The terms "vacuum permittivity" or its variants, such as "permittivity in/of vacuum",[8] [9] "permittivity of empty space",[10] or "permittivity of free space"[11] are widespread. Standards Organizations worldwide now use "electric constant" as a uniform term for this quantity,[5] and official standards documents have adopted the term (although they continue to list the older terms as synonyms).[12] [13]

Another historical synonym was "dielectric constant of vacuum", as "dielectric constant" was sometimes used in the past for the absolute permittivity.[14] [15] However, in modern usage "dielectric constant" typically refers exclusively to a relative permittivity $\varepsilon/\varepsilon_0$ and even this usage is considered "obsolete" by some standards bodies in favor of relative static permittivity.[13] [16] Hence, the term "dielectric constant of vacuum" for the electric constant ε_0 is considered obsolete by most modern authors, although occasional examples of continuing usage can be found.

As for notation, the constant can be denoted by either ε_0 or ϵ_0, using either of the common glyphs for the letter epsilon.

Historical origin of the parameter ε_0

As indicated above, the parameter ε_0 is a measurement-system constant. Its presence in the equations now used to define electromagnetic quantities is the result of the so-called "rationalization" process described below. But the method of allocating a value to it is a consequence of the result that Maxwell's equations predict that, in free space, electromagnetic waves move with the speed of light. Understanding why ε_0 has the value it does requires a brief understanding of the history of how electromagnetic measurement systems developed.

Rationalization of units

The experiments of Coulomb and others showed that the force F between two equal point-like "amounts" of electricity, situated a distance r apart in free space, should be given by a formula that has the form

$$F = k_\mathrm{e} Q^2/r^2,$$

where Q is a quantity that represents the amount of electricity present at each of the two points, and k_e is Coulomb's constant. If one is starting with no constraints, then the value of k_e may be chosen arbitrarily.[17] For each different choice of k_e there is a different "interpretation" of Q: to avoid confusion, each different "interpretation" has to be allocated a distinctive name and symbol.

In one of the systems of equations and units agreed in the late 19th century, called the "centimetre-gram-second electrostatic system of units" (the cgs esu system), the constant k_e was taken equal to 1, and a quantity now called "gaussian electric charge" q_s was defined by the resulting equation

$$F = {q_s}^2/r^2.$$

The unit of gaussian charge, the statcoulomb, is such that two units, a distance of 1 centimetre apart, repel each other with a force equal to the cgs unit of force, the dyne. Thus the unit of gaussian charge can also be written 1 dyne$^{1/2}$ cm. "Gaussian electric charge" is not the same mathematical quantity as modern (rmks) electric charge and is not measured in coulombs.

The idea subsequently developed that it would be better, in situations of spherical geometry, to include a factor 4π in equations like Coulomb's law, and write it in the form:

$$F = k'_\mathrm{e} {q'_s}^2/4\pi r^2.$$

This idea is called "rationalization". The quantities q'_s and k_e' are not the same as those in the older convention. Putting $k_\mathrm{e}'=1$ generates a unit of electricity of different size, but it still has the same dimensions as the cgs esu system.

The next step was to treat the quantity representing "amount of electricity" as a fundamental quantity in its own right, denoted by the symbol q, and to write Coulomb's Law in its modern form:

$$F = q^2/4\pi\epsilon_0 r^2.$$

The system of equations thus generated is known as the rationalized metre-kilogram-second (rmks) equation system, or "metre-kilogram-second-ampere (mksa)" equation system. This is the system used to define the SI units.[18] The new quantity q is given the name "rmks electric charge", or (nowadays) just "electric charge". Clearly, the quantity q_s used in the old cgs esu system is related to the new quantity q by

$$q_s = q/(k'_\mathrm{e}\epsilon_0)^{1/2}.$$

Determination of a value for ε_0

One now adds the requirement that one wants force to be measured in newtons, distance in metres, and charge to be measured in the engineers' practical unit, the coulomb, which is defined as the charge accumulated when a current of 1 ampere flows for one second. This shows that the parameter ε_0 should be allocated the unit $C^2 \cdot N^{-1} \cdot m^{-2}$ (or equivalent units - in practice "farads per metre").

In order to establish the numerical value of ε_0, one makes use of the fact that if one uses the rationalized forms of Coulomb's law and Ampère's force law (and other ideas) to develop Maxwell's equations, then the relationship stated above is found to exist between ε_0, μ_0 and c_0. In principle, one has a choice of deciding whether to make the coulomb or the ampere the fundamental unit of electricity and magnetism. The decision was taken internationally to use the ampere. This means that the value of ε_0 is determined by the values of c_0 and μ_0, as stated above. For a brief explanation of how the value of μ_0 is decided, see the article about μ_0.

Permittivity of real media

By convention, the electric constant ε_0 appears in the relationship that defines the electric displacement field **D** in terms of the electric field **E** and classical electrical polarization density **P** of the medium. In general, this relationship has the form

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}.$$

For a linear dielectric, **P** is assumed to be proportional to **E**, so one has

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_r \varepsilon_0 \mathbf{E}$$

where ε is the permittivity and ε_r the relative static permittivity. In vacuum, the polarization $\mathbf{P} = \mathbf{0}$, so $\varepsilon_r = 1$ and $\varepsilon = \varepsilon_0$.

Notes

[1] "electric constant" (http://www.electropedia.org/iev/iev.nsf/display?openform&ievref=121-11-03). *Electropedia: International Electrotechnical Vocabulary (IEC 60050)*. Geneva: International Electrotechnical Commission. . Retrieved 2010-04-02.

[2] Quote from NIST: "The symbol c_0 (or sometimes simply c) is the conventional symbol for the speed of light in vacuum. " See NIST *Special Publication 330*, p. 18 (http://physics.nist.gov/Pubs/SP330/sp330.pdf)

[3] See the last sentence of the NIST definition of ampere (http://physics.nist.gov/cuu/Units/ampere.html).

[4] See the last sentence of the NIST definition of meter (http://physics.nist.gov/cuu/Units/meter.html).

[5] Mohr, Peter J.; Taylor, Barry N.; Newell, David B. (2008). "CODATA Recommended Values of the Fundamental Physical Constants: 2006" (http://physics.nist.gov/cuu/Constants/codata.pdf). *Rev. Mod. Phys.* **80**: 633–730. Bibcode 2008RvMP...80..633M. doi:10.1103/RevModPhys.80.633. . Direct link to value (http://physics.nist.gov/cgi-bin/cuu/Value?ep0)..

[6] A summary of the definitions of c_0, μ_0 and ε_0 is provided in the 2006 CODATA Report: CODATA report, pp. 6-7 (http://physics.nist.gov/cuu/Constants/codata.pdf)

[7] "Recommendation E1" (http://www.bipm.org/utils/common/pdf/CCEM25.pdf). *Report of the 25th meeting (15–16 March 2007), Consultative Committee for Electricity and Magnetism (CCEM)*. Sèvres, France: International Bureau for Weights and Measures. .

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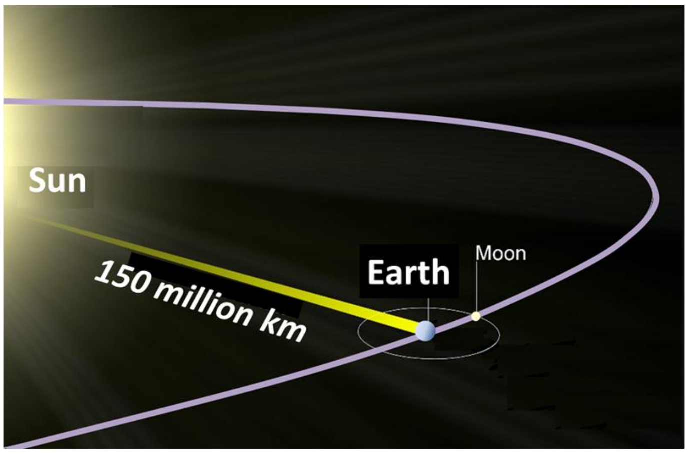
Speed of light

Speed of light



Sunlight takes about 8 minutes, 19 seconds to reach the Earth (based on the average distance).

Exact values	
Metres per second	299792458
Planck units	1
Approximate values	
kilometres per second	300,000
kilometres per hour	1,080 million
miles per second	186,000
miles per hour	671 million
astronomical units per day	173
Approximate light signal travel times	
Distance	**Time**
one foot	1.0 ns
one metre	3.3 ns
one kilometre	3.3 µs
one statute mile	5.4 µs
from geostationary orbit to Earth	119 ms
the length of Earth's equator	134 ms
from Moon to Earth	1.3 s
from Sun to Earth (1 AU)	8.3 min
from Voyager 1 to Earth	16.5 hours[1]

one parsec	3.26 years
from Proxima Centauri to Earth	4.24 years
from Alpha Centauri to Earth	4.37 years
from the nearest galaxy (the Canis Major Dwarf Galaxy) to Earth	25,000 years
across the Milky Way	100,000 years
from the Andromeda Galaxy to Earth	2.5 million years
from the furthest observed galaxy to Earth	13 billion years

The **speed of light** in vacuum, usually denoted by ***c***, is a physical constant important in many areas of physics. Its value is 299,792,458 metres per second, a figure that is exact since the length of the metre is defined from this constant and the international standard for time.[2] In imperial units this speed is approximately 186,282 miles per second.

According to special relativity, c is the maximum speed at which all energy, matter, and information in the universe can travel. It is the speed of all massless particles and associated fields—including electromagnetic radiation such as light—in vacuum, and it is predicted by the current theory to be the speed of gravity (that is, gravitational waves). Such particles and waves travel at c regardless of the motion of the source or the inertial frame of reference of the observer. In the theory of relativity, c interrelates space and time, and appears in the famous equation of mass–energy equivalence $E = mc^2$.[3]

The speed at which light propagates through transparent materials, such as glass or air, is less than c. The ratio between c and the speed v at which light travels in a material is called the refractive index n of the material ($n = c / v$). For example, for visible light the refractive index of glass is typically around 1.5, meaning that light in glass travels at $c / 1.5 \approx 200000$ km/s; the refractive index of air for visible light is about 1.0003, so the speed of light in air is about 90 km/s slower than c.

In most practical cases, light can be thought of as moving instantaneously, but for long distances and very sensitive measurements the finite speed of light has noticeable effects. In communicating with distant space probes, it can take minutes to hours for the message to get from Earth to the spacecraft and back. The light we see from stars left them many years ago, allowing us to study the history of the universe by looking at distant objects. The finite speed of light also limits the theoretical maximum speed of computers, since information must be sent within the computer from chip to chip. Finally, the speed of light can be used with time of flight measurements to measure large distances to high precision.

Ole Rømer first demonstrated in 1676 that light travelled at a finite speed (as opposed to instantaneously) by studying the apparent motion of Jupiter's moon Io. In 1865, James Clerk Maxwell proposed that light was an electromagnetic wave, and therefore traveled at the speed c appearing in his theory of electromagnetism. In 1905, Albert Einstein postulated that the speed of light with respect to any inertial frame is independent of the motion of the light source,[4] and explored the consequences of that postulate by deriving the special theory of relativity and showing that the parameter c had relevance outside of the context of light and electromagnetism. After centuries of increasingly precise measurements, in 1975 the speed of light was known to be 299792458 m/s with a relative measurement uncertainty of 4 parts per billion. In 1983, the metre was redefined in the International System of Units (SI) as the distance travelled by light in vacuum in $^{1}\!/_{299792458}$ of a second. As a result, the numerical value of c in metres per second is now fixed exactly by the definition of the metre.[5]

Numerical value, notation, and units

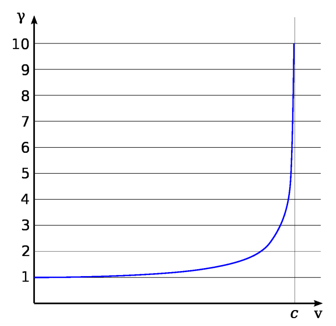
The speed of light in vacuum is usually denoted by c, for "constant" or the Latin *celeritas* (meaning "swiftness"). Originally, the symbol V was used, introduced by James Clerk Maxwell in 1865. In 1856, Wilhelm Eduard Weber and Rudolf Kohlrausch used c for a constant later shown to equal $\sqrt{2}$ times the speed of light in vacuum. In 1894, Paul Drude redefined c with its modern meaning. Einstein used V in his original German-language papers on special relativity in 1905, but in 1907 he switched to c, which by then had become the standard symbol.[6] [7]

Sometimes c is used for the speed of waves in *any* material medium, and c_0 for the speed of light in vacuum.[8] This subscripted notation, which is endorsed in official SI literature,[5] has the same form as other related constants: namely, μ_0 for the vacuum permeability or magnetic constant, ε_0 for the vacuum permittivity or electric constant, and Z_0 for the impedance of free space. This article uses c exclusively for the speed of light in vacuum.

In the International System of Units (SI), the metre is defined as the distance light travels in vacuum in $^1/_{299792458}$ of a second. This definition fixes the speed of light in vacuum at exactly 299792458 m/s.[9] [10] [11] As a dimensional physical constant, the numerical value of c is different for different unit systems.[12] In branches of physics in which c appears often, such as in relativity, it is common to use systems of natural units of measurement in which $c = 1$.[13] [14] Using these units, c does not appear explicitly because multiplication or division by 1 does not affect the result.

Fundamental role in physics

The speed at which light waves propagate in vacuum is independent both of the motion of the wave source and of the inertial frame of reference of the observer.[15] This invariance of the speed of light was postulated by Einstein in 1905,[4] after being motivated by Maxwell's theory of electromagnetism and the lack of evidence for the luminiferous aether;[16] it has since been consistently confirmed by many experiments. It is only possible to verify experimentally that the two-way speed of light (for example, from a source to a mirror and back again) is frame-independent, because it is impossible to measure the one-way speed of light (for example, from a source to a distant detector) without some convention as to how clocks at the source and at the detector should be synchronized. However, by adopting Einstein synchronization for the clocks, the one-way speed of light becomes equal to the two-way speed of light by definition.[14] [17] The special theory of relativity explores the consequences of this invariance of c with the assumption that the laws of physics are the same in all inertial frames of reference.[18] [19] One consequence is that c is the speed at which all massless particles and waves, including light, must travel in vacuum.



The Lorentz factor γ as a function of velocity. It starts at 1 and approaches infinity as v approaches c.

Special relativity has many counterintuitive and experimentally verified implications.[20] These include the equivalence of mass and energy ($E = mc^2$), length contraction (moving objects shorten),[21] and time dilation (moving clocks run slower). The factor γ by which lengths contract and times dilate, is known as the Lorentz factor and is given by $\gamma = (1 - v^2/c^2)^{-1/2}$, where v is the speed of the object. The difference of γ from 1 is negligible for speeds much slower than c, such as most everyday speeds—in which case special relativity is closely approximated by Galilean relativity—but it increases at relativistic speeds and diverges to infinity as v approaches c.

The results of special relativity can be summarized by treating space and time as a unified structure known as spacetime (with c relating the units of space and time), and requiring that physical theories satisfy a special symmetry called Lorentz invariance, whose mathematical formulation contains the parameter c.[22] Lorentz invariance is an almost universal assumption for modern physical theories, such as quantum electrodynamics, quantum chromodynamics, the Standard Model of particle physics, and general relativity. As such, the parameter c is ubiquitous in modern physics, appearing in many contexts that are unrelated to light. For example, general relativity predicts that c is also the speed of gravity and of gravitational waves.[23] [24] In non-inertial frames of reference (gravitationally curved space or accelerated reference frames), the *local* speed of light is constant and equal to c, but the speed of light along a trajectory of finite length can differ from c, depending on how distances and times are defined.[25]

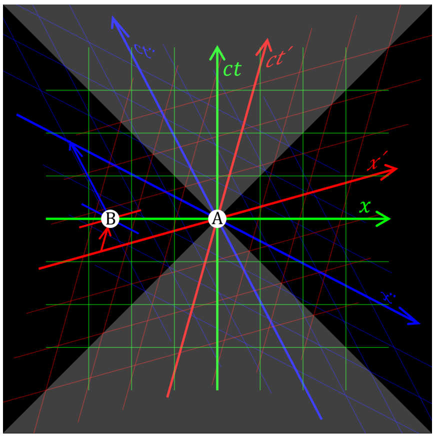
It is generally assumed that fundamental constants such as c have the same value throughout spacetime, meaning that they do not depend on location and do not vary with time. However, it has been suggested in various theories that the speed of light may have changed over time.[26] [27] No conclusive evidence for such changes has been found, but they remain the subject of ongoing research.[28] [29]

It also is generally assumed that the speed of light is isotropic, meaning that it has the same value regardless of the direction in which it is measured. Observations of the emissions from nuclear energy levels as a function of the orientation of the emitting nuclei in a magnetic field (see Hughes–Drever experiment), and of rotating optical resonators (see Resonator experiments) have put stringent limits on the possible two-way anisotropy.[30] [31]

Upper limit on speeds

According to special relativity, the energy of an object with rest mass m and speed v is given by γmc^2, where γ is the Lorentz factor defined above. When v is zero, γ is equal to one, giving rise to the famous $E = mc^2$ formula for mass-energy equivalence. Since the γ factor approaches infinity as v approaches c, it would take an infinite amount of energy to accelerate an object with mass to the speed of light. The speed of light is the upper limit for the speeds of objects with positive rest mass.[32]

More generally, it is normally impossible for information or energy to travel faster than c. One argument for this follows from the counter-intuitive implication of special relativity known as the relativity of simultaneity. If the spatial distance between two events A and B is greater than the time interval between them multiplied by c then there are frames of reference in which A precedes B, others in which B precedes A, and others in which they are simultaneous. As a result, if something were travelling faster than c relative to an inertial frame of reference, it would be travelling backwards in time relative to another frame, and causality would be violated.[33] [34] In such a frame of reference, an "effect" could be observed before its "cause". Such a violation of causality has never been recorded,[17] and would lead to paradoxes such as the tachyonic antitelephone.[35]



Event A precedes B in the red frame, is simultaneous with B in the green frame, and follows B in the blue frame.

Faster-than-light observations and experiments

There are situations in which it may seem that matter, energy, or information travels at speeds greater than c, but they do not. For example, as is discussed in the propagation of light in a medium section below, many wave velocities can exceed c. For example, the phase velocity of X-rays through most glasses can routinely exceed c,[36] but such waves do not convey any information.[37]

If a laser beam is swept quickly across a distant object, the spot of light can move faster than c, although the initial movement of the spot is delayed because of the time it takes light to get to the distant object at the speed c. However, the only physical entities that are moving are the laser and its emitted light, which travels at the speed c from the laser to the various positions of the spot. Similarly, a shadow projected onto a distant object can be made to move faster than c, after a delay in time.[38] In neither case does any matter, energy, or information travel faster than light.[39]

The rate of change in the distance between two objects in a frame of reference with respect to which both are moving (their closing speed) may have a value in excess of *c*. However, this does not represent the speed of any single object as measured in a single inertial frame.[39]

Certain quantum effects appear to be transmitted instantaneously and therefore faster than *c*, as in the EPR paradox. An example involves the quantum states of two particles that can be entangled. Until either of the particles is observed, they exist in a superposition of two quantum states. If the particles are separated and one particle's quantum state is observed, the other particle's quantum state is determined instantaneously (i.e., faster than light could travel from one particle to the other). However, it is impossible to control which quantum state the first particle will take on when it is observed, so information cannot be transmitted in this manner.[39] [40]

Another quantum effect that predicts the occurrence of faster-than-light speeds is called the Hartman effect; under certain conditions the time needed for a virtual particle to tunnel through a barrier is constant, regardless of the thickness of the barrier.[41] [42] This could result in a virtual particle crossing a large gap faster-than-light. However, no information can be sent using this effect.[43]

So-called superluminal motion is seen in certain astronomical objects,[44] such as the relativistic jets of radio galaxies and quasars. However, these jets are not moving at speeds in excess of the speed of light: the apparent superluminal motion is a projection effect caused by objects moving near the speed of light and approaching Earth at a small angle to the line of sight: since the light which was emitted when the jet was farther away took longer to reach the Earth, the time between two successive observations corresponds to a longer time between the instants at which the light rays were emitted.[45]

In models of the expanding universe, the farther galaxies are from each other, the faster they drift apart. This receding is not due to motion *through* space, but rather to the expansion of space itself.[39] For example, galaxies far away from Earth appear to be moving away from the Earth with a speed proportional to their distances. Beyond a boundary called the Hubble sphere, the rate at which their distance from Earth increases becomes greater than the speed of light.[46]

In late September 2011, physicists working at the OPERA experiment published results that seemed to suggest beams of neutrinos had travelled from CERN (in Geneva, Switzerland) to LNGS (at the Gran Sasso, Italy) faster than the speed of light, arriving (57.8 ± 7.8 (stat.) + 8.3/-5.9 (sys.)) nanoseconds early (corresponding to about 18 metres in a total distance of 730 kilometres)[47] These findings have yet to be independently verified.[48] and the OPERA researchers say they are going to "investigate possible still unknown systematic effects that could explain the observed anomaly" and "deliberately do not attempt any theoretical or phenomenological interpretation of the results."[47] Mid-November 2011 an updated version of the OPERA paper was released, addressing various issues raised by the community.[47]

Propagation of light

In classical physics, light is described as a type of electromagnetic wave. The classical behaviour of the electromagnetic field is described by Maxwell's equations, which predict that the speed *c* with which electromagnetic waves (such as light) propagate through the vacuum is related to the electric constant ε_0 and the magnetic constant μ_0 by the equation $c = 1/\sqrt{\varepsilon_0\mu_0}$.[49] In modern quantum physics, the electromagnetic field is described by the theory of quantum electrodynamics (QED). In this theory, light is described by the fundamental excitations (or quanta) of the electromagnetic field, called photons. In QED, photons are massless particles and thus, according to special relativity, they travel at the speed of light in vacuum.

Extensions of QED in which the photon has a mass have been considered. In such a theory, its speed would depend on its frequency, and the invariant speed *c* of special relativity would then be the upper limit of the speed of light in vacuum.[25] No variation of the speed of light with frequency has been observed in rigorous testing,[50] [51] [52] putting stringent limits on the mass of the photon. The limit obtained depends on the model used: if the massive photon is described by Proca theory,[53] the experimental upper bound for its mass is about 10^{-57} grams;[54] if photon mass is generated by a Higgs mechanism, the experimental upper limit is less sharp, $m \leq 10^{-14}$ eV/c^2 [53] (roughly 2×10^{-47} g).

Another reason for the speed of light to vary with its frequency would be the failure of special relativity to apply to arbitrarily small scales, as predicted by some proposed theories of quantum gravity. In 2009, the observation of the spectrum of gamma-ray burst GRB 090510 did not find any difference in the speeds of photons of different energies, confirming that Lorentz invariance is verified at least down to the scale of the Planck length ($l_P = \sqrt{\hbar G/c^3} \approx 1.6163 \times 10^{-35}$ m) divided by 1.2.[55]

In a medium

In a medium, light usually does not propagate at a speed equal to *c*; further, different types of light wave will travel at different speeds. The speed at which the individual crests and troughs of a plane wave (a wave filling the whole space, with only one frequency) propagate is called the phase velocity v_p. An actual physical signal with a finite extent (a pulse of light) travels at a different speed. The largest part of the pulse travels at the group velocity v_g, and its earliest part travels at the front velocity v_f.

The blue dot moves at the speed of the ripples, the phase velocity; the green dot moves with the speed of the envelope, the group velocity; and the red dot moves with the speed of the foremost part of the pulse, the front velocity

The phase velocity is important in determining how a light wave travels through a material or from one material to another. It is often represented in terms of a *refractive index*. The refractive index of a material is defined as the ratio of *c* to the phase velocity v_p in the material: larger indices of refraction indicate lower speeds. The refractive index of a material may depend on the light's frequency, intensity, polarization, or direction of propagation; in many cases, though, it can be treated as a material-dependent constant. The refractive index of air is approximately 1.0003.[56] Denser media, such as water,[57] glass,[58] and diamond,[59] have refractive indexes of around 1.3, 1.5 and 2.4 respectively for visible light.

In transparent materials, the refractive index generally is greater than 1, meaning that the phase velocity is less than *c*. In other materials, it is possible for the refractive index to become smaller than 1 for some frequencies; in some exotic materials it is even possible for the index of refraction to become negative.[60] The requirement that causality is not violated implies that the real and imaginary parts of the dielectric constant of any material, corresponding respectively to the index of refraction and to the attenuation coefficient, are linked by the Kramers–Kronig relations.[61] In practical terms, this means that in a material with refractive index less than 1, the absorption of the wave is so quick that no signal can be sent faster than *c*.

A pulse with different group and phase velocities (which occurs if the phase velocity is not the same for all the frequencies of the pulse) smears out over time, a process known as dispersion. Certain materials have an exceptionally low (or even zero) group velocity for light waves, a phenomenon called slow light, which has been confirmed in various experiments.[62] [63] [64] [65] The opposite, group velocities exceeding *c*, has also been shown in experiment.[66] It should even be possible for the group velocity to become infinite or negative, with pulses travelling instantaneously or backwards in time.[67]

None of these options, however, allow information to be transmitted faster than *c*. It is impossible to transmit information with a light pulse any faster than the speed of the earliest part of the pulse (the front velocity). It can be shown that this is (under certain assumptions) always equal to *c*.[67]

It is possible for a particle to travel through a medium faster than the phase velocity of light in that medium (but still slower than *c*). When a charged particle does that in an electrical insulator, the electromagnetic equivalent of a shock wave, known as Cherenkov radiation, is emitted.[68]

Practical effects of finiteness

The finiteness of the speed of light has implications for various sciences and technologies. In some cases, it is a hindrance: for example, *c*, being the upper limit of the speed with which signals can be sent, provides a theoretical upper limit for the operating speed of microprocessors.[69] On the other hand, some techniques depend on it, for example in distance measurements. Also, earth-based controllers must wait for round-trip communication lag that increases as spacecraft get farther away; NASA must wait several hours for information from a probe orbiting Jupiter, and if it needs to correct a navigation error, the fix will not arrive at the spacecraft for an equal amount of time, creating a risk of the correction not arriving in time.

The speed of light is of relevance to communications. For example, given the equatorial circumference of the Earth is about 40,075 km and *c* about 300,000 km/s, the theoretical shortest time for a piece of information to travel half the globe along the surface is about 67 milliseconds. When light is travelling around the globe in an optical fibre, the actual transit time is longer, in part because the speed of light is slower by about 35% in an optical fibre, depending on its refractive index *n*.[70] Furthermore, straight lines rarely occur in global communications situations, and delays are created when the signal passes through an electronic switch or signal regenerator.[71]

A beam of light is depicted travelling between the Earth and the Moon in the time it takes a light pulse to move between them: 1.255 seconds at their mean orbital (surface-to-surface) distance. The relative sizes and separation of the Earth–Moon system are shown to scale.

Another consequence of the finite speed of light is that communications between the Earth and spacecraft are not instantaneous. There is a brief delay from the source to the receiver, which becomes more noticeable as distances increase. This delay was significant for communications between ground control and Apollo 8 when it became the first manned spacecraft to orbit the Moon: for every question, the ground control station had to wait at least three seconds for the answer to arrive.[72] The communications delay between Earth and Mars can vary between five and twenty minutes depending upon the relative positions of the two planets. As a consequence of this, if a robot on the surface of Mars were to encounter a problem, its human controllers would not be aware of it until at least five minutes later, and possibly up to twenty minutes later; it would then take a further five to twenty minutes for instructions to travel from Earth to Mars.

The speed of light can also be of concern over very short distances. In supercomputers, the speed of light imposes a limit on how quickly data can be sent between processors. If a processor operates at 1 gigahertz, a signal can only travel a maximum of about 30 centimetres (1 ft) in a single cycle. Processors must therefore be placed close to each other to minimize communication latencies; this can cause difficulty with cooling. If clock frequencies continue to increase, the speed of light will eventually become a limiting factor for the internal design of single chips.[69]

Distance measurement

Radar systems measure the distance to a target by the time it takes a radio-wave pulse to return to the radar antenna after being reflected by the target: the distance to the target is half the round-trip transit time multiplied by the speed of light. A Global Positioning System (GPS) receiver measures its distance to GPS satellites based on how long it takes for a radio signal to arrive from each satellite, and from these distances calculates the receiver's position. Because light travels about 300,000 kilometres (186,000 miles) in one second, these measurements of small fractions of a second must be very precise. The Lunar Laser Ranging Experiment, radar astronomy and the Deep Space Network determine distances to the Moon,[73] planets[74] and spacecraft,[75] respectively, by measuring round-trip transit times.

Astronomy

The finite speed of light is important in astronomy. Due to the vast distances involved, it can take a very long time for light to travel from its source to Earth. For example, it has taken 13 billion (13×10^9) years for light to travel to Earth from the faraway galaxies viewed in the Hubble Ultra Deep Field images.[76] [77] Those photographs, taken today, capture images of the galaxies as they appeared 13 billion years ago, when the universe was less than a billion years old.[76] The fact that more distant objects appear to be younger, due to the finite speed of light, allows astronomers to infer the evolution of stars, of galaxies, and of the universe itself.

Astronomical distances are sometimes expressed in light-years, especially in popular science publications and media.[78] A light-year is the distance light travels in one year, around 9461 billion kilometres, 5879 billion miles, or 0.3066 parsecs. Proxima Centauri, the closest star to Earth after the Sun, is around 4.2 light-years away.[79]

Measurement

There are different ways to determine the value of c. One way is to measure the actual speed at which light waves propagate, which can be done in various astronomical and earth-based setups. However, it is also possible to determine c from other physical laws where it appears, for example, by determining the values of the electromagnetic constants ε_0 and μ_0 and using their relation to c. Historically, the most accurate results have been obtained by separately determining the frequency and wavelength of a light beam, with their product equaling c.

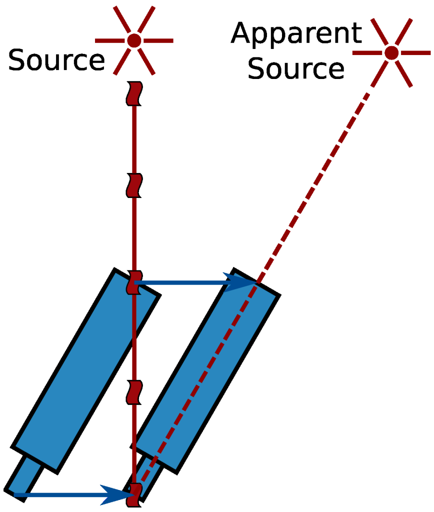
In 1983 the metre was defined as "the length of the path travelled by light in vacuum during a time interval of 1⁄299,792,458 of a second",[80] fixing the value of the speed of light at 299792458 m/s by definition, as described below. Consequently, accurate measurements of the speed of light yield an accurate realization of the metre rather than an accurate value of c.

Astronomical measurements

Outer space is a natural setting for measuring the speed of light because of its large scale and nearly perfect vacuum. Typically, one measures the time needed for light to traverse some reference distance in the solar system, such as the radius of the Earth's orbit. Historically, such measurements could be made fairly accurately, compared to how accurately the length of the reference distance is known in Earth-based units. It is customary to express the results in astronomical units (AU) per day. An astronomical unit is approximately the average distance between the Earth and Sun; it is not based on the International System of Units.[81] Because the AU determines an actual length, and is not based upon time-of-flight like the SI units, modern measurements of the speed of light in astronomical units per day can be compared with the defined value of c in the International System of Units.

Ole Christensen Rømer used an astronomical measurement to make the first quantitative estimate of the speed of light.[82] [83] When measured from Earth, the periods of moons orbiting a distant planet are shorter when the Earth is approaching the planet than when the Earth is receding from it. The distance travelled by light from the planet (or its moon) to Earth is shorter when the Earth is at the point in its orbit that is closest to its planet than when the Earth is at the farthest point in its orbit, the difference in distance being the diameter of the Earth's orbit around the Sun. The observed change in the moon's orbital period is actually the difference in the time it takes light to traverse the shorter or longer distance. Rømer observed this effect for Jupiter's innermost moon Io and deduced that light takes 22 minutes to cross the diameter of the Earth's orbit.

Another method is to use the aberration of light, discovered and explained by James Bradley in the 18th century.[84] This effect results from the vector addition of the velocity of light arriving from a distant source (such as a star) and the velocity of its observer (see diagram on the right). A moving observer thus sees the light coming from a slightly different direction and consequently sees the source at a position shifted from its original position. Since the direction of the Earth's velocity changes continuously as the Earth orbits the Sun, this effect causes the apparent position of stars to move around. From the angular difference in the position of stars (maximally 20.5 arcseconds)[85] it is possible to express the speed of light in terms of the Earth's velocity around the Sun, which with the known length of a year can be easily converted to the time needed to travel from the Sun to the Earth. In 1729, Bradley used this method to derive that light travelled 10,210 times faster than the Earth in its orbit (the modern figure is 10,066 times faster) or, equivalently, that it would take light 8 minutes 12 seconds to travel from the Sun to the Earth.[84]



Aberration of light: light from a distant source appears to be from a different location for a moving telescope due to the finite speed of light.

Nowadays, the "light time for unit distance"—the inverse of c, expressed in seconds per astronomical unit—is measured by comparing the time for radio signals to reach different spacecraft in the Solar System, with their position calculated from the gravitational effects of the Sun and various planets. By combining many such measurements, a best fit value for the light time per unit distance is obtained. As of 2009, the best estimate, as approved by the International Astronomical Union (IAU), is:[86] [87]

light time for unit distance: 499.004783836(10) s

c = 0.00200398880410(4) AU/s = 173.144632674(3) AU/day.

The relative uncertainty in these measurements is 0.02 parts per billion (2×10^{-11}), equivalent to the uncertainty in Earth-based measurements of length by interferometry.[88] [89] Since the metre is defined to be the length travelled by light in a certain time interval, the measurement of the light time for unit distance can also be interpreted as measuring the length of an AU in metres.[90]

Time of flight techniques

A method of measuring the speed of light is to measure the time needed for light to travel to a mirror at a known distance and back. This is the working principle behind the Fizeau–Foucault apparatus developed by Hippolyte Fizeau and Léon Foucault.

The setup as used by Fizeau consists of a beam of light directed at a mirror 8 kilometres (5 mi) away. On the way from the source to the mirror, the beam passes through a rotating cogwheel. At a certain rate of rotation, the beam passes through one gap on the way out and another on the way back, but at slightly higher or lower rates, the beam strikes a tooth and does not pass through the wheel. Knowing the distance between the wheel and the mirror, the number of teeth on the wheel, and the rate of rotation, the speed of light can be calculated.[91]

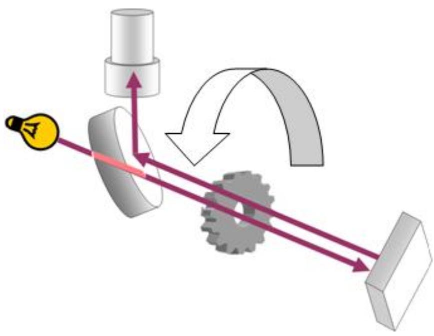


Diagram of the Fizeau apparatus

The method of Foucault replaces the cogwheel by a rotating mirror. Because the mirror keeps rotating while the light travels to the distant mirror and back, the light is reflected from the rotating mirror at a different angle on its way out than it is on its way back. From this difference in angle, the known speed of rotation and the distance to the distant mirror the speed of light may be calculated.[92]

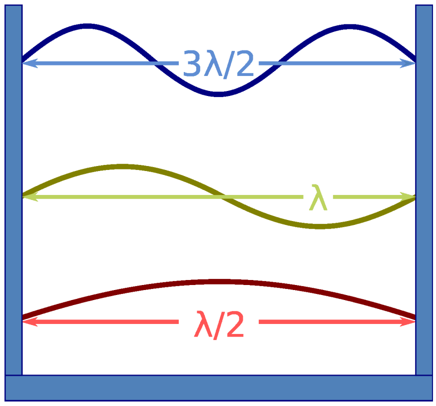
Nowadays, using oscilloscopes with time resolutions of less than one nanosecond, the speed of light can be directly measured by timing the delay of a light pulse from a laser or an LED reflected from a mirror. This method is less precise (with errors of the order of 1%) than other modern techniques, but it is sometimes used as a laboratory experiment in college physics classes.[93] [94] [95]

Electromagnetic constants

An option for deriving c that does not directly depend on a measurement of the propagation of electromagnetic waves is to use the relation between c and the vacuum permittivity ε_0 and vacuum permeability μ_0 established by Maxwell's theory: $c^2 = 1/(\varepsilon_0\mu_0)$. The vacuum permittivity may be determined by measuring the capacitance and dimensions of a capacitor, whereas the value of the vacuum permeability is fixed at exactly 4×10^{-7} H·m^{-1} through the definition of the ampere. Rosa and Dorsey used this method in 1907 to find a value of 299710 ± 22 km/s.[96] [97]

Cavity resonance

Another way to measure the speed of light is to independently measure the frequency f and wavelength λ of an electromagnetic wave in vacuum. The value of c can then be found by using the relation $c = f\lambda$. One option is to measure the resonance frequency of a cavity resonator. If the dimensions of the resonance cavity are also known, these can be used determine the wavelength of the wave. In 1946, Louis Essen and A.C. Gordon-Smith establish the frequency for a variety of normal modes of microwaves of a microwave cavity of precisely known dimensions. The dimensions were established to an accuracy of about ±0.8 μm using gauges calibrated by interferometry.[96] As the wavelength of the modes was known from the geometry of the cavity and from electromagnetic theory, knowledge of the associated frequencies enabled a calculation of the speed of light.[96] [98]

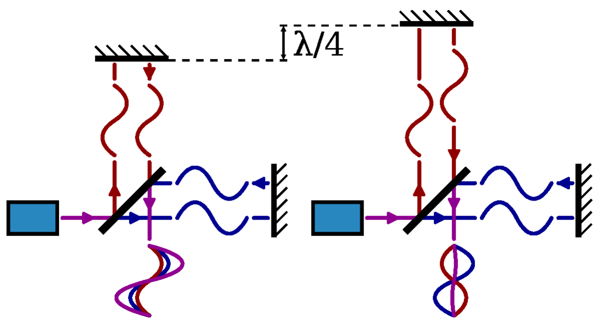


Electromagnetic standing waves in a cavity.

The Essen–Gordon-Smith result, 299792 ± 9 km/s, was substantially more precise than those found by optical techniques.[96] By 1950, repeated measurements by Essen established a result of 299792.5 ± 3.0 km/s.[99]

A household demonstration of this technique is possible, using a microwave oven and food such as marshmallows or margarine: if the turntable is removed so that the food does not move, it will cook the fastest at the antinodes (the points at which the wave amplitude is the greatest), where it will begin to melt. The distance between two such spots is half the wavelength of the microwaves; by measuring this distance and multiplying the wavelength by the microwave frequency (usually displayed on the back of the oven, typically 2450 MHz), the value of c can be calculated, "often with less than 5% error".[100] [101]

Interferometry



An interferometric determination of length. Left: constructive interference; Right: destructive interference.

Interferometry is another method to find the wavelength of electromagnetic radiation for determining the speed of light.[102] A coherent beam of light (e.g. from a laser), with a known frequency (f), is split to follow two paths and then recombined. By adjusting the path length while observing the interference pattern and carefully measuring the change in path length, the wavelength of the light (λ) can be determined. The speed of light is then calculated using the equation $c = \lambda f$.

Before the advent of laser technology, coherent radio sources were used for interferometry measurements of the speed of light.[103] However interferometric determination of wavelength becomes less precise with wavelength and the experiments were thus limited in precision by the long wavelength (~0.4 cm) of the radiowaves. The precision can be improved by using light with a shorter wavelength, but then it becomes difficult to directly measure the frequency of the light. One way around this problem is to start with a low frequency signal of which the frequency can be precisely measured, and from this signal progressively synthesize higher frequency signals whose frequency can then be linked to the original signal. A laser can then be locked to the frequency, and its wavelength can be determined using interferometry.[104] This technique was due to a group at the National Bureau of Standards (NBS) (which later became NIST). They used it in 1972 to measure the speed of light in vacuum with a fractional uncertainty of 3.5×10^{-9}.[104] [105]

History

History of measurements of *c* (in km/s)

1675	Rømer and Huygens, moons of Jupiter	220000[83] [106]
1729	James Bradley, aberration of light	301000[91]
1849	Hippolyte Fizeau, toothed wheel	315000[91]
1862	Léon Foucault, rotating mirror	298000 ± 500[91]
1907	Rosa and Dorsey, EM constants	299710 ± 30[96] [97]
1926	Albert Michelson, rotating mirror	299796 ± 4[107]
1950	Essen and Gordon-Smith, cavity resonator	299792.5 ± 3.0[99]
1958	K.D. Froome, radio interferometry	299792.50 ± 0.10[103]
1972	Evenson *et al.*, laser interferometry	299792.4562 ± 0.0011[105]
1983	17th CGPM, definition of the metre	299792.458 (exact)[80]

Until the early modern period, it was not known whether light travelled instantaneously or at a very fast finite speed. The first extant recorded examination of this subject was in ancient Greece. Empedocles was the first to claim that the light has a finite speed.[108] He maintained that light was something in motion, and therefore must take some time to travel. Aristotle argued, to the contrary, that "light is due to the presence of something, but it is not a movement".[109] Euclid and Ptolemy advanced the emission theory of vision, where light is emitted from the eye, thus enabling sight. Based on that theory, Heron of Alexandria argued that the speed of light must be infinite because distant objects such as stars appear immediately upon opening the eyes.

Early Islamic philosophers initially agreed with the Aristotelian view that light had no speed of travel. In 1021, Alhazen (Ibn al-Haytham) published the *Book of Optics*, in which he presented a series of arguments dismissing the emission theory in favour of the now accepted intromission theory of vision, in which light moves from an object into the eye.[110] This led Alhazen to propose that light must have a finite speed,[109] [111] [112] and that the speed of light is variable, decreasing in denser bodies.[112] [113] He argued that light is substantial matter, the propagation of which requires time, even if this is hidden from our senses.[114]

Also in the 11th century, Abū Rayhān al-Bīrūnī agreed that light has a finite speed, and observed that the speed of light is much faster than the speed of sound.[115] Roger Bacon argued that the speed of light in air was not infinite, using philosophical arguments backed by the writing of Alhazen and Aristotle.[116] [117] In the 1270s, Witelo considered the possibility of light travelling at infinite speed in vacuum, but slowing down in denser bodies.[118]

In the early 17th century, Johannes Kepler believed that the speed of light was infinite, since empty space presents no obstacle to it. René Descartes argued that if the speed of light were finite, the Sun, Earth, and Moon would be noticeably out of alignment during a lunar eclipse. Since such misalignment had not been observed, Descartes concluded the speed of light was infinite. Descartes speculated that if the speed of light were found to be finite, his whole system of philosophy might be demolished.[109]

First measurement attempts

In 1629, Isaac Beeckman proposed an experiment in which a person observes the flash of a cannon reflecting off a mirror about one mile (1.6 km) away. In 1638, Galileo Galilei proposed an experiment, with an apparent claim to having performed it some years earlier, to measure the speed of light by observing the delay between uncovering a lantern and its perception some distance away. He was unable to distinguish whether light travel was instantaneous or not, but concluded that if it were not, it must nevertheless be extraordinarily rapid.[119] [120] Galileo's experiment was carried out by the Accademia del Cimento of Florence, Italy, in 1667, with the lanterns separated by about one mile, but no delay was observed. The actual delay in this experiment would have been about 11 microseconds.

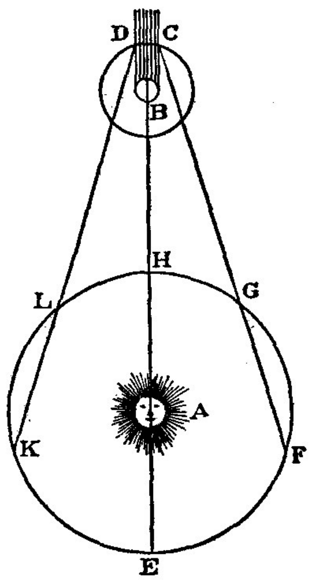


Fig. 70.

Rømer's observations of the occultations of Io from Earth

The first quantitative estimate of the speed of light was made in 1676 by Rømer (see Rømer's determination of the speed of light).[82] [83] From the observation that the periods of Jupiter's innermost moon Io appeared to be shorter when the Earth was approaching Jupiter than when receding from it, he concluded that light travels at a finite speed, and estimated that it takes light 22 minutes to cross the diameter of Earth's orbit. Christiaan Huygens combined this estimate with an estimate for the diameter of the Earth's orbit to obtain an estimate of speed of light of 220000 km/s, 26% lower than the actual value.[106]

In his 1704 book *Opticks*, Isaac Newton reported Rømer's calculations of the finite speed of light and gave a value of "seven or eight minutes" for the time taken for light to travel from the Sun to the Earth (the modern value is 8 minutes 19 seconds).[121] Newton queried whether Rømer's eclipse shadows were coloured; hearing that they were not, he concluded the different colours travelled at the same speed. In 1729, James Bradley discovered the aberration of light.[84] From this effect he determined that light must travel 10,210 times faster than the Earth in its orbit (the modern figure is 10,066 times faster) or, equivalently, that it would take light 8 minutes 12 seconds to travel from the Sun to the Earth.[84]

19th and early 20th century

In the 19th century Hippolyte Fizeau developed a method to determine the speed of light based on time-of-flight measurements on Earth and reported a value of 315000 km/s. His method was improved upon by Léon Foucault who obtained a value of 298000 km/s in 1862.[91] In the year 1856, Wilhelm Eduard Weber and Rudolf Kohlrausch measured the ratio of the electromagnetic and electrostatic units of charge, $1/\sqrt{\varepsilon_0\mu_0}$, by discharging a Leyden jar, and found that its numerical value was very close to the speed of light as measured directly by Fizeau. The following year Gustav Kirchhoff calculated that an electric signal in a resistanceless wire travels along the wire at this speed.[122] In the early 1860s, Maxwell showed that according to the theory of electromagnetism which he was working on, that electromagnetic waves propagate in empty space[123] [124] [125] at a speed equal to the above Weber/Kohrausch ratio, and drawing attention to the numerical proximity of this value to the speed of light as measured by Fizeau, he proposed that light is in fact an electromagnetic wave.[126]

It was thought at the time that empty space was filled with a background medium called the luminiferous aether in which the electromagnetic field existed. Some physicists thought that this aether acted as a preferred frame of reference for the propagation of light and therefore it should be possible to measure the motion of the Earth with respect to this medium, by measuring the isotropy of the speed of light. Beginning in the 1880s several experiments were performed to try to detect this motion, the most famous of which is the experiment performed by Albert Michelson and Edward Morley in 1887.[127] The detected motion was always less than the observational error. Modern experiments indicate that the two-way speed of light is isotropic (the same in every direction) to within 6 nanometres per second.[128] Because of this experiment Hendrik Lorentz proposed that the motion of the apparatus through the aether may cause the apparatus to contract along its length in the direction of motion, and he further assumed, that the time variable for moving systems must also be changed accordingly ("local time"), which led to the formulation of the Lorentz transformation. Based on Lorentz's aether theory, Henri Poincaré (1900) showed that this local time (to first order in v/c) is indicated by clocks moving in the aether, which are synchronized under the assumption of constant light speed. In 1904, he speculated that the speed of light could be a limiting velocity in dynamics, provided that the assumptions of Lorentz's theory are all confirmed. In 1905, Poincaré brought Lorentz's aether theory into full observational agreement with the principle of relativity.[129] [130]

In 1905 Einstein postulated from the outset that the speed of light in vacuum, measured by a non-accelerating observer, is independent of the motion of the source or observer. Using this and the principle of relativity as a basis he derived the special theory of relativity, in which the speed of light in vacuum c featured as a fundamental parameter, also appearing in contexts unrelated to light. This made the concept of the stationary aether (to which Lorentz and Poincaré still adhered) useless and revolutionized the concepts of space and time.[131] [132]

Increased accuracy of *c* and redefinition of the metre

In the second half of the 20th century much progress was made in increasing the accuracy of measurements of the speed of light, first by cavity resonance techniques and later by laser interferometer techniques. In 1972, using the latter method and the 1960 definition of the metre in terms of a particular spectral line of krypton-86, a group at NBS in Boulder, Colorado determined the speed of light in vacuum to be c = 299792456.2 ± 1.1 m/s. This was 100 times less uncertain than the previously accepted value. The remaining uncertainty was mainly related to the definition of the metre.[133] [105] Since similar experiments found comparable results for c, the 15th Conférence Générale des Poids et Mesures (CGPM) in 1975 recommended using the value 299792458 m/s for the speed of light.[134]

Because the previous definition was deemed inadequate for the needs of various experiments, the 17th CGPM in 1983 decided to redefine the metre.[135] The new (and current) definition reads: "The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."[80] As a result of this definition, the value of the speed of light in vacuum is exactly 299792458 m/s[136] [137] and has become a defined constant in the SI system of units.[11] Improved experimental techniques do not affect the value of the speed of light in SI units, but instead allow for a more precise realization of the definition of the metre.[138] [139]

Notes

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Further reading

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External links

- Speed of light in vacuum (http://physics.nist.gov/cgi-bin/cuu/Value?c) (National Institute of Standards and Technology, NIST)
- Definition of the metre (http://www.bipm.org/en/si/si_brochure/chapter2/2-1/metre.html) (International Bureau of Weights and Measures, BIPM)
- Data Gallery: Michelson Speed of Light (Univariate Location Estimation) (http://www.itl.nist.gov/div898/bayesian/datagall/michelso.htm) (download data gathered by A.A. Michelson)
- Subluminal (http://gregegan.customer.netspace.net.au/APPLETS/20/20.html) (Java applet demonstrating group velocity information limits)
- De Mora Luminis (http://www.mathpages.com/rr/s3-03/3-03.htm) at MathPages
- Light discussion on adding velocities (http://www.ertin.com/sloan_on_speed_of_light.html)
- Speed of Light (http://www.colorado.edu/physics/2000/waves_particles/lightspeed-1.html) (University of Colorado Department of Physics)
- c: Speed of Light (http://sixtysymbols.com/videos/light.htm) (Sixty Symbols, University of Nottingham Department of Physics [video])
- Usenet Physics FAQ (http://math.ucr.edu/home/baez/physics/)
- The Fizeau "Rapidly Rotating Toothed Wheel" Method (http://njsas.org/projects/speed_of_light/fizeau/)

Related phenomena

Dielectric

A **dielectric** is an electrical insulator that can be polarized by an applied electric field. When a dielectric is placed in an electric field, electric charges do not flow through the material, as in a conductor, but only slightly shift from their average equilibrium positions causing **dielectric polarization**. Because of dielectric polarization, positive charges are displaced toward the field and negative charges shift in the opposite direction. This creates an internal electric field which reduces the overall field within the dielectric itself.[1] If a dielectric is composed of weakly bonded molecules, those molecules not only become polarized, but also reorient so that their symmetry axis aligns to the field.[1]

Although the term "insulator" implies low electrical conduction, "dielectric" is typically used to describe materials with a high polarizability. The latter is expressed by a number called the dielectric constant. A common, yet notable example of a dielectric is the electrically insulating material between the metallic plates of a capacitor. The polarization of the dielectric by the applied electric field increases the capacitor's surface charge.[1]

The study of dielectric properties is concerned with the storage and dissipation of electric and magnetic energy in materials.[2] It is important to explain various phenomena in electronics, optics, and solid-state physics.

The term "dielectric" was coined by William Whewell (from "dia-electric") in response to a request from Michael Faraday.[3]

Electric susceptibility

The **electric susceptibility** χ_e of a dielectric material is a measure of how easily it polarizes in response to an electric field. This, in turn, determines the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.

It is defined as the constant of proportionality (which may be a tensor) relating an electric field **E** to the induced dielectric polarization density **P** such that

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E},$$

where ε_0 is the electric permittivity of free space.

The susceptibility of a medium is related to its relative permittivity ε_r by

$$\chi_e = \varepsilon_r - 1.$$

So in the case of a vacuum,

$$\chi_e = 0.$$

The electric displacement **D** is related to the polarization density **P** by

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon_r \varepsilon_0 \mathbf{E}.$$

Dispersion and causality

In general, a material cannot polarize instantaneously in response to an applied field. The more general formulation as a function of time is

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{t} \chi_e(t - t')\mathbf{E}(t')\, dt'.$$

That is, the polarization is a convolution of the electric field at previous times with time-dependent susceptibility given by $\chi_e(\Delta t)$. The upper limit of this integral can be extended to infinity as well if one defines $\chi_e(\Delta t) = 0$ for $\Delta t < 0$. An instantaneous response corresponds to Dirac delta function susceptibility $\chi_e(\Delta t) = \chi_e \delta(\Delta t)$.

It is more convenient in a linear system to take the Fourier transform and write this relationship as a function of frequency. Due to the convolution theorem, the integral becomes a simple product,

$$\mathbf{P}(\omega) = \varepsilon_0 \chi_e(\omega)\mathbf{E}(\omega).$$

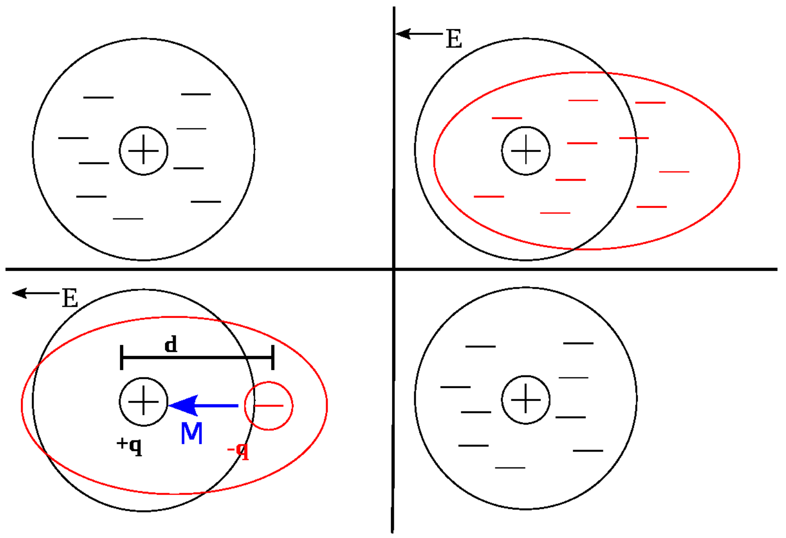
Note the simple frequency dependence of the susceptibility, or equivalently the permittivity. The shape of the susceptibility with respect to frequency characterizes the dispersion properties of the material.

Moreover, the fact that the polarization can only depend on the electric field at previous times (i.e. $\chi_e(\Delta t) = 0$ for $\Delta t < 0$), a consequence of causality, imposes Kramers–Kronig constraints on the susceptibility $\chi_e(0)$.

Dielectric polarization

Basic atomic model

In the classical approach to the dielectric model, a material is made up of atoms. Each atom consists of a cloud of negative charge (Electrons) bound to and surrounding a positive point charge at its center. In the presence of an electric field the charge cloud is distorted, as shown in the top right of the figure.



Electric field interaction with an atom under the classical dielectric model.

This can be reduced to a simple dipole using the superposition principle. A dipole is characterized by its dipole moment, a vector quantity shown in the figure as the blue arrow labeled *M*. It is the relationship between the electric field and the dipole moment that gives rise to the behavior of the dielectric. (Note that the dipole moment is shown to be pointing in the same direction as the electric field. This isn't always correct, and it is a major simplification, but it is suitable for many materials.)

When the electric field is removed the atom returns to its original state. The time required to do so is the so-called relaxation time; an exponential decay.

This is the essence of the model in physics. The behavior of the dielectric now depends on the situation. The more complicated the situation the richer the model has to be in order to accurately describe the behavior. Important questions are:

- Is the electric field constant or does it vary with time?

  - If the electric field does vary, at what rate?
- What are the characteristics of the material?
  - Is the direction of the field important (isotropy)?
  - Is the material the same all the way through (homogeneous)?
  - Are there any boundaries/interfaces that have to be taken into account?
- Is the system linear or do nonlinearities have to be taken into account?

The relationship between the electric field **E** and the dipole moment **M** gives rise to the behavior of the dielectric, which, for a given material, can be characterized by the function **F** defined by the equation:

$$\mathbf{M} = \mathbf{F}(\mathbf{E}).$$

When both the type of electric field and the type of material have been defined, one then chooses the simplest function *F* that correctly predicts the phenomena of interest. Examples of phenomena that can be so modeled include:

- Refractive index
- Group velocity dispersion
- Birefringence
- Self-focusing
- Harmonic generation

Dipolar polarization

Dipolar polarization is a polarization that is either inherent to polar molecules (**orientation polarization**), or can be induced in any molecule in which the asymmetric distortion of the nuclei is possible (**distortion polarization**). Orientation polarization results from a permanent dipole, e.g. that arising from the *ca.* 104^{o} angle between the asymmetric bonds between oxygen and hydrogen atoms in the water molecule, which retains polarization in the absence of an external electric field. The assembly of these dipoles forms a macroscopic polarization.

When an external electric field is applied, the distance between charges, which is related to chemical bonding, remains constant in orientation polarization; however, the polarization itself rotates. This rotation occurs on a timescale which depends on the torque and the surrounding local viscosity of the molecules. Because the rotation is not instantaneous, dipolar polarizations lose the response to electric fields at the lowest frequency in polarizations. A molecule rotates about 1ps per radian in a fluid, thus this loss occurs at about 10^{11} Hz (in the microwave region). The delay of the response to the change of the electric field causes friction and heat.

When an external electric field is applied in the infrared, a molecule is bent and stretched by the field and the molecular moment changes in response. The molecular vibration frequency is approximately the inverse of the time taken for the molecule to bend, and the **distortion polarization** disappears above the infrared.

Ionic polarization

Ionic polarization is polarization which is caused by relative displacements between positive and negative ions in ionic crystals (for example, NaCl).

If crystals or molecules do not consist of only atoms of the same kind, the distribution of charges around an atom in the crystals or molecules leans to positive or negative. As a result, when lattice vibrations or molecular vibrations induce relative displacements of the atoms, the centers of positive and negative charges might be in different locations. These center positions are affected by the symmetry of the displacements. When the centers don't correspond, polarizations arise in molecules or crystals. This polarization is called **ionic polarization**.

Ionic polarization causes ferroelectric transition as well as dipolar polarization. The transition, which is caused by the order of the directional orientations of permanent dipoles along a particular direction, is called **order-disorder phase transition**. The transition which is caused by ionic polarizations in crystals is called **displacive phase**

transition.

Dielectric dispersion

In physics, **dielectric dispersion** is the dependence of the permittivity of a dielectric material on the frequency of an applied electric field. Because there is always a lag between changes in polarization and changes in an electric field, the permittivity of the dielectric is a complicated, complex-valued function of frequency of the electric field. It is very important for the application of dielectric materials and the analysis of polarization systems.

This is one instance of a general phenomenon known as material dispersion: a frequency-dependent response of a medium for wave propagation.

When the frequency becomes higher:

1. it becomes impossible for dipolar polarization to follow the electric field in the microwave region around 10^{10} Hz;
2. in the infrared or far-infrared region around 10^{13} Hz, ionic polarization and molecular distortion polarization lose the response to the electric field;
3. electronic polarization loses its response in the ultraviolet region around 10^{15} Hz.

In the frequency region above ultraviolet, permittivity approaches the constant ε_0 in every substance, where ε_0 is the permittivity of the free space. Because permittivity indicates the strength of the relation between an electric field and polarization, if a polarization process loses its response, permittivity decreases.

Dielectric relaxation

Dielectric relaxation is the momentary delay (or lag) in the dielectric constant of a material. This is usually caused by the delay in molecular polarization with respect to a changing electric field in a dielectric medium (e.g. inside capacitors or between two large conducting surfaces). Dielectric relaxation in changing electric fields could be considered analogous to hysteresis in changing magnetic fields (for inductors or transformers). Relaxation in general is a delay or lag in the response of a linear system, and therefore dielectric relaxation is measured relative to the expected linear steady state (equilibrium) dielectric values. The time lag between electrical field and polarization implies an irreversible degradation of free energy(G).

In physics, **dielectric relaxation** refers to the relaxation response of a dielectric medium to an external electric field of microwave frequencies. This relaxation is often described in terms of permittivity as a function of frequency, which can, for ideal systems, be described by the Debye equation. On the other hand, the distortion related to ionic and electronic polarization shows behavior of the resonance or oscillator type. The character of the distortion process depends on the structure, composition, and surroundings of the sample.

The number of possible wavelengths of emitted radiation due to dielectric relaxation can be equated using Hemmings' first law (named after Mark Hemmings)

$$n = \frac{l^2 - l}{2}$$

where

n is the number of different possible wavelengths of emitted radiation

l is the number of energy levels (including ground level).

Debye relaxation

Debye relaxation is the dielectric relaxation response of an ideal, noninteracting population of dipoles to an alternating external electric field. It is usually expressed in the complex permittivity ε of a medium as a function of the field's frequency ω:

$$\hat{\varepsilon}(\omega) = \varepsilon_{\infty} + \frac{\Delta\varepsilon}{1 + i\omega\tau},$$

where ε_{∞} is the permittivity at the high frequency limit, $\Delta\varepsilon = \varepsilon_s - \varepsilon_{\infty}$ where ε_s is the static, low frequency permittivity, and τ is the characteristic relaxation time of the medium.

This relaxation model was introduced by and named after the chemist Peter Debye (1913).[4]

Variants of the Debye equation

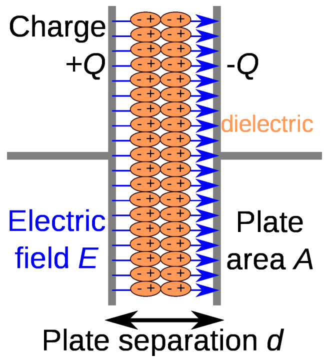
- Cole–Cole equation
- Cole–Davidson equation
- Havriliak–Negami relaxation
- Kohlrausch–Williams–Watts function (Fourier transform of stretched exponential function)

Applications

Capacitors

Commercially manufactured capacitors typically use a solid dielectric material with high permittivity as the intervening medium between the stored positive and negative charges. This material is often referred to in technical contexts as the "capacitor dielectric".[5]

The most obvious advantage to using such a dielectric material is that it prevents the conducting plates on which the charges are stored from coming into direct electrical contact. More significant, however, a high permittivity allows a greater charge to be stored at a given voltage. This can be seen by treating the case of a linear dielectric with permittivity ε and thickness d between two conducting plates with uniform charge density σ_{ε}. In this case the charge density is given by



Charge separation in a parallel-plate capacitor causes an internal electric field. A dielectric (orange) reduces the field and increases the capacitance.

$$\sigma_{\epsilon} = \epsilon \frac{V}{d}$$

and the capacitance per unit area by

$$c = \frac{\sigma_{\epsilon}}{V} = \frac{\epsilon}{d}$$

From this, it can easily be seen that a larger ε leads to greater charge stored and thus greater capacitance.

Dielectric materials used for capacitors are also chosen such that they are resistant to ionization. This allows the capacitor to operate at higher voltages before the insulating dielectric ionizes and begins to allow undesirable current.

Dielectric resonator

A *dielectric resonator oscillator* (DRO) is an electronic component that exhibits resonance for a narrow range of frequencies, generally in the microwave band. It consists of a "puck" of ceramic that has a large dielectric constant and a low dissipation factor. Such resonators are often used to provide a frequency reference in an oscillator circuit. An unshielded dielectric resonator can be used as a Dielectric Resonator Antenna (DRA).

Some practical dielectrics

Dielectric materials can be solids, liquids, or gases. In addition, a high vacuum can also be a useful, lossless dielectric even though its relative dielectric constant is only unity.

Solid dielectrics are perhaps the most commonly used dielectrics in electrical engineering, and many solids are very good insulators. Some examples include porcelain, glass, and most plastics. Air, nitrogen and sulfur hexafluoride are the three most commonly used gaseous dielectrics.

- Industrial coatings such as parylene provide a dielectric barrier between the substrate and its environment.
- Mineral oil is used extensively inside electrical transformers as a fluid dielectric and to assist in cooling. Dielectric fluids with higher dielectric constants, such as electrical grade castor oil, are often used in high voltage capacitors to help prevent corona discharge and increase capacitance.
- Because dielectrics resist the flow of electricity, the surface of a dielectric may retain *stranded* excess electrical charges. This may occur accidentally when the dielectric is rubbed (the triboelectric effect). This can be useful, as in a Van de Graaff generator or electrophorus, or it can be potentially destructive as in the case of electrostatic discharge.
- Specially processed dielectrics, called electrets (which should not be confused with ferroelectrics), may retain excess internal charge or "frozen in" polarization. Electrets have a semipermanent external electric field, and are the electrostatic equivalent to magnets. Electrets have numerous practical applications in the home and industry.
- Some dielectrics can generate a potential difference when subjected to mechanical stress, or change physical shape if an external voltage is applied across the material. This property is called piezoelectricity. Piezoelectric materials are another class of very useful dielectrics.
- Some ionic crystals and polymer dielectrics exhibit a spontaneous dipole moment which can be reversed by an externally applied electric field. This behavior is called the ferroelectric effect. These materials are analogous to the way ferromagnetic materials behave within an externally applied magnetic field. Ferroelectric materials often have very high dielectric constants, making them quite useful for capacitors.

References

[1] Quote from Encyclopedia Britannica**:** "*Dielectric, insulating material or a very poor conductor of electric current. When dielectrics are placed in an electric field, practically no current flows in them because, unlike metals, they have no loosely bound, or free, electrons that may drift through the material*".

- "Dielectrics (physics)" (http://www.britannica.com/EBchecked/topic/162630/dielectric). *Britannica*. 2009. pp. 1. . Retrieved 2009-08-12.

[2] Arthur R. von Hippel, in his seminal work, *Dielectric Materials and Applications*, stated: "*Dielectrics*... are not a narrow class of so-called insulators, but the broad expanse of *nonmetals* considered from the standpoint of their interaction with electric, magnetic, or electromagnetic fields. Thus we are concerned with gases as well as with liquids and solids, and with the storage of electric and magnetic energy as well as its dissipation." (Technology Press of MIT and John Wiley, NY, 1954).

[3] J. Daintith (1994). *Biographical Encyclopedia of Scientists*. CRC Press. p. 943. ISBN 0750302879.

[4] P. Debye (1913), Ver. Deut. Phys. Gesell. 15, 777; reprinted 1954 in collected papers of Peter J.W. Debye Interscience, New York

[5] States7113388 United States patent 7113388 (http://worldwide.espacenet.com/textdoc?DB=EPODOC&IDX=United), Mussig & Hans-Joachim, "Semiconductor capacitor with praseodymium oxide as dielectric", published 2003-11-06, issued 2004-10-18, assigned to IHP GmbH- Innovations for High Performance Microelectronics/Institute Fur Innovative Mikroelektronik

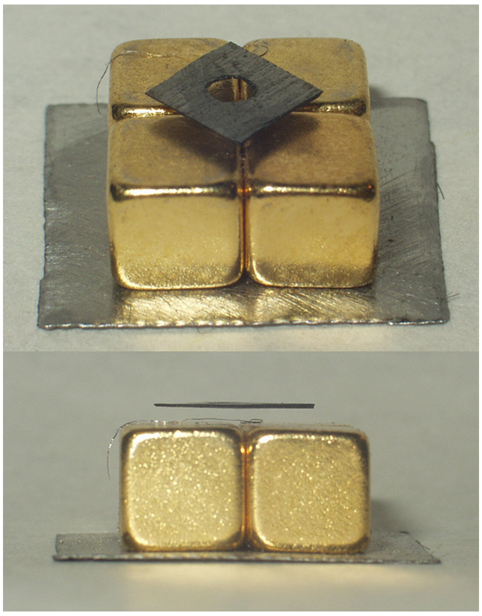
Further reading

- Jackson, John David (August 10, 1998). *Classical Electrodynamics* (http://books.google.com/books?id=U3LBQgAACAAJ&dq=Classical+Electrodynamics) (3 rd ed.). John Wiley & Sons. ISBN 9780471309321. 808 or 832 pages.

External links

- Electromagnetism (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html) - A chapter from an online textbook
- Dielectric Sphere in an Electric Field (http://wiki.4hv.org/index.php/Dielectric_Sphere_in_Electric_Field)
- DoITPoMS Teaching and Learning Package "Dielectric Materials" (http://www.doitpoms.ac.uk/tlplib/dielectrics/index.php)

Diamagnetic



Levitating pyrolytic carbon

Diamagnetism is the property of an object which causes it to create a magnetic field in opposition to an externally applied magnetic field, thus causing a repulsive effect. Specifically, an external magnetic field alters the orbital velocity of electrons around their nuclei, thus changing the magnetic dipole moment. According to Lenz's law, these electrons will oppose the magnetic field *changes* provided by the applied field, preventing them from building up. The result is that lines of magnetic flux curve away from the material. In most materials, diamagnetism is a weak effect, but under conditions of superconductivity, which only some materials may obtain, a strong quantum effect can emerge wherein the lines are completely blocked, excluding a very thin layer at the material's surface. Diamagnets are materials with a magnetic permeability less than μ_0 (a relative permeability less than 1).

History

In 1778, Sebald Justinus Brugmans was the first individual to observe that bismuth and antimony were repelled by magnetic fields. However, the term *diamagnetism* was coined by Michael Faraday in September 1845, when he realized that all materials in nature possessed some form of diamagnetic response to an applied magnetic field.

Diamagnetic materials

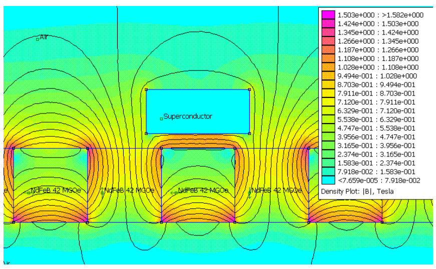
Notable diamagnetic materials[1]

Material	χ_v (10^{-5})
Superconductor	-10^5
Pyrolytic carbon	−40.0
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Carbon (graphite)	−1.6
Copper	−1.0
Water	−0.91

Diamagnetism is a very general phenomenon, because all electrons, including the electrons of an atom, will always make a weak contribution to the material's response. However, for materials that show some other form of magnetism (such as ferromagnetism or paramagnetism), the diamagnetism is completely overpowered. Substances that mostly display diamagnetic behaviour are termed diamagnetic materials, or diamagnets. Materials that are said to be diamagnetic are those that are usually considered by non-physicists to be *non-magnetic*, and include water, wood, most organic compounds such as petroleum and some plastics, and many metals including copper, particularly the heavy ones with many core electrons, such as mercury, gold and bismuth. The magnetic susceptibility of various molecular fragments are called Pascal's constants.

Diamagnetic materials have a relative magnetic permeability that is less than or equal to 1, and therefore a magnetic susceptibility which is less than 0 since susceptibility is defined as $\chi_v = \mu_v - 1$. This means that diamagnetic materials are repelled by magnetic fields. However, since diamagnetism is such a weak property its effects are not observable in everyday life. For example, the magnetic susceptibility of diamagnets such as water is $\chi_v = -9.05\times10^{-6}$. The most strongly diamagnetic material is bismuth, $\chi_v = -1.66\times10^{-4}$, although pyrolytic carbon may have a susceptibility of $\chi_v = -4.00\times10^{-4}$ in one plane. Nevertheless, these values are orders of magnitudes smaller than the magnetism exhibited by paramagnets and ferromagnets. Note that because χ_v is derived from the ratio of the internal magnetic field to the applied field, it is a dimensionless value.

Superconductors may be considered to be perfect diamagnets ($\chi_v = -1$), since they expel all fields (except in a thin surface layer) due to the Meissner effect. However this effect is not due to eddy currents, as in ordinary diamagnetic materials (see the article on superconductivity).



A superconductor acts as an essentially perfect diamagnetic material when placed in a magnetic field and it excludes the field, and the flux lines avoid the region

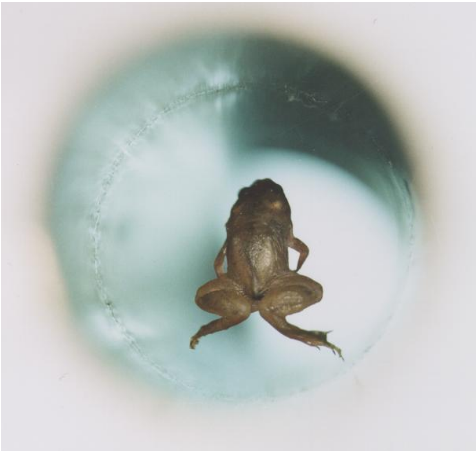
Additionally, all conductors exhibit an effective diamagnetism when they experience a changing magnetic field. The Lorentz force on electrons causes them to circulate around forming eddy currents. The eddy currents then produce an induced magnetic field which opposes the applied field, resisting the conductor's motion.

Demonstrations of diamagnetism

Curving water surfaces

If a powerful magnet (such as a supermagnet) is covered with a layer of water (that is thin compared to the diameter of the magnet) then the field of the magnet significantly repels the water. This causes a slight dimple in the water's surface that may be seen by its reflection.[2] [3]

Diamagnetic levitation



A live frog levitates inside a 32 mm diameter vertical bore of a Bitter solenoid in a magnetic field of about 16 teslas at the Nijmegen High Field Magnet Laboratory.[4]

Diamagnets may be levitated in stable equilibrium in a magnetic field, with no power consumption. Earnshaw's theorem seems to preclude the possibility of static magnetic levitation. However, Earnshaw's theorem only applies to objects with positive moments, such as ferromagnets (which have a permanent positive moment) and paramagnets (which induce a positive moment). These are attracted to field maxima, which do not exist in free space. Diamagnets (which induce a negative moment) are attracted to field minima, and there can be a field minimum in free space.

A thin slice of pyrolytic graphite, which is an unusually strong diamagnetic material, can be stably floated in a magnetic field, such as that from rare earth permanent magnets. This can be done with all components at room temperature, making a visually effective demonstration of diamagnetism.

The Radboud University Nijmegen, the Netherlands, has conducted experiments where water and other substances were successfully levitated. Most spectacularly, a live frog (see figure) was levitated.[5]

In September 2009, NASA's Jet Propulsion Laboratory in Pasadena, California announced they had successfully levitated mice using a superconducting magnet,[6] an important step forward since mice are closer biologically to humans than frogs.[7] They hope to perform experiments regarding the effects of microgravity on bone and muscle mass.

Recent experiments studying the growth of protein crystals has led to a technique using powerful magnets to allow growth in ways that counteract Earth's gravity.[8]

A simple homemade device for demonstration can be constructed out of bismuth plates and a few permanent magnets that will levitate a permanent magnet.[9]

Theory of diamagnetism

The Bohr–van Leeuwen theorem proves that there cannot be any diamagnetism or paramagnetism in a purely classical system. Yet the classical theory for Langevin diamagnetism gives the same prediction as the quantum theory.[10] The classical theory is given below.

Langevin diamagnetism

The Langevin theory of diamagnetism applies to materials containing atoms with closed shells (see dielectrics). A field with intensity B, applied to an electron with charge e and mass m, gives rise to Larmor precession with frequency $\omega = \frac{eB}{2m}$. The number of revolutions per unit time is $\frac{1}{2\pi}\omega$, so the current for an atom with Z electrons is (in SI units)[10]

$$I = -\frac{Ze^2B}{4\pi m}.$$

The magnetic moment of a current loop is equal to the current times the area of the loop. Suppose the field is aligned with the z axis. The average loop area can be given as $\pi\left\langle\rho^2\right\rangle$, where $\left\langle\rho^2\right\rangle$ is the mean square distance of the electrons perpendicular to the z axis. The magnetic moment is therefore

$$\mu = -\frac{Ze^2B}{4m}\langle\rho^2\rangle.$$

If the distribution of charge is spherically symmetric, we can suppose that the distribution of x, y, z coordinates are independent and identically distributed. Then $\left\langle x^2\right\rangle = \left\langle y^2\right\rangle = \left\langle z^2\right\rangle = \frac{1}{3}\left\langle r^2\right\rangle$, where $\left\langle r^2\right\rangle$ is the mean square distance of the electrons from the nucleus. Therefore $\left\langle\rho^2\right\rangle = \left\langle x^2\right\rangle + \left\langle y^2\right\rangle = \frac{2}{3}\left\langle r^2\right\rangle$. If N is the number of atoms per unit volume, the diamagnetic susceptibility is

$$\chi = \frac{\mu_0 N\mu}{B} = -\frac{\mu_0 N Z e^2}{6m}\langle r^2\rangle.$$

Diamagnetism in metals

The Langevin theory does not apply to metals because they have non-localized electrons. The theory for the diamagnetism of a free electron gas is called Landau diamagnetism, and instead considers the weak counter-acting field that forms when their trajectories are curved due to the Lorentz force. Landau diamagnetism, however, should be contrasted with Pauli paramagnetism, an effect associated with the polarization of delocalized electrons' spins.[11]

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External links

- Video of a museum-style magnetic elevation train model which makes use of diamagnetism (http://www.youtube.com/watch?v=8tFsrGRwOOM)
- Videos of frogs and other diamagnets levitated in a strong magnetic field (http://www.ru.nl/hfml/research/levitation/diamagnetic/)
- Video of levitating pyrolytic graphite (http://www.grand-illusions.com/images/articles/toyshop/diamagnetic_levitation_2/diamagnetic_levitation_2.wmv)
- Video of Meissner-Ochsenfeld effect involving liquid nitrogen (http://www.science.tv/watch/e257e44aa9d5bade97ba/liquid-nitrogen-and-superconductor)
- Video of a piece of neodymium magnet levitating between blocks of bismuth. (http://netti.nic.fi/~054028/images/LevitorMK1.0-1.mpg)
  - Website about this device, with images (in Finnish). (http://netti.nic.fi/~054028/)

Electromagnetic induction

Electromagnetic induction is the production of an electric current across a conductor moving through a magnetic field. It underlies the operation of generators, transformers, induction motors, electric motors, synchronous motors, and solenoids.

Michael Faraday is generally credited with the discovery of induction in 1831 though it may have been anticipated by the work of Francesco Zantedeschi in 1829. Around 1830[1] to 1832[2] Joseph Henry made a similar discovery, but did not publish his findings until later.

Overview

Michael Faraday stated that electromotive force (EMF) produced around a closed path is proportional to the rate of change of the magnetic flux through any surface bounded by that path. In practice, this means that an electric current will be induced in any closed circuit when the magnetic flux through a surface bounded by the conductor changes. This applies whether the field itself changes in strength or the conductor is moved through it.

In mathematical form, Faraday's law states that:

$$\mathcal{E} = -\frac{d\Phi_B}{dt},$$

where

ε is the electromotive force

Φ_B is the magnetic flux.

For the special case of a coil of wire, composed of *N* loops with the same area, the equation becomes

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}.$$

A corollary of Faraday's Law, together with Ampère's law and Ohm's law is Lenz's law: The EMF induced in an electric circuit always acts in such a direction that the current it drives around the circuit opposes the change in magnetic flux which produces the EMF.[3]

Applications

The principles of electromagnetic induction are applied in many devices and systems, including:

- Current clamp
- Electrical generators
- Electromagnetic forming
- Graphics tablet
- Hall effect meters
- Induction cookers
- Induction motors
- Induction sealing
- Induction welding
- Inductors
- Magnetic flow meters
- Mechanically powered flashlight
- Pickups
- Rowland ring
- Transcranial magnetic stimulation
- Transformers
- Wireless energy transfer

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External links

- A free java simulation on motional EMF (http://www.phy.hk/wiki/englishhtm/Induction.htm)
- Two videos demonstrating Faraday's and Lenz's laws at EduMation (http://msdaif.googlepages.com/physics)
- European Commission (http://ec.europa.eu/CEmarking): Blue Guide (http://ec.europa.eu/enterprise/policies/single-market-goods/files/blue-guide/guidepublic_en.pdf)

Electromagnetic radiation

Electromagnetic radiation (often abbreviated **E-M radiation** or **EMR**) is a form of energy that exhibits wave-like behavior as it travels through space. EMR has both electric and magnetic field components, which oscillate in phase perpendicular to each other and perpendicular to the direction of energy propagation.

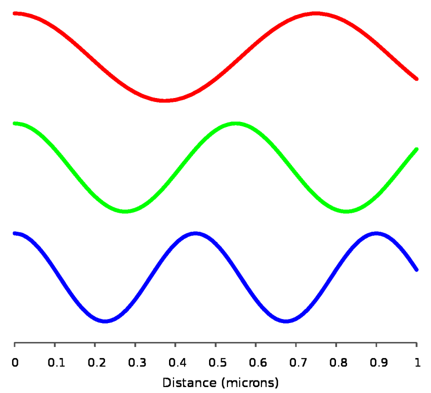
Electromagnetic radiation is classified according to the frequency of its wave. The electromagnetic spectrum, in order of increasing frequency and decreasing wavelength, consists of radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays. The eyes of various organisms sense a small and somewhat variable window of frequencies called the visible spectrum. The photon is the quantum of the electromagnetic interaction and the basic "unit" of light and all other forms of electromagnetic radiation and is also the force carrier for the electromagnetic force.

Electromagnetic radiation carries energy - sometimes called radiant energy - and momentum that may be imparted to matter with which it interacts, through absorption of electromagnetic radiation.

Physics

Theory

James Clerk Maxwell first formally postulated *electromagnetic waves*. These were subsequently confirmed by Heinrich Hertz. Maxwell derived a wave form of the electric and magnetic equations, thus uncovering the wave-like nature of electric and magnetic fields, and their symmetry. Because the speed of EM waves predicted by the wave equation coincided with the measured speed of light, Maxwell concluded that light itself is an EM wave.

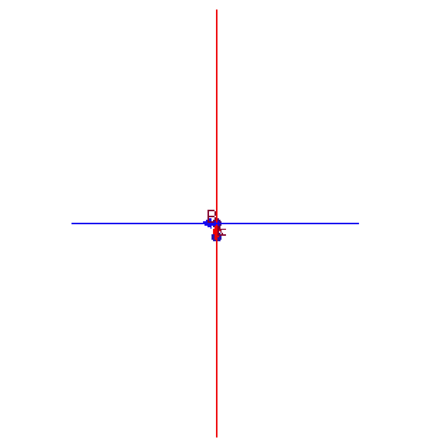


Shows the relative wavelengths of the electromagnetic waves of three different colors of light (blue, green, and red) with a distance scale in micrometres along the x-axis.

According to Maxwell's equations, a spatially varying electric field causes the magnetic field to change over time. Likewise, a spatially varying magnetic field causes changes over time in the electric field. In an electromagnetic wave, the changes induced by the electric field shift the wave in the magnetic field in one direction; the action of the magnetic field shifts the electric field in the same direction. Together, these fields form a propagating electromagnetic wave. This view of propagating electromagnetic waves makes sense from a *local* perspective,[1] but note that some prefer instead to look into the past for the source charge(s) that were the *original* cause of the wave.[2]

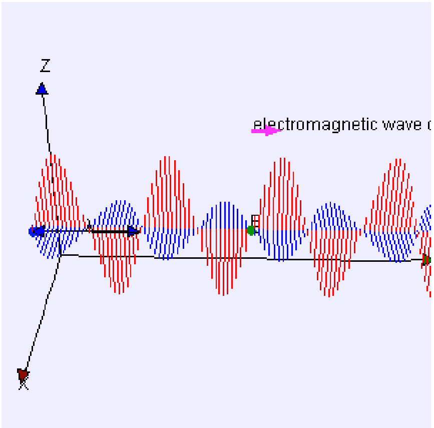
A quantum theory of the interaction between electromagnetic radiation and matter such as electrons is described by the theory of quantum electrodynamics.

Properties



The electric field is in a vertical plane and the magnetic field in a horizontal plane

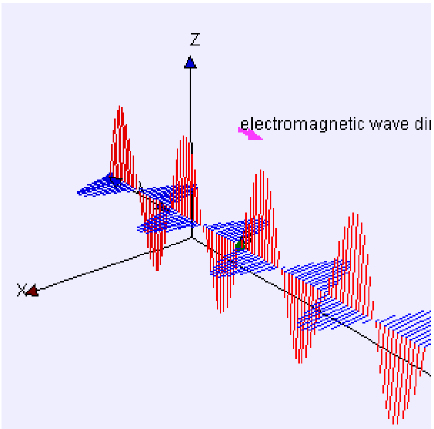
The physics of electromagnetic radiation is electrodynamics. Electromagnetism is the physical phenomenon associated with the theory of electrodynamics. Electric and magnetic fields obey the properties of superposition. Thus, a field due to any particular particle or time-varying electric or magnetic field contributes to the fields present in the same space due to other causes. Further, as they are vector fields, all magnetic and electric field vectors add together according to vector addition. For example, in optics two or more coherent lightwaves may interact and by constructive or destructive interference yield a resultant irradiance deviating from the sum of the component irradiances of the individual lightwaves.



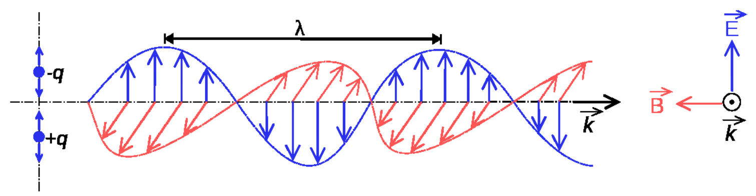
Electromagnetic waves can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. This 3D diagram shows a plane linearly polarized wave propagating from left to right

Since light is an oscillation it is not affected by travelling through static electric or magnetic fields in a linear medium such as a vacuum. However in nonlinear media, such as some crystals, interactions can occur between light and static electric and magnetic fields — these interactions include the Faraday effect and the Kerr effect.

In refraction, a wave crossing from one medium to another of different density alters its speed and direction upon entering the new medium. The ratio of the refractive indices of the media determines the degree of refraction, and is summarized by Snell's law. Light of composite wavelengths (natural sunlight) disperses into a visible spectrum passing through a prism, because of the wavelength dependent refractive index of the prism material (dispersion); that is, each component wave within the composite light is bent a different amount.



This 3D diagram shows a plane linearly polarized wave propagating from left to right



Electromagnetic waves can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. This diagram shows a plane linearly polarized wave propagating from left to right. The electric field is in a vertical plane and the magnetic field in a horizontal plane.

EM radiation exhibits both wave properties and particle properties at the same time (see wave-particle duality). Both wave and particle characteristics have been confirmed in a large number of experiments. Wave characteristics are more apparent when EM radiation is measured over relatively large timescales and over large distances while particle characteristics are more evident when measuring small timescales and distances. For example, when electromagnetic radiation is absorbed by matter, particle-like properties will be more obvious when the average number of photons in the cube of the relevant wavelength is much smaller than 1. Upon absorption of light, it is not too difficult to experimentally observe non-uniform deposition of energy. However, this alone is not evidence of "particulate" behavior of light. Rather, it reflects the quantum nature of *matter*.[3]

There are experiments in which the wave and particle natures of electromagnetic waves appear in the same experiment, such as the self-interference of a single photon. *True* single-photon experiments (in a quantum optical sense) can be done today in undergraduate-level labs.[4] When a single photon is sent through an interferometer, it passes through both paths, interfering with itself, as waves do, yet is detected by a photomultiplier or other sensitive detector only once.

Wave model

Electromagnetic radiation is a transverse wave meaning that the oscillations of the waves are perpendicular to the direction of energy transfer and travel. An important aspect of the nature of light is frequency. The frequency of a wave is its rate of oscillation and is measured in hertz, the SI unit of frequency, where one hertz is equal to one oscillation per second. Light usually has a spectrum of frequencies that sum to form the resultant wave. Different frequencies undergo different angles of refraction.

A wave consists of successive troughs and crests, and the distance between two adjacent crests or troughs is called the wavelength. Waves of the electromagnetic spectrum vary in size, from very long radio waves the size of buildings to very short gamma rays smaller than atom nuclei. Frequency is inversely proportional to wavelength, according to the equation:

$$v = f\lambda$$

where v is the speed of the wave (c in a vacuum, or less in other media), f is the frequency and λ is the wavelength. As waves cross boundaries between different media, their speeds change but their frequencies remain constant.

Interference is the superposition of two or more waves resulting in a new wave pattern. If the fields have components in the same direction, they constructively interfere, while opposite directions cause destructive interference.

The energy in electromagnetic waves is sometimes called radiant energy.

Particle model

Because energy of an electromagnetic wave is quantized (see second quantization), electromagnetic energy is emitted and absorbed as discrete packets of energy, or quanta, called photons. The energy of the photons is proportional to the frequency of the wave.[5] On the converse, in a first-quantized treatment, because a photon acts as a transporter of energy, it is associated with a probability wave with frequency proportional to the energy carried. In both treatments, the energy per photon is related to the frequency via the Planck–Einstein equation:[6]

$$E = hf$$

where E is the energy, h is Planck's constant, and f is frequency. The energy is commonly expressed in the unit of electronvolt (eV). This photon-energy expression is a particular case of the energy levels of the more general *electromagnetic oscillator*, whose average energy, which is used to obtain Planck's radiation law, can be shown to differ sharply from that predicted by the equipartition principle at low temperature, thereby establishes a failure of equipartition due to quantum effects at low temperature.[7]

As a photon is absorbed by an atom, it excites the atom, elevating an electron to a higher energy level. If the energy is great enough, so that the electron jumps to a high enough energy level, it may escape the positive pull of the nucleus and be liberated from the atom in a process called photoionisation. However, an electron that descends to a lower energy level in an atom emits a photon of light equal to the energy difference. Since the energy levels of electrons in atoms are discrete, each element emits and absorbs its own characteristic frequencies.

Together, these effects explain the emission and absorption spectra of light. The dark bands in the absorption spectrum are due to the atoms in the intervening medium absorbing different frequencies of the light. The composition of the medium through which the light travels determines the nature of the absorption spectrum. For instance, dark bands in the light emitted by a distant star are due to the atoms in the star's atmosphere. These bands correspond to the allowed energy levels in the atoms. A similar phenomenon occurs for emission. As the electrons descend to lower energy levels, a spectrum is emitted that represents the jumps between the energy levels of the electrons. This is manifested in the emission spectrum of nebulae. Today, scientists use this phenomenon to observe what elements a certain star is composed of. It is also used in the determination of the distance of a star, using the red shift.

Speed of propagation

Any electric charge that accelerates, or any changing magnetic field, produces electromagnetic radiation. Electromagnetic information about the charge travels at the speed of light. Accurate treatment thus incorporates a concept known as retarded time (as opposed to advanced time, which is not physically possible in light of causality), which adds to the expressions for the electrodynamic electric field and magnetic field. These extra terms are responsible for electromagnetic radiation. When any wire (or other conducting object such as an antenna) conducts alternating current, electromagnetic radiation is propagated at the same frequency as the electric current. At the quantum level, electromagnetic radiation is produced when the wavepacket of a charged particle oscillates or otherwise accelerates. Charged particles in a stationary state do not move, but a superposition of such states may result in oscillation, which is responsible for the phenomenon of radiative transition between quantum states of a charged particle.

Depending on the circumstances, electromagnetic radiation may behave as a wave or as particles. As a wave, it is characterized by a velocity (the speed of light), wavelength, and frequency. When considered as particles, they are known as photons, and each has an energy related to the frequency of the wave given by Planck's relation $E = h\nu$, where E is the energy of the photon, $h = 6.626 \times 10^{-34}$ J·s is Planck's constant, and ν is the frequency of the wave.

One rule is always obeyed regardless of the circumstances: EM radiation in a vacuum always travels at the speed of light, *relative to the observer*, regardless of the observer's velocity. (This observation led to Albert Einstein's development of the theory of special relativity.)

In a medium (other than vacuum), velocity factor or refractive index are considered, depending on frequency and application. Both of these are ratios of the speed in a medium to speed in a vacuum.

Thermal radiation and electromagnetic radiation as a form of heat

The basic structure of matter involves charged particles bound together in many different ways. When electromagnetic radiation is incident on matter, it causes the charged particles to oscillate and gain energy. The ultimate fate of this energy depends on the situation. It could be immediately re-radiated and appear as scattered, reflected, or transmitted radiation. It may also get dissipated into other microscopic motions within the matter, coming to thermal equilibrium and manifesting itself as thermal energy in the material. With a few exceptions such as fluorescence, harmonic generation, photochemical reactions and the photovoltaic effect, absorbed electromagnetic radiation simply deposits its energy by heating the material. This happens both for infrared and for non-infrared radiation. Intense radio waves can thermally burn living tissue and can cook food. In addition to infrared lasers, sufficiently intense visible and ultraviolet lasers can also easily set paper afire. Ionizing electromagnetic radiation can create high-speed electrons in a material and break chemical bonds, but after these electrons collide many times with other atoms in the material eventually most of the energy gets downgraded to thermal energy, this whole process happening in a tiny fraction of a second. That infrared radiation is a form of heat and other electromagnetic radiation is not, is a widespread misconception in physics. *Any* electromagnetic radiation can heat a material when it is absorbed.

The inverse or time-reversed process of absorption is responsible for thermal radiation. Much of the thermal energy in matter consists of random motion of charged particles, and this energy can be radiated away from the matter. The resulting radiation may subsequently be absorbed by another piece of matter, with the deposited energy heating the material. Radiation is an important mechanism of heat transfer.

The electromagnetic radiation in an opaque cavity at thermal equilibrium is effectively a form of thermal energy, having maximum radiation entropy. The thermodynamic potentials of electromagnetic radiation can be well-defined as for matter. Thermal radiation in a cavity has energy density (see Planck's Law) of

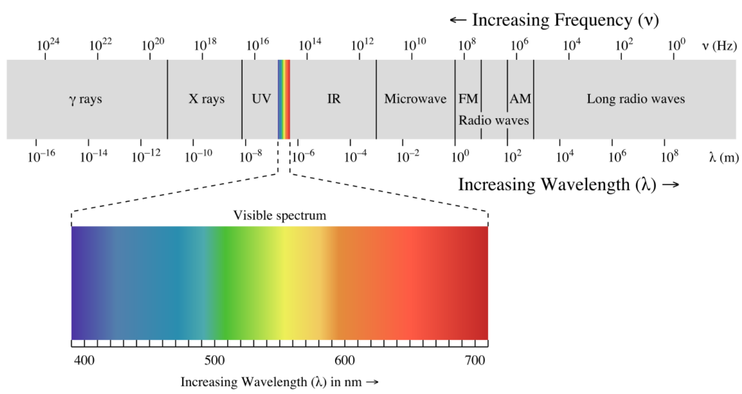
$$\frac{U}{V} = \frac{8\pi^5 (kT)^4}{15 (hc)^3},$$

Differentiating the above with respect to temperature, we may say that the electromagnetic radiation field has an effective volumetric heat capacity given by

$$\frac{32\pi^5 k^4 T^3}{15 (hc)^3},$$

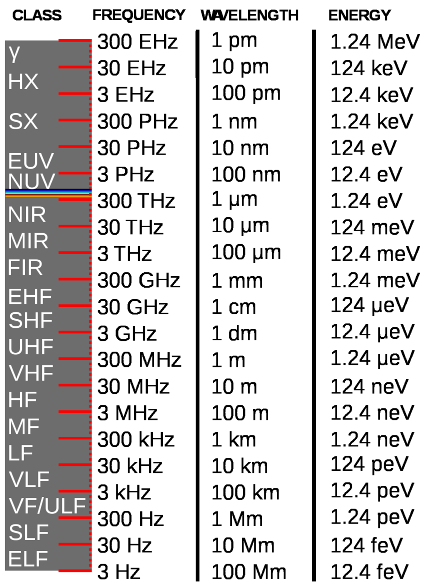
Electromagnetic spectrum

In general, EM radiation (the designation 'radiation' excludes static electric and magnetic and near fields) is classified by wavelength into radio, microwave, infrared, the visible region we perceive as light, ultraviolet, X-rays, and gamma rays. Arbitrary electromagnetic waves can always be expressed by Fourier analysis in terms of sinusoidal monochromatic waves, which can be classified into these regions of the spectrum.



Electromagnetic spectrum with light highlighted

The behavior of EM radiation depends on its wavelength. Higher frequencies have shorter wavelengths, and lower frequencies have longer wavelengths. When EM radiation interacts with single atoms and molecules, its behavior depends on the amount of energy per quantum it carries. Spectroscopy can detect a much wider region of the EM spectrum than the visible range of 400 nm to 700 nm. A common laboratory spectroscope can detect wavelengths from 2 nm to 2500 nm. Detailed information about the physical properties of objects, gases, or even stars can be obtained from this type of device. It is widely used in astrophysics. For example, hydrogen atoms emit radio waves of wavelength 21.12 cm.



Legend: γ = Gamma rays HX = Hard X-rays SX = Soft X-Rays EUV = Extreme-ultraviolet NUV = Near-ultravioletVisible light NIR = Near-infrared MIR = Moderate-infrared FIR = Far-infrared**Radio waves:** EHF = Extremely high frequency (Microwaves) SHF = Super-high frequency (Microwaves) UHF = Ultrahigh frequency VHF = Very high frequency HF = High frequency MF = Medium frequency LF = Low frequency VLF = Very low frequency VF = Voice frequency ULF = Ultra-low frequency SLF = Super-low frequency ELF = Extremely low frequency

Soundwaves are not electromagnetic radiation. At the lower end of the electromagnetic spectrum, about 20 Hz to about 20 kHz, are frequencies that might be considered in the audio range. However, electromagnetic waves cannot be directly perceived by human ears. Sound waves are the oscillating compression of molecules. To be heard, electromagnetic radiation must be converted to air pressure waves, or if the ear is submerged, water pressure waves.

Light

EM radiation with a wavelength between approximately 400 nm and 700 nm is directly detected by the human eye and perceived as visible light. Other wavelengths, especially nearby infrared (longer than 700 nm) and ultraviolet (shorter than 400 nm) are also sometimes referred to as light, especially when visibility to humans is not relevant.

If radiation having a frequency in the visible region of the EM spectrum reflects off of an object, say, a bowl of fruit, and then strikes our eyes, this results in our visual perception of the scene. Our brain's visual system processes the multitude of reflected frequencies into different shades and hues, and through this not-entirely-understood psychophysical phenomenon, most people perceive a bowl of fruit.

At most wavelengths, however, the information carried by electromagnetic radiation is not directly detected by human senses. Natural sources produce EM radiation across the spectrum, and our technology can also manipulate a broad range of wavelengths. Optical fiber transmits light, which, although not suitable for direct viewing, can carry data that can be translated into sound or an image. To be meaningful both transmitter and receiver must use some agreed-upon encoding system - especially so if the transmission is digital as opposed to the analog nature of the waves.

Radio waves

Radio waves can be made to carry information by varying the amplitude, frequency or phase.

When EM radiation impinges upon a conductor, it couples to the conductor, travels along it, and induces an electric current on the surface of that conductor by exciting the electrons of the conducting material. This effect (the skin effect) is used in antennas. EM radiation may also cause certain molecules to absorb energy and thus to heat up; this is exploited in microwave ovens. Radio waves are *not* ionizing radiation, as the energy per photon is too small.

Derivation

Electromagnetic waves as a general phenomenon were predicted by the classical laws of electricity and magnetism, known as Maxwell's equations. Inspection of Maxwell's equations without sources (charges or currents) results in, along with the possibility of nothing happening, nontrivial solutions of changing electric and magnetic fields. Beginning with Maxwell's equations in free space:

$$\nabla \cdot \mathbf{E} = 0 \qquad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (3)$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad (4)$$

where

∇ is a vector differential operator (see Del).

One solution,

$$\mathbf{E} = \mathbf{B} = \mathbf{0},$$

is trivial.

For a more useful solution, we utilize vector identities, which work for any vector, as follows:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

To see how we can use this, take the curl of equation (2):

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \qquad (5)$$

Evaluating the left hand side:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E} \qquad (6)$$

where we simplified the above by using equation (1).

Evaluate the right hand side:

$$\nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (7)$$

Equations (6) and (7) are equal, so this results in a vector-valued differential equation for the electric field, namely

$$\boxed{\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}}$$

Applying a similar pattern results in similar differential equation for the magnetic field:

$$\boxed{\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.}$$

These differential equations are equivalent to the wave equation:

$$\nabla^2 f = \frac{1}{{c_0}^2} \frac{\partial^2 f}{\partial t^2}$$

where

c_0 is the speed of the wave in free space and

f describes a displacement

Or more simply:

$$\Box f = 0$$

where $\Box$ is d'Alembertian:

$$\Box = \nabla^2 - \frac{1}{{c_0}^2}\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{{c_0}^2}\frac{\partial^2}{\partial t^2}$$

Notice that, in the case of the electric and magnetic fields, the speed is:

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

This is the speed of light in vacuum. Maxwell's equations have unified the vacuum permittivity ϵ_0, the vacuum permeability μ_0, and the speed of light itself, c_0. Before this derivation it was not known that there was such a strong relationship between light and electricity and magnetism.

But these are only two equations and we started with four, so there is still more information pertaining to these waves hidden within Maxwell's equations. Let's consider a generic vector wave for the electric field.

$$\mathbf{E} = \mathbf{E}_0 f\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right)$$

Here, $\mathbf{E}_0$ is the constant amplitude, f is any second differentiable function, $\hat{\mathbf{k}}$ is a unit vector in the direction of propagation, and $\mathbf{x}$ is a position vector. We observe that $f\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right)$ is a generic solution to the wave equation. In other words

$$\nabla^2 f\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = \frac{1}{{c_0}^2}\frac{\partial^2}{\partial t^2} f\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right),$$

for a generic wave traveling in the $\hat{\mathbf{k}}$ direction.

This form will satisfy the wave equation, but will it satisfy all of Maxwell's equations, and with what corresponding magnetic field?

$$\nabla \cdot \mathbf{E} = \hat{\mathbf{k}} \cdot \mathbf{E}_0 f'\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = 0$$

$$\mathbf{E} \cdot \hat{\mathbf{k}} = 0$$

The first of Maxwell's equations implies that electric field is orthogonal to the direction the wave propagates.

$$\nabla \times \mathbf{E} = \hat{\mathbf{k}} \times \mathbf{E}_0 f'\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\mathbf{B} = \frac{1}{c_0}\hat{\mathbf{k}} \times \mathbf{E}$$

The second of Maxwell's equations yields the magnetic field. The remaining equations will be satisfied by this choice of $\mathbf{E}, \mathbf{B}$.

Not only are the electric and magnetic field waves traveling at the speed of light but they have a special restricted orientation and proportional magnitudes, $E_0 = c_0 B_0$, which can be seen immediately from the Poynting vector. The electric field, magnetic field, and direction of wave propagation are all orthogonal, and the wave propagates in the same direction as $\mathbf{E} \times \mathbf{B}$.

From the viewpoint of an electromagnetic wave traveling forward, the electric field might be oscillating up and down, while the magnetic field oscillates right and left; but this picture can be rotated with the electric field oscillating right and left and the magnetic field oscillating down and up. This is a different solution that is traveling in the same direction. This arbitrariness in the orientation with respect to propagation direction is known as polarization. On a quantum level, it is described as photon polarization. The direction of the polarization is defined as the direction of the electric field.

More general forms of the second-order wave equations given above are available, allowing for both non-vacuum propagation media and sources. A great many competing derivations exist, all with varying levels of approximation and intended applications. One very general example is a form of the electric field equation,[8] which was factorized into a pair of explicitly directional wave equations, and then efficiently reduced into a single uni-directional wave equation by means of a simple slow-evolution approximation.

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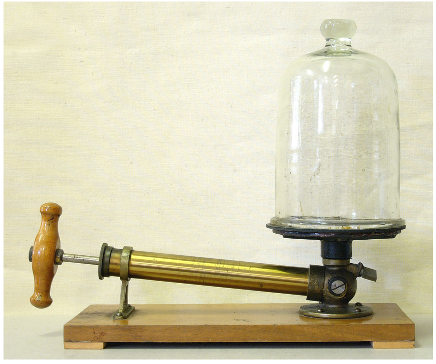
External links

- Electromagnetism (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html) - a chapter from an online textbook
- Electromagnetic Radiation (http://www.learnemc.com/tutorials/Radiation/EM_Radiation.html) - an introduction for electrical engineers
- *Electromagnetic Waves from Maxwell's Equations* (http://www.physnet.org/modules/pdf_modules/m210.pdf) on Project PHYSNET (http://www.physnet.org).
- Radiation of atoms? e-m wave, Polarisation, ... (http://www.hydrogenlab.de/elektronium/HTML/einleitung_hauptseite_uk.html)
- An Introduction to The Wigner Distribution in Geometric Optics (http://scripts.mit.edu/~raskar/lightfields/index.php?title=An_Introduction_to_The_Wigner_Distribution_in_Geometric_Optics)
- The windows of the electromagnetic spectrum, on Astronoo (http://www.astronoo.com/articles/electromagneticSpectrum-en.html)

Vacuum


Pump to demonstrate vacuum

In everyday usage, **vacuum** is a volume of space that is essentially empty of matter, such that its gaseous pressure is much less than atmospheric pressure.[1] The word comes from the Latin term for "empty". A **perfect vacuum** would be one with no particles in it at all, which is impossible to achieve in practice. Physicists often discuss ideal test results that would occur in a perfect vacuum, which they simply call "vacuum" or "free space", and use the term **partial vacuum** to refer to an actual imperfect vacuum as one might have in a laboratory or in space. The Latin term **in vacuo** is also used to describe an object as being in what would otherwise be a vacuum.

The *quality* of a vacuum refers to how closely it approaches a perfect vacuum. Other things equal, lower gas pressure means higher-quality vacuum. For example, a typical vacuum cleaner produces enough suction to reduce air pressure by around 20%.[2] Much higher-quality vacuums are possible. Ultra-high vacuum chambers, common in chemistry, physics, and engineering, operate below one trillionth (10^{-12}) of atmospheric pressure (100 nPa), and can reach around 100 particles/cm^3.[3] Outer space is an even higher-quality vacuum, with the equivalent of just a few hydrogen atoms per cubic meter on average.[4] However, even if every single atom and particle could be removed from a volume, it would still not be "empty" due to vacuum fluctuations, dark energy, and other phenomena in quantum physics. In modern Particle Physics, the vacuum is considered as the ground state of matter.

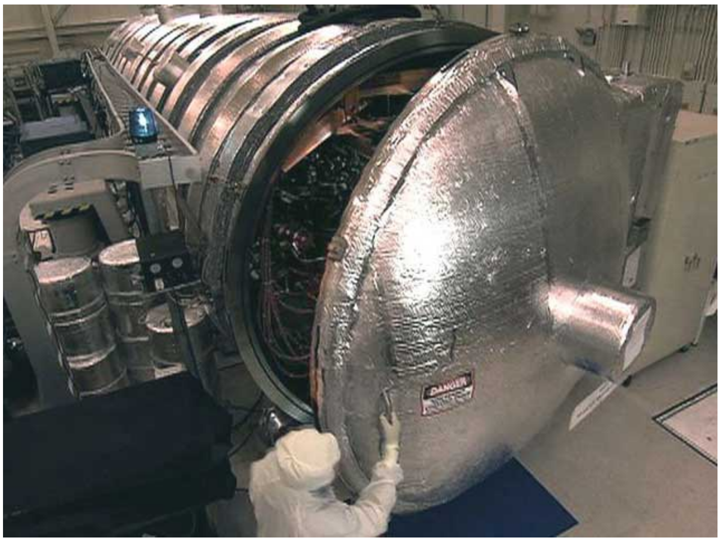
Vacuum has been a frequent topic of philosophical debate since ancient Greek times, but was not studied empirically until the 17th century. Evangelista Torricelli produced the first laboratory vacuum in 1643, and other experimental techniques were developed as a result of his theories of atmospheric pressure. A **torricellian vacuum** is created by filling with mercury a tall glass container closed at one end and then inverting the container into a bowl to contain the mercury.[5]

Vacuum became a valuable industrial tool in the 20th century with the introduction of incandescent light bulbs and vacuum tubes, and a wide array of vacuum technology has since become available. The recent development of human spaceflight has raised interest in the impact of vacuum on human health, and on life forms in general.

Etymology

From Latin **vacuum** (*an empty space, void*) noun use of neuter of *vacuus* (*empty*) related to *vacare* (*be empty*).

"Vacuum" is one of the few words in the English language that contains two consecutive 'u's[6] .



A large vacuum chamber

Uses

Vacuum is useful in a variety of processes and devices. Its first widespread use was in the incandescent light bulb to protect the filament from chemical degradation. The chemical inertness produced by a vacuum is also useful for electron beam welding, cold welding, vacuum packing and vacuum frying. Ultra-high vacuum is used in the study of atomically clean substrates, as only a very good vacuum preserves atomic-scale clean surfaces for a reasonably long time (on the order of minutes to days). High to ultra-high vacuum removes the obstruction of air, allowing particle beams to deposit or remove materials without contamination. This is the principle behind chemical vapor deposition, physical vapor deposition, and dry etching which are essential to the fabrication of semiconductors and optical coatings, and to surface science. The reduction of convection provides the thermal insulation of thermos bottles. Deep vacuum lowers the boiling point of liquids and promotes low temperature outgassing which is used in freeze drying, adhesive preparation, distillation, metallurgy, and process purging. The electrical properties of vacuum make electron microscopes and vacuum tubes possible, including cathode ray tubes. The elimination of air friction is useful for flywheel energy storage and ultracentrifuges.

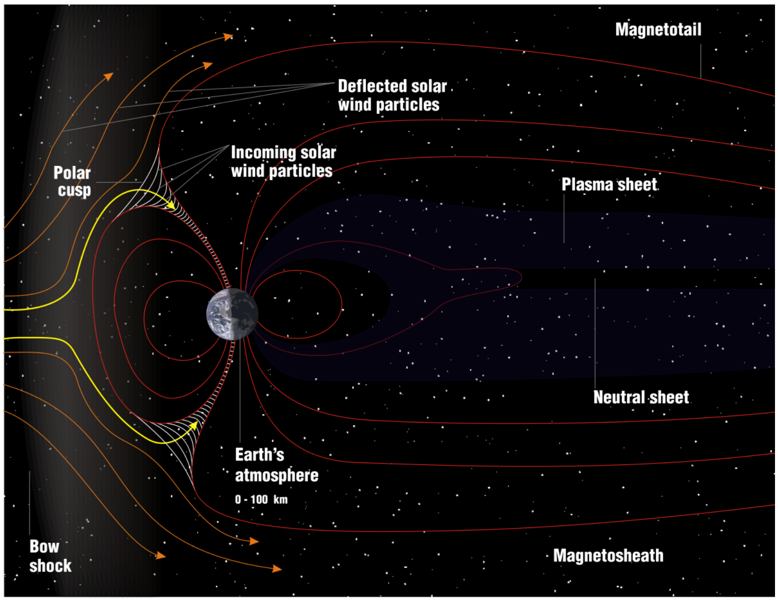
Light bulbs contain a partial vacuum, usually backfilled with argon, which protects the tungsten filament

Vacuum driven machines

Vacuums are commonly used to produce suction, which has an even wider variety of applications. The Newcomen steam engine used vacuum instead of pressure to drive a piston. In the 19th century, vacuum was used for traction on Isambard Kingdom Brunel's experimental atmospheric railway. Vacuum brakes were once widely used on trains in the UK but, except on heritage railways, they have been replaced by air brakes.

Manifold vacuum can be used to drive accessories on automobiles. The best-known application is the vacuum servo, used to provide power assistance for the brakes. Obsolete applications include vacuum-driven windscreen wipers and fuel pumps.

Outer space



Outer space is not a perfect vacuum, but a tenuous plasma awash with charged particles, electromagnetic fields, and the occasional star.

Outer space has very low density and pressure, and is the closest physical approximation of a perfect vacuum. It has effectively no friction, allowing stars, planets and moons to move freely along ideal gravitational trajectories. But no vacuum is truly perfect, not even in interstellar space, where there are still a few hydrogen atoms per cubic centimeter.[4]

Stars, planets and moons keep their atmospheres by gravitational attraction, and as such, atmospheres have no clearly delineated boundary: the density of atmospheric gas simply decreases with distance from the object. The Earth's atmospheric pressure drops to about 3.2×10^{-2} Pa at 100 kilometres (62 mi) of altitude,[7] the Kármán line, which is a common definition of the boundary with outer space. Beyond this line, isotropic gas pressure rapidly becomes insignificant when compared to radiation pressure from the sun and the dynamic pressure of the solar wind, so the definition of pressure becomes difficult to interpret. The thermosphere in this range has large gradients of pressure, temperature and composition, and varies greatly due to space weather. Astrophysicists prefer to use number density to describe these environments, in units of particles per cubic centimetre.

But although it meets the definition of outer space, the atmospheric density within the first few hundred kilometers above the Kármán line is still sufficient to produce significant drag on satellites. Most artificial satellites operate in this region called low earth orbit and must fire their engines every few days to maintain orbit. The drag here is low enough that it could theoretically be overcome by radiation pressure on solar sails, a proposed propulsion system for interplanetary travel. Planets are too massive for their trajectories to be significantly affected by these forces, although their atmospheres are eroded by the solar winds.

All of the observable universe is filled with large numbers of photons, the so-called cosmic background radiation, and quite likely a correspondingly large number of neutrinos. The current temperature of this radiation is about 3 K, or -270 degrees Celsius or -454 degrees Fahrenheit.

Effects on humans and animals

This painting, An Experiment on a Bird in the Air Pump by Joseph Wright of Derby, 1768, depicts an experiment performed by Robert Boyle in 1660.

Humans and animals exposed to vacuum will lose consciousness after a few seconds and die of hypoxia within minutes, but the symptoms are not nearly as graphic as commonly depicted in media and popular culture. The reduction in pressure lowers the temperature at which blood and other body fluids boil, but the elastic pressure of blood vessels ensures that this boiling point remains above the internal body temperature of 37°C.[8] Although the blood will not boil, the formation of gas bubbles in bodily fluids at reduced pressures, known as ebullism, is still a concern. The steam may bloat the body to twice its normal size and slow circulation, but tissues are elastic and porous enough to prevent rupture.[9] Swelling and ebullism can be restrained by containment in a flight suit. Shuttle astronauts wear a fitted elastic garment called the Crew Altitude Protection Suit (CAPS) which prevents ebullism at pressures as low as 2 kPa (15 Torr).[10] Rapid boiling will cool the skin and create frost, particularly in the mouth, but this is not a significant hazard.

Animal experiments show that rapid and complete recovery is normal for exposures shorter than 90 seconds, while longer full-body exposures are fatal and resuscitation has never been successful.[11] There is only a limited amount of data available from human accidents, but it is consistent with animal data. Limbs may be exposed for much longer if breathing is not impaired.[12] Robert Boyle was the first to show in 1660 that vacuum is lethal to small animals.

During 1942, in one of a series of experiments on human subjects for the Luftwaffe, the Nazi regime experimented on prisoners in Dachau concentration camp by exposing them to low pressure.[13]

Cold or oxygen-rich atmospheres can sustain life at pressures much lower than atmospheric, as long as the density of oxygen is similar to that of standard sea-level atmosphere. The colder air temperatures found at altitudes of up to 3 km generally compensate for the lower pressures there.[12] Above this altitude, oxygen enrichment is necessary to prevent altitude sickness in humans that did not undergo prior acclimatization, and spacesuits are necessary to prevent ebullism above 19 km.[12] Most spacesuits use only 20 kPa (150 Torr) of pure oxygen, just enough to sustain full consciousness. This pressure is high enough to prevent ebullism, but simple evaporation of blood can still cause decompression sickness and gas embolisms if not managed.

Rapid decompression can be much more dangerous than vacuum exposure itself. Even if the victim does not hold his or her breath, venting through the windpipe may be too slow to prevent the fatal rupture of the delicate alveoli of the lungs.[12] Eardrums and sinuses may be ruptured by rapid decompression, soft tissues may bruise and seep blood, and the stress of shock will accelerate oxygen consumption leading to hypoxia.[14] Injuries caused by rapid decompression are called barotrauma. A pressure drop of 13 kPa (100 Torr), which produces no symptoms if it is gradual, may be fatal if it occurs suddenly.[12]

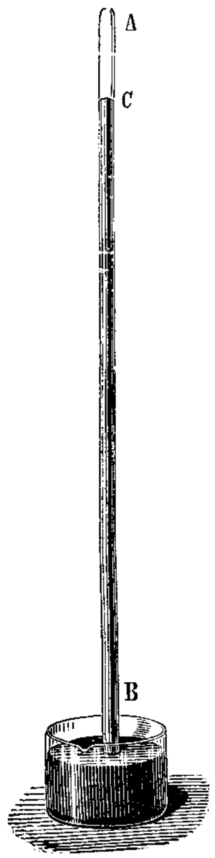
Some extremophile microrganisms, such as tardigrades, can survive vacuum for a period of days.

Historical interpretation

Historically, there has been much dispute over whether such a thing as a vacuum can exist. Ancient Greek philosophers did not like to admit the existence of a vacuum, asking themselves "how can 'nothing' be something?". Plato found the idea of a vacuum inconceivable. He believed that all physical things were instantiations of an abstract Platonic ideal, and he could not conceive of an "ideal" form of a vacuum. Similarly, Aristotle considered the creation of a vacuum impossible — nothing could not be something. Later Greek philosophers thought that a vacuum could exist outside the cosmos, but not within it. Hero of Alexandria was the first to challenge this belief in the first century AD, but his attempts to create an artificial vacuum failed.[15]

In the Roman city of Pompeii, a dual-action suction pump was found, proving that the ancient Romans had access to this kind of technology. Used for raising water, this pump had two cylinders, alternately operated by a walking-beam pump. In the suction phase, a lower valve opened, permitting the entry of water into the cylinder, while an upper valve remained closed. When the piston went down, the lower valve closed and the upper one opened.[16]

In the medieval Islamic world, the Muslim physicist and philosopher, Al-Farabi (Alpharabius, 872-950), conducted a small experiment concerning the existence of vacuum, in which he investigated handheld plungers in water.[17] He concluded that air's volume can expand to fill available space, and he suggested that the concept of perfect vacuum was incoherent.[18] However, the Muslim physicist Ibn al-Haytham (Alhazen, 965-1039) and the Mu'tazili theologians disagreed with Aristotle and Al-Farabi, and they supported the existence of a void. Using geometry, Ibn al-Haytham mathematically demonstrated that place (*al-makan*) is the imagined three-dimensional void between the inner surfaces of a containing body.[19] Abū Rayhān al-Bīrūnī also states that "there is no observable evidence that rules out the possibility of vacuum".[20] The suction pump was described in 1206 by the Muslim engineer and inventor, Al-Jazari. The suction pump later appeared in Europe from the 15th century.[21] [22] [23] Taqi al-Din's six-cylinder 'Monobloc' pump, invented in 1551, could also create a partial vacuum.



Torricelli's mercury barometer produced one of the first sustained vacuums in a laboratory.

In medieval Europe, the Catholic Church regarded the idea of a vacuum as against nature or even heretical; the absence of anything implied the absence of God, and harkened back to the void prior to the creation story in the Book of Genesis.[24] Medieval thought experiments into the idea of a vacuum considered whether a vacuum was present, if only for an instant, between two flat plates when they were rapidly separated.[24] There was much discussion of whether the air moved in quickly enough as the plates were separated, or, as Walter Burley postulated, whether a 'celestial agent' prevented the vacuum arising. The commonly held view that nature abhorred a vacuum was called *horror vacui*. Speculation that even God could not create a vacuum if he wanted to was shut down by the 1277 Paris condemnations of Bishop Etienne Tempier, which required there to be no restrictions on the powers of God, which led to the conclusion that God could create a vacuum if he so wished.[25] René Descartes also argued against the existence of a vacuum, arguing along the following lines: "Space is identical with extension, but extension is connected with bodies; thus there is no space without bodies and hence no empty space (vacuum)." In spite of this, opposition to the idea of a vacuum existing in nature continued into the Scientific Revolution, with scholars such as Paolo Casati taking an anti-vacuist position. Jean Buridan reported in the 14th century that teams of ten horses could not pull open bellows when the port was sealed, apparently because of horror vacui.[15]

The Crookes tube, used to discover and study cathode rays, was an evolution of the Geissler tube.

The belief in horror vacui was overthrown in the 17th century. Water pump designs had improved by then to the point that they produced measurable vacuums, but this was not immediately understood. What was known was that suction pumps could not pull water beyond a certain height: 18 Florentine yards according to a measurement taken around 1635. (The conversion to metres is uncertain, but it would be about 9 or 10 metres.) This limit was a concern to irrigation projects, mine drainage, and decorative water fountains planned by the Duke of Tuscany, so the Duke commissioned Galileo to investigate the problem. Galileo advertised the puzzle to other scientists, including Gasparo Berti who replicated it by building the first water barometer in Rome in 1639.[26] Berti's barometer produced a vacuum above the water column, but he could not explain it. The breakthrough was made by Evangelista Torricelli in 1643. Building upon Galileo's notes, he built the first mercury barometer and wrote a convincing argument that the space at the top was a vacuum. The height of the column was then limited to the maximum weight that atmospheric pressure could support. Some people believe that although Torricelli's experiment was crucial, it was Blaise Pascal's experiments that proved the top space really contained vacuum.

In 1654, Otto von Guericke invented the first vacuum pump[27] and conducted his famous Magdeburg hemispheres experiment, showing that teams of horses could not separate two hemispheres from which the air had been (partially) evacuated. Robert Boyle improved Guericke's design and conducted experiments on the properties of vacuum. Robert Hooke also helped Boyle produce an air pump which helped to produce the vacuum. The study of vacuum then lapsed until 1850 when August Toepler invented the Toepler Pump. Then in 1855 Heinrich Geissler invented the mercury displacement pump and achieved a record vacuum of about 10 Pa (0.1 Torr). A number of electrical properties become observable at this vacuum level, and this renewed interest in vacuum. This, in turn, led to the development of the vacuum tube. Shortly after this Hermann Sprengel invented the Sprengel Pump in 1865.

While outer space has been likened to a vacuum, early theories of the nature of light relied upon the existence of an invisible, aetherial medium which would convey waves of light. (Isaac Newton relied on this idea to explain refraction and radiated heat).[28] This evolved into the luminiferous aether of the 19th century, but the idea was known to have significant shortcomings - specifically, that if the Earth were moving through a material medium, the medium would have to be both extremely tenuous (because the Earth is not detectably slowed in its orbit), and extremely rigid (because vibrations propagate so rapidly). An 1891 article by William Crookes noted: "the [freeing of] occluded gases into the vacuum of space".[29] Even up until 1912, astronomer Henry Pickering commented: "While the interstellar absorbing medium may be simply the ether, [it] is characteristic of a gas, and free gaseous molecules are certainly there".[30]

In 1887, the Michelson-Morley experiment, using an interferometer to attempt to detect the change in the speed of light caused by the Earth moving with respect to the aether, was a famous null result. Many misinterpreted the results, which neither proved nor disproved the existence of the aether, as showing that there really was no static, pervasive medium throughout space and through which the Earth moved as though through a wind.[31] [32] As a simplification, one can assume there no aether, and no such entity is required for the propagation of light. Besides the various particles which comprise cosmic radiation, there is a cosmic background of photonic radiation (electromagnetic radiation), including the cosmic microwave background (CMB), the thermal remnant of the Big Bang at about 2.7 K. However, none of these findings affect the outcome of the Michelson-Morley experiment to any significant degree.

Einstein argued that physical objects are not located in space, but rather have a spatial extent. Seen this way, the concept of empty space loses its meaning.[33] Rather, space is an abstraction, based on the relationships between local objects. Nevertheless, the general theory of relativity admits a pervasive gravitational field, which, in Einstein's words,[34] may be regarded as an "aether", with properties varying from one location to another. One must take care, though, to not ascribe to it material properties such as velocity and so on.

In 1930, Paul Dirac proposed a model of vacuum as an infinite sea of particles possessing negative energy, called the Dirac sea. This theory helped refine the predictions of his earlier formulated Dirac equation, and successfully predicted the existence of the positron, discovered two years later in 1932. Despite this early success, the idea was soon abandoned in favour of the more elegant quantum field theory.

The development of quantum mechanics has complicated the modern interpretation of vacuum by requiring indeterminacy. Niels Bohr and Werner Heisenberg's uncertainty principle and Copenhagen interpretation, formulated in 1927, predict a fundamental uncertainty in the instantaneous measurability of the position and momentum of any particle, and which, not unlike the gravitational field, questions the emptiness of space between particles. In the late 20th century, this principle was understood to also predict a fundamental uncertainty in the number of particles in a region of space, leading to predictions of virtual particles arising spontaneously out of the void. In other words, there is a lower bound on the vacuum, dictated by the lowest possible energy state of the quantized fields in any region of space.

In electromagnetism

In classical electromagnetism, *free space* or *perfect vacuum* is a standard reference medium for electromagnetic effects.[35] [36]

In the theory of classical electromagnetism, free space has the following properties:

- Electromagnetic radiation travels without obstructions, at the speed of light.
- The superposition principle is always exactly true.[37] For example, the electric potential generated by two charges is the simple addition of the potentials generated by each charge in isolation. The value of the electric field at any point around these two charges is found by calculating the vector sum of the two electric fields from each of the charges acting alone.
- The permittivity and permeability are exactly ε_0 and μ_0 respectively (in SI units), or exactly 1 (in Gaussian units).
- The characteristic impedance (η) equals the impedance of free space $Z_0 \approx 376.73\ \Omega$.

In quantum mechanics

In quantum mechanics and quantum field theory, the vacuum is defined as the state (i.e. solution to the equations of the theory) with the lowest possible energy (the ground state of the Hilbert space). This is a state with no matter particles (hence the name), and also no photons, no gravitons, etc. As described above, this state is impossible to achieve experimentally. (Even if every matter particle could somehow be removed from a volume, it would be impossible to eliminate all the blackbody photons.)

This hypothetical vacuum state often has interesting and complex properties. For example, it contains vacuum fluctuations (virtual particles that hop into and out of existence). It also, relatedly, has a finite energy, called vacuum energy. Vacuum fluctuations are an essential and ubiquitous part of quantum field theory. Some readily-apparent effects of vacuum fluctuations include the Casimir effect and Lamb shift.[25]

There can be more than one possible vacuum state. The starting and ending of cosmological inflation is thought to have arisen from transitions between different vacuum states. For theories obtained by quantization of a classical theory, each stationary point of the energy in the configuration space gives rise to a single vacuum. String theory is believed to have a huge number of vacua - the so-called string theory landscape.

In the superfluid vacuum theory the physical vacuum is described as the quantum superfluid which is essentially non-relativistic whereas the Lorentz symmetry is an approximate emerging symmetry valid only for the small fluctuations of the superfluid background. An observer who resides inside such vacuum and is capable of creating and/or measuring the small fluctuations would observe them as relativistic objects - unless their energy and momentum are sufficiently high (as compared to the background ones) to make the Lorentz-breaking corrections detectable. It was shown that the relativistic gravity arises as the small-amplitude collective excitation mode whereas the relativistic elementary particles can be described by the particle-like modes in the low-momentum limit.

Pumping and ambient air pressure

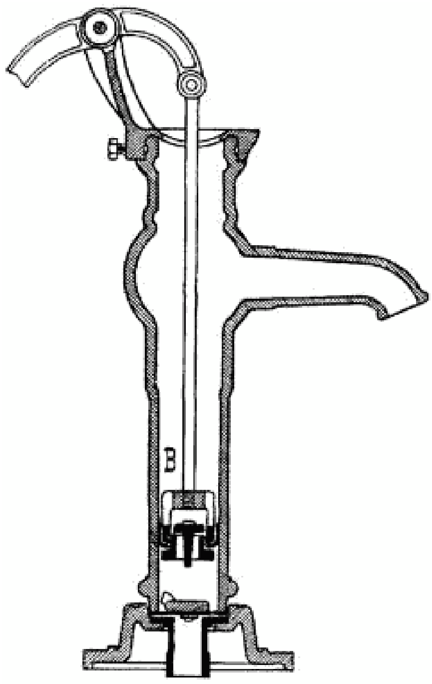
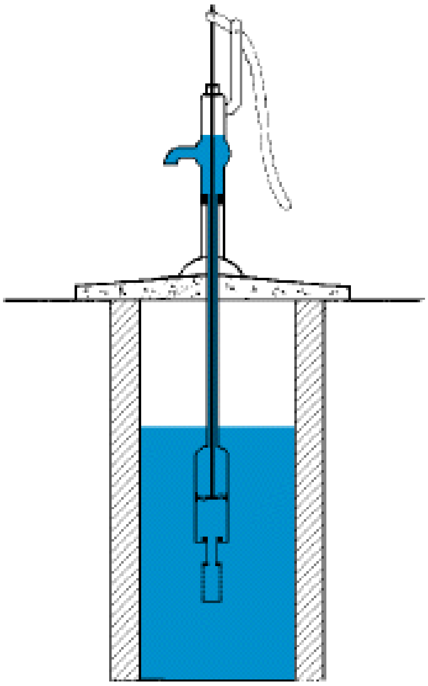


Fig. 9.

This shallow water well pump reduces atmospheric air pressure inside the pump chamber. Atmospheric pressure extends down into the well, and forces water up the pipe into the pump to balance the reduced pressure. Above-ground pump chambers are only effective to a depth of approximately 9 meters due to the water column weight balancing the atmospheric pressure.

Fluids cannot generally be pulled, so a vacuum cannot be created by suction. Suction can spread and dilute a vacuum by letting a higher pressure push fluids into it, but the vacuum has to be created first before suction can occur. The easiest way to create an artificial vacuum is to expand the volume of a container. For example, the diaphragm muscle expands the chest cavity, which causes the volume of the lungs to increase. This expansion reduces the pressure and creates a partial vacuum, which is soon filled by air pushed in by atmospheric pressure.

To continue evacuating a chamber indefinitely without requiring infinite growth, a compartment of the vacuum can be repeatedly closed off, exhausted, and expanded again. This is the principle behind positive displacement pumps, like the manual water pump for example. Inside the pump, a mechanism expands a small sealed cavity to create a vacuum. Because of the pressure differential, some fluid from the chamber (or the well, in our example) is pushed into the pump's small cavity. The pump's cavity is then sealed from the chamber, opened to the atmosphere, and squeezed back to a minute size.



Deep wells have the pump chamber down in the well close to the water surface, or in the water. A "sucker rod" extends from the handle down the center of the pipe deep into the well to operate the plunger. The pump handle acts as a heavy counterweight against both the sucker rod weight and the weight of the water column standing on the upper plunger up to ground level.

The above explanation is merely a simple introduction to vacuum pumping, and is not representative of the entire range of pumps in use. Many variations of the positive displacement pump have been developed, and many other pump designs rely on fundamentally different principles. Momentum transfer pumps, which bear some similarities to dynamic pumps used at higher pressures, can achieve much higher quality vacuums than positive displacement pumps. Entrapment pumps can capture gases in a solid or absorbed state, often with no moving parts, no seals and no vibration. None of these pumps are universal; each type has important performance limitations. They all share a difficulty in pumping low molecular weight gases, especially hydrogen, helium, and neon.

A cutaway view of a turbomolecular pump, a momentum transfer pump used to achieve high vacuum

The lowest pressure that can be attained in a system is also dependent on many things other than the nature of the pumps. Multiple pumps may be connected in series, called stages, to achieve higher vacuums. The choice of seals, chamber geometry, materials, and pump-down procedures will all have an impact. Collectively, these are called *vacuum technique*. And sometimes, the final pressure is not the only relevant characteristic. Pumping systems differ in oil contamination, vibration, preferential pumping of certain gases, pump-down speeds, intermittent duty cycle, reliability, or tolerance to high leakage rates.

In ultra high vacuum systems, some very "odd" leakage paths and outgassing sources must be considered. The water absorption of aluminium and palladium becomes an unacceptable source of outgassing, and even the adsorptivity of hard metals such as stainless steel or titanium must be considered. Some oils and greases will boil off in extreme vacuums. The permeability of the metallic chamber walls may have to be considered, and the grain direction of the metallic flanges should be parallel to the flange face.

The lowest pressures currently achievable in laboratory are about 10^{-13} torr (13 pPa).[38] However, pressures as low as 5×10^{-17} Torr (6.7 fPa) have been indirectly measured in a 4 K cryogenic vacuum system.[3] This corresponds to ≈100 particles/cm^3.

Outgassing

Evaporation and sublimation into a vacuum is called outgassing. All materials, solid or liquid, have a small vapour pressure, and their outgassing becomes important when the vacuum pressure falls below this vapour pressure. In man-made systems, outgassing has the same effect as a leak and can limit the achievable vacuum. Outgassing products may condense on nearby colder surfaces, which can be troublesome if they obscure optical instruments or react with other materials. This is of great concern to space missions, where an obscured telescope or solar cell can ruin an expensive mission.

The most prevalent outgassing product in man-made vacuum systems is water absorbed by chamber materials. It can be reduced by desiccating or baking the chamber, and removing absorbent materials. Outgassed water can condense in the oil of rotary vane pumps and reduce their net speed drastically if gas ballasting is not used. High vacuum systems must be clean and free of organic matter to minimize outgassing.

Ultra-high vacuum systems are usually baked, preferably under vacuum, to temporarily raise the vapour pressure of all outgassing materials and boil them off. Once the bulk of the outgassing materials are boiled off and evacuated, the system may be cooled to lower vapour pressures and minimize residual outgassing during actual operation. Some systems are cooled well below room temperature by liquid nitrogen to shut down residual outgassing and simultaneously cryopump the system.

Quality

The quality of a vacuum is indicated by the amount of matter remaining in the system, so that a high quality vacuum is one with very little matter left in it. Vacuum is primarily measured by its absolute pressure, but a complete characterization requires further parameters, such as temperature and chemical composition. One of the most important parameters is the **mean free path** (MFP) of residual gases, which indicates the average distance that molecules will travel between collisions with each other. As the gas density decreases, the MFP increases, and when the MFP is longer than the chamber, pump, spacecraft, or other objects present, the continuum assumptions of fluid mechanics do not apply. This vacuum state is called *high vacuum*, and the study of fluid flows in this regime is called particle gas dynamics. The MFP of air at atmospheric pressure is very short, 70 nm, but at 100 mPa (~1×10^{-3} Torr) the MFP of room temperature air is roughly 100 mm, which is on the order of everyday objects such as vacuum tubes. The Crookes radiometer turns when the MFP is larger than the size of the vanes.

Vacuum quality is subdivided into ranges according to the technology required to achieve it or measure it. These ranges do not have universally agreed definitions, but a typical distribution is as follows:[39] [40]

	pressure (Torr)	pressure (Pa)
Atmospheric pressure	760	101.3 kPa
Low vacuum	760 to 25	100 kPa to 3 kPa
Medium vacuum	25 to 1×10^{-3}	3 kPa to 100 mPa
High vacuum	1×10^{-3} to 1×10^{-9}	100 mPa to 100 nPa
Ultra high vacuum	1×10^{-9} to 1×10^{-12}	100 nPa to 100 pPa
Extremely high vacuum	$<1\times10^{-12}$	<100 pPa
Outer Space	1×10^{-6} to $<3\times10^{-17}$	100 µPa to <3fPa
Perfect vacuum	0	0 Pa

- **Atmospheric pressure** is variable but standardized at 101.325 kPa (760 Torr)
- **Low vacuum**, also called *rough vacuum* or *coarse vacuum*, is vacuum that can be achieved or measured with rudimentary equipment such as a vacuum cleaner and a liquid column manometer.
- **Medium vacuum** is vacuum that can be achieved with a single pump, but the pressure is too low to measure with a liquid or mechanical manometer. It can be measured with a McLeod gauge, thermal gauge or a capacitive gauge.
- **High vacuum** is vacuum where the MFP of residual gases is longer than the size of the chamber or of the object under test. High vacuum usually requires multi-stage pumping and ion gauge measurement. Some texts differentiate between high vacuum and *very high vacuum.*
- **Ultra high vacuum** requires baking the chamber to remove trace gases, and other special procedures. British and German standards define ultra high vacuum as pressures below 10^{-6} Pa (10^{-8} Torr).[41] [42]
- **Deep space** is generally much more empty than any artificial vacuum. It may or may not meet the definition of high vacuum above, depending on what region of space and astronomical bodies are being considered. For example, the MFP of interplanetary space is smaller than the size of the solar system, but larger than small planets and moons. As a result, solar winds exhibit continuum flow on the scale of the solar system, but must be considered as a bombardment of particles with respect to the Earth and Moon.
- **Perfect vacuum** is an ideal state of no particles at all. It cannot be achieved in a laboratory, although there may be small volumes which, for a brief moment, happen to have no particles of matter in them. Even if all particles of matter were removed, there would still be photons and gravitons, as well as dark energy, virtual particles, and other aspects of the quantum vacuum.
- **Hard vacuum** and **Soft vacuum** are terms that are defined with a dividing line defined differently by different sources, such as 5 psia,[43] one Torr,[44] or 0.1 Torr[45] the common denominator being that a hard vacuum is a higher vacuum than a soft one.

Examples

	pressure (Pa)	pressure (Torr)	mean free path	molecules per cm^3
Vacuum cleaner	approximately 80 kPa	600	70 nm	10^{19}
liquid ring vacuum pump	approximately 3.2 kPa	24	1.75 μm	10^{18}
freeze drying	100 to 10 Pa	1 to 0.1	100 μm to 1 mm	10^{16} to 10^{15}
rotary vane pump	100 Pa to 100 mPa	1 to 10^{-3}	100 μm to 10 cm	10^{16} to 10^{13}
Incandescent light bulb	10 to 1 Pa	0.1 to 0.01	1 mm to 1 cm	10^{15} to 10^{14}
Thermos bottle	1 to 0.01 Pa[1]	10^{-2} to 10^{-4}	1 cm to 1 m	10^{14} to 10^{12}
Earth thermosphere	1 Pa to 100 nPa	10^{-2} to 10^{-9}	1 cm to 100 km	10^{14} to 10^{7}
Vacuum tube	10 μPa to 10 nPa	10^{-7} to 10^{-10}	1 to 1,000 km	10^{9} to 10^{6}
Cryopumped MBE chamber	100 nPa to 1 nPa	10^{-9} to 10^{-11}	100 to 10,000 km	10^{7} to 10^{5}
Pressure on the Moon	approximately 1 nPa	10^{-11}	10,000 km	4×10^{5} [46]
Interplanetary space				10 [1]
Interstellar space				1 [47]
Intergalactic space				10^{-6} [1]

Measurement

Relative versus absolute measurement

Vacuum is measured in units of pressure, typically as a subtraction relative to ambient atmospheric pressure on Earth. But the amount of relative measurable vacuum varies with local conditions. On the surface of Jupiter, where ground level atmospheric pressure is much higher than on Earth, much higher relative vacuum readings would be possible. On the surface of the moon with almost no atmosphere, it would be extremely difficult to create a measurable vacuum relative to the local environment.

Similarly, much higher than normal relative vacuum readings are possible deep in the Earth's ocean. A submarine maintaining an internal pressure of 1 atmosphere submerged to a depth of 10 atmospheres (98 meters; a 9.8 meter column of seawater has the equivalent weight of 1 atm) is effectively a vacuum chamber keeping out the crushing exterior water pressures, though the 1 atm inside the submarine would not normally be considered a vacuum.

Therefore to properly understand the following discussions of vacuum measurement, it is important that the reader assumes the relative measurements are being done on Earth at sea level, at exactly 1 atmosphere of ambient atmospheric pressure.

Vacuum measurements relative to 1 atm

The SI unit of pressure is the pascal (symbol Pa), but vacuum is usually measured in torrs, named for Torricelli, an early Italian physicist (1608–1647). A torr is equal to the displacement of a millimeter of mercury (mmHg) in a manometer with 1 torr equaling 133.3223684 pascals above absolute zero pressure. Vacuum is often also measured using inches of mercury on the barometric scale or as a percentage of atmospheric pressure in bars or atmospheres. Low vacuum is often measured in inches of mercury (inHg), millimeters of mercury (mmHg) or kilopascals (kPa) below atmospheric pressure. "Below atmospheric" means that the absolute pressure is equal to the current atmospheric pressure (e.g. 29.92 inHg) minus the vacuum pressure in the same units. Thus a vacuum of 26 inHg is equivalent to an absolute pressure of 4 inHg (29.92 inHg − 26 inHg).

In other words, most low vacuum gauges that read, for example, −28 inHg at full vacuum are actually reporting 2 inHg, or 50.79 Torr. Many inexpensive low vacuum gauges have a margin of error and may report a vacuum of −30 inHg, or 0 Torr but in practice this generally requires a two stage rotary vane or other medium type of vacuum pump to go much beyond (lower than) 25 torr.

Many devices are used to measure the pressure in a vacuum, depending on what range of vacuum is needed.[48]

A glass McLeod gauge, drained of mercury

Hydrostatic gauges (such as the mercury column manometer) consist of a vertical column of liquid in a tube whose ends are exposed to different pressures. The column will rise or fall until its weight is in equilibrium with the pressure differential between the two ends of the tube. The simplest design is a closed-end U-shaped tube, one side of which is connected to the region of interest. Any fluid can be used, but mercury is preferred for its high density and low vapour pressure. Simple hydrostatic gauges can measure pressures ranging from 1 torr (100 Pa) to above atmospheric. An important variation is the McLeod gauge which isolates a known volume of vacuum and compresses it to multiply the height variation of the liquid column. The McLeod gauge can measure vacuums as high as 10^{-6} torr (0.1 mPa), which is the lowest direct measurement of pressure that is possible with current technology. Other vacuum gauges can measure lower pressures, but only indirectly by measurement of other pressure-controlled properties. These indirect measurements must be calibrated via a direct measurement, most commonly a McLeod gauge.[49]

Mechanical or **elastic** gauges depend on a Bourdon tube, diaphragm, or capsule, usually made of metal, which will change shape in response to the pressure of the region in question. A variation on this idea is the **capacitance manometer**, in which the diaphragm makes up a part of a capacitor. A change in pressure leads to the flexure of the diaphragm, which results in a change in capacitance. These gauges are effective from 10^{+3} torr to 10^{-4} torr, and beyond.

Thermal conductivity gauges rely on the fact that the ability of a gas to conduct heat decreases with pressure. In this type of gauge, a wire filament is heated by running current through it. A thermocouple or Resistance Temperature Detector (RTD) can then be used to measure the temperature of the filament. This temperature is dependent on the rate at which the filament loses heat to the surrounding gas, and therefore on the thermal conductivity. A common variant is the Pirani gauge which uses a single platimum filament as both the heated element and RTD. These gauges are accurate from 10 torr to 10^{-3} torr, but they are sensitive to the chemical composition of the gases being measured.

Ion gauges are used in ultrahigh vacuum. They come in two types: hot cathode and cold cathode. In the hot cathode version an electrically heated filament produces an electron beam. The electrons travel through the gauge and ionize gas molecules around them. The resulting ions are collected at a negative electrode. The current depends on the number of ions, which depends on the pressure in the gauge. Hot cathode gauges are accurate from 10^{-3} torr to 10^{-10} torr. The principle behind cold cathode version is the same, except that electrons are produced in a discharge created by a high voltage electrical discharge. Cold cathode gauges are accurate from 10^{-2} torr to 10^{-9} torr. Ionization gauge calibration is very sensitive to construction geometry, chemical composition of gases being measured, corrosion and surface deposits. Their calibration can be invalidated by activation at atmospheric pressure or low vacuum. The composition of gases at high vacuums will usually be unpredictable, so a mass spectrometer must be used in conjunction with the ionization gauge for accurate measurement.[50]

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External links

- VIDEO on the nature of vacuum (http://spacegeek.org/ep9_QT.shtml) by Canadian astrophysicist Doctor P
- The Foundations of Vacuum Coating Technology (http://www.svc.org/H/H_HistoryArticle.html)
- American Vacuum Society (http://www.avs.org/)
- Journal of Vacuum Science and Technology A (http://scitation.aip.org/jvsta/)
- Journal of Vacuum Science and Technology B (http://scitation.aip.org/jvstb/)
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- Vacuum Energy in High Energy Physics (http://www.arXiv.org/abs/hep-th/0012062)
- Vacuum, Production of Space (http://void.mit.edu/~4.396/wiki/index.php?title=Main_Page)
- "Much Ado About Nothing" by Professor John D. Barrow, Gresham College (http://www.gresham.ac.uk/event.asp?PageId=4&EventId=258)
- Free pdf copy of The Structured Vacuum - thinking about nothing (http://www.physics.arizona.edu/~rafelski/Books/StructVacuumE.pdf) by Johann Rafelski and Berndt Muller (1985) ISBN 3-87144-889-3.

People

André-Marie Ampère

André-Marie Ampère

André-Marie Ampère (1775-1836)

Born	20 January 1775 Parish of St. Nizier, Lyon, France
Died	10 June 1836 (aged 61) Marseille, France
Residence	France
Nationality	French
Fields	Physics
Institutions	Bourg-en-Bresse École Polytechnique
Known for	Ampere's Law

Signature

André-Marie Ampère (20 January 1775 – 10 June 1836) was a French physicist and mathematician who is generally regarded as one of the main discoverers of electromagnetism. The SI unit of measurement of electric current, the ampere, is named after him.

Early days

Ampère was born in Lyon, France on 20 January 1775. He spent his childhood and adolescence at the family property at Poleymieux-au-Mont-d'Or near Lyon.[1] His father began to teach him Latin, until he discovered the boy's preference and aptitude for mathematical studies. The young Ampère, however, soon resumed his Latin lessons, to enable him to master the works of Euler and Bernoulli. In later life Ampère claimed that he knew as much about mathematics and science when he was eighteen as ever he knew; but, a polymath, his reading embraced history, travels, poetry, philosophy, and the natural sciences.

During the French Revolution, Ampere's father stayed at Lyon expecting to be safer there. Nevertheless, after the revolutionaries had taken the city he was captured and executed. This death was a great shock to Ampère.

In 1796 Ampère met Julie Carron, and in 1799 they were married. From about 1796, Ampère gave private lessons at Lyon in mathematics, chemistry, and languages. In 1801 he moved to Bourg-en-Bresse, as professor of physics and chemistry, leaving his ailing wife and his infant son (Jean-Jacques Ampère) at Lyon. Her death, in July 1803, troubled Ampère for the rest of his life. Also in 1804, Ampère was appointed professor of mathematics at the University of Lyon.

Ampère claimed that "at eighteen years he found three culminating points in his life, his First Communion, the reading of Antoine Leonard Thomas's "Eulogy of Descartes", and the Taking of the Bastille. On the day of his wife's death he wrote two verses from the Psalms, and the prayer, 'O Lord, God of Mercy, unite me in Heaven with those whom you have permitted me to love on earth.' Serious doubts harassed him at times, and made him very unhappy. Then he would take refuge in the reading of the Bible and the Fathers of the Church."[2]

For a time he took into his family the young student Antoine-Frédéric Ozanam (1813-1853), one of the founders of the Conference of Charity, later known as the Society of Saint Vincent de Paul. Through Ampère, Ozanam had contact with leaders of the neo-Catholic movement, such as François-René de Chateaubriand, Jean-Baptiste Henri Lacordaire, and Charles Forbes René de Montalembert. Ozanam was beatified by Pope John Paul II in 1997.

Physics and further studies

Jean Baptiste Joseph Delambre's recommendation obtained for Ampère the Lyon appointment, and afterwards (1805) a minor position in the polytechnic school at Paris, where he was appointed professor of mathematics in 1809. Here Ampère continued to pursue his scientific research and his diverse studies with unabated diligence. He was admitted as a member of the Institute in 1814.

Ampère's fame mainly rests on his establishing the relations between electricity and magnetism, and in developing the science of electromagnetism, or, as he called it, electrodynamics. On 11 September 1820 he heard of H. C. Ørsted's discovery that a magnetic needle is acted on by a voltaic current. Only a week later, on 18 September, Ampère presented a paper to the Academy containing a much more complete exposition of that and kindred phenomena. On the same day, Ampère also demonstrated before the Academy that parallel wires carrying currents attract or repel each other, depending on whether currents are in the same (attraction) or in opposite directions (repulsion). This laid the foundation of electrodynamics.

The topic of electromagnetism thus begun, Ampère developed a mathematical theory which not only described the electromagnetic phenomena already observed, but also predicted many new ones.

In 1828, he was elected a foreign member of the Royal Swedish Academy of Science.

Writings

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- André-Marie Ampère: *Recueil d'observations électro-dynamiques.* contenant divers mémoires, notices, extraits de lettres ou d'ouvrages périodiques sur les sciences, relatifs a l'action mutuelle de deux courans électriques, à celle qui existe entre un courant électrique et un aimant ou le globe terrestre, et à celle de deux aimans l'un sur l'autre Chez Crochard, 21 November 2011 (*André-Marie Ampère* [4] at Google Books ; as at: 2010-9-26).
- André-Marie Ampère, Babinet (Jacques, M.): *Exposé des nouvelles découvertes sur l'électricité et le magneétisme.* Chez Méquignon-Marvis, 21 November 2011 (*André-Marie Ampère* [4] at Google Books ; as at: 2010-9-26).
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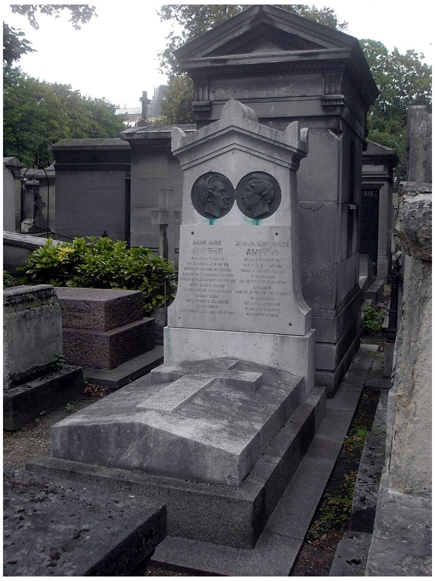
  - André-Marie Ampère: *Essai sur la philosophie des sciences.* Bd. 2, Bachelier, 21 November 2011 (*André-Marie Ampère* [4] at Google Books ; as at: 2010-9-26).

Last years

Ampère died at Marseille and was buried in the Cimetière de Montmartre, Paris. The great amiability and childlike simplicity of his character are well brought out in his *Journal et correspondence* (Paris, 1872).



Grave of Ampère and his son

Ampère's final work, published posthumously, was *Essai sur la philosophie des sciences, ou exposition analytique d'une classification naturelle de toutes les connaissances humaines ("Essay on the philosophy of science or analytical exposition on the natural classification of human knowledge").*

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[5] http://www.archive.org/details/thoriemathmatiq00ampgoog

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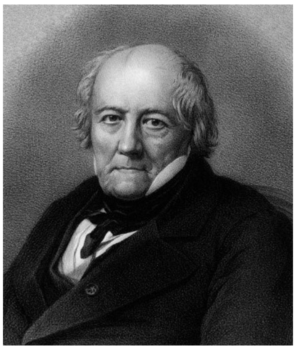
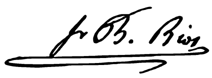
External links

- Ampère and the history of electricity (http://www.ampere.cnrs.fr) - a French-language, edited by CNRS, site with Ampère's correspondence (full text and critical edition with links to manuscripts pictures, more than 1000 letters), an Ampère bibliography, experiments, and 3D simulations
- Ampère Museum (http://musee-ampere.univ-lyon1.fr) - a French-language site from the museum in Poleymieux-au-Mont-d'or, near Lyon, France
- O'Connor, John J.; Robertson, Edmund F., "André-Marie Ampère" (http://www-history.mcs.st-andrews.ac.uk/Biographies/Ampere.html), *MacTutor History of Mathematics archive*, University of St Andrews.
- Weisstein, Eric W., *Ampère, André (1775-1836)* (http://scienceworld.wolfram.com/biography/Ampere.html) from ScienceWorld.
- Catholic Encyclopedia on André Marie Ampère (http://www.newadvent.org/cathen/01437c.htm)
- This article incorporates text from a publication now in the public domain: Chisholm, Hugh, ed (1911). *Encyclopædia Britannica* (11th ed.). Cambridge University Press.

- André-Marie Ampère: The Founder of Electromagnetism (http://www.juliantrubin.com/bigten/ampereexperiments.html) - Background information and related experiments

Jean-Baptiste Biot

Jean-Baptiste Biot	
 Jean-Baptiste Biot	
Born	Paris
Died	3 February 1862 (aged 87) Paris
Nationality	French
Fields	Physics, astronomy, and mathematics
Known for	Biot-Savart law
Influenced	Louis Pasteur, William Ritchie
Signature 	

Jean-Baptiste Biot (21 April 1774 – 3 February 1862) was a French physicist, astronomer, and mathematician who established the reality of meteorites, made an early balloon flight, and studied the polarization of light.

Biography

Jean-Baptiste Biot was born in Paris, France on 21 April 1774 and died in Paris on 3 February 1862. He had one son, Edouard Constant Biot, in 1803. Biot served in the artillery before he was appointed professor of mathematics at Beauvais in 1797. He later went on to become a professor of physics at the Collège de France around 1800, and three years later was elected as a member of the Academy of Sciences. In 1804 Biot was on board for the first scientific hot-air balloon ride with Gay-Lussac (NNDB 2009, O'Connor and Robertson 1997). Biot was also a member of the Legion of Honor; he was elected chevalier in 1814 and commander in 1849. In 1816, he was elected a foreign member of the Royal Swedish Academy of Sciences. In addition, Biot received the Rumford Medal [link], awarded by the Royal Society in the field of thermal or optic properties of matter, in 1840 (O'Connor and Robertson 1997).

Biot's Work

Jean-Baptiste Biot made many contributions to the scientific community in his lifetime – most notably in optics, magnetism, and astronomy. The Biot-Savart Law in magnetism is named after Biot and his colleague Félix Savart for their work in 1820. In their experiment they showed a connection between electricity and magnetism by "starting with a long vertical wire and a magnetic needle some horizontal distance apart [and showing] that running a current through the wire caused the needle to move" (Parsley).

In 1803 Biot was sent by the French Academy to report back on 3000 meteorites that fell on L'aigle, France. He found that the meteorites, or stones at the time, were from outer space. With his report, Biot helped support Ernst Florens Friedrich Chladni's argument that meteorites were debris from space, which he had published in 1794. Biot also helped further the field of optics in 1815 with a study in polarized light. In his experiment Biot studied the effects of polarized light as it penetrated organic substances and determined that light "could be rotated clockwise or counterclockwise, dependent upon the optical axis of the material" (Molecular).

Meteorites

Prior to Biot's thorough investigation of the meteorites that fell near l'Aigle, France in 1803, very few truly believed that rocks found on Earth could have extraterrestrial origins. There were anecdotal tales of unusual rocks found on the ground after fireballs had been seen in the sky, but such stories were often dismissed as fantasy. Serious debate concerning the unusual rocks began in 1794 when German physicist Chladni published a book claiming that rocks had an extraterrestrial origin (Westrum). Only after Biot was able to analyze the rocks at l'Aigle was it commonly accepted that the fireballs seen in the sky were meteors falling through the atmosphere. Since Biot's time, analysis of meteorites has resulted in accurate measurements of the chemical composition of the solar system. The composition and position of meteors in the solar system have also given astronomers clues as to how the solar system formed.

Polarized light

In 1812, Biot turned his attention to the study of optics, particularly the polarization of light. Prior to the 19th century, light was believed to consist of discrete packets called corpuscles. During the early 19th century, many scientists began to disregard the corpuscular theory in favor of the wave theory of light. Biot began his work on polarization to show that the results he was obtaining could appear only if light were made of corpuscles. His work in chromatic polarization and rotary polarization greatly advanced the field of optics, although it was later shown that his findings could also be obtained using the wave theory of light (Frankel). Biot's work on the polarization of light has led to many breakthroughs in the field of optics. Liquid crystal displays (LCDs), such as television and computer screens, use light that is polarized by a filter as it enters the liquid crystal to provide a clearer picture. Polarizing filters are used extensively in photography to cut out unwanted reflections or to enhance reflection.

Selected writings

- Traité élémentaire d'astronomie physique [1] (Klostermann, 1810–1811)
- Traité de physique expérimentale et mathématique [2](Deterville, 1816)
- Précis de l'histoire de l'astronomie chinoise [3] (impr. impériale, 1861)
- Études sur l'astronomie indienne et sur l'astronomie chinoise [4] (Lévy frères, 1862)
- Mélanges scientifiques et littéraires [5] (Lévy frères, 1858)
- Recherches sur plusieurs points de l'astronomie égyptienne [6] (Didot, 1823)

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- Levitt, Theresa (Sep 2003). "Biot's paper and Arago's plates. Photographic practice and the transparency of representation". *Isis* **94** (3): 456–476. PMID 14626764.

External links

- Catholic Encyclopedia article [8]
- Encyclopædia Britannica article [9]
- Short Biography [10] Pasteur Brewing.
- O'Connor, John J.; Robertson, Edmund F., "Jean-Baptiste Biot" [11], *MacTutor History of Mathematics archive*, University of St Andrews.

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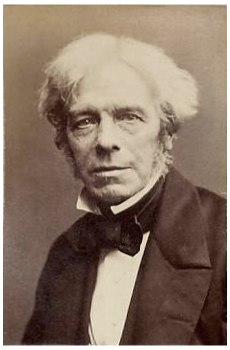
[4] http://gallica.bnf.fr/notice?N=FRBNF35395244
[5] http://books.google.com/books?id=jm4tAAAAMAAJ&pg=PA1&dq=%22Jean-Baptiste+Biot%22
[6] http://books.google.com/books?id=KhoGAAAAQAAJ&pg=PA1&dq=%22Jean-Baptiste+Biot%22#PPR3,M1
[7] http://books.google.com/?id=7SvtVoa1W-cC&printsec=frontcover&dq=The+history+of+meteoritics+and+key+meteorite+collections#PPA73,M2
[8] http://www.newadvent.org/cathen/02576a.htm
[9] http://www.britannica.com/EBchecked/topic/66209/Jean-Baptiste-Biot
[10] http://www.pasteurbrewing.com/colleagues/biographies/jean-baptiste-biot-1774-1862.html
[11] http://www-history.mcs.st-andrews.ac.uk/Biographies/Biot.html

Michael Faraday

Michael Faraday	
 Michael Faraday	
Born	22 September 1791 Newington Butts, England
Died	25 August 1867 (aged 75) Hampton Court, Middlesex, England
Residence	England
Nationality	British
Fields	Physics and chemistry
Institutions	Royal Institution
Known for	Faraday's law of induction Electrochemistry Faraday effect Faraday cage Faraday constant Faraday cup Faraday's laws of electrolysis Faraday paradox Faraday rotator Faraday-efficiency effect Faraday wave Faraday wheel Lines of force
Influences	Humphry Davy William Thomas Brande
Notable awards	Royal Medal (1835 & 1846) Copley Medal (1832 & 1838) Rumford Medal (1846)
Signature 	

Michael Faraday, FRS (22 September 1791 – 25 August 1867) was an English chemist and physicist (or *natural philosopher*, in the terminology of the time) who contributed to the fields of electromagnetism and electrochemistry.

Faraday studied the magnetic field around a conductor carrying a DC electric current. While conducting these studies, Faraday established the basis for the electromagnetic field concept in physics, subsequently enlarged upon by James Maxwell. He similarly discovered electromagnetic induction, diamagnetism, and laws of electrolysis. He established that magnetism could affect rays of light and that there was an underlying relationship between the two phenomena.[1] [2] His inventions of electromagnetic rotary devices formed the foundation of electric motor technology, and it was largely due to his efforts that electricity became viable for use in technology.

As a chemist, Faraday discovered benzene, investigated the clathrate hydrate of chlorine, invented an early form of the Bunsen burner and the system of oxidation numbers, and popularised terminology such as anode, cathode, electrode, and ion.

Although Faraday received little formal education and knew little of higher mathematics, such as calculus, he was one of the most influential scientists in history.[3] Historians[4] of science refer to him as the best experimentalist in the history of science.[5] The SI unit of capacitance, the farad, is named after him, as is the Faraday constant, the charge on a mole of electrons (about 96,485 coulombs). Faraday's law of induction states that magnetic flux changing in time creates a proportional electromotive force.

Faraday was the first and foremost Fullerian Professor of Chemistry at the Royal Institution of Great Britain, a life-time position.

Albert Einstein kept a photograph of Faraday on his study wall alongside pictures of Isaac Newton and James Clerk Maxwell.[6]

Faraday was highly religious; he was a member of the Sandemanian Church, a Christian sect founded in 1730 that demanded total faith and commitment. Biographers have noted that "a strong sense of the unity of God and nature pervaded Faraday's life and work."[7]

Early years

Faraday was born in Newington Butts,[8] now part of the London Borough of Southwark; but then a suburban part of Surrey, one mile south of London Bridge.[9] His family was not well off. His father, James, was a member of the Glassite sect of Christianity. James Faraday moved his wife and two children to London during the winter of 1790–1 from Outhgill in Westmorland, where he had been an apprentice to the village blacksmith.[10] Michael was born the autumn of that year. The young Michael Faraday, the third of four children, having only the most basic of school educations, had to largely educate himself.[11] At fourteen he became apprenticed to a local bookbinder and bookseller George Riebau in Blandford St[12] and, during his seven-year apprenticeship, he read many books, including Isaac Watts' *The Improvement of the Mind*, and he enthusiastically implemented the principles and suggestions that it contained. He developed an interest in science, especially in electricity. In particular, he was inspired by the book *Conversations on Chemistry* by Jane Marcet.[13]


Portrait of Faraday in his late thirties

At the age of twenty, in 1812, at the end of his apprenticeship, Faraday attended lectures by the eminent English chemist Humphry Davy of the Royal Institution and Royal Society, and John Tatum, founder of the City Philosophical Society. Many tickets for these lectures were given to Faraday by William Dance (one of the founders of the Royal Philharmonic Society). Afterwards, Faraday sent Davy a three hundred page book based on notes taken during the lectures. Davy's reply was immediate, kind, and favourable. When Davy damaged his eyesight in an accident with nitrogen trichloride, he decided to employ Faraday as a secretary. When John Payne, one of the Royal Institution's assistants, was sacked, Sir Humphry Davy was asked to find a replacement. He appointed Faraday as Chemical Assistant at the Royal Institution on 1 March 1813.[1]

In the class-based English society of the time, Faraday was not considered a gentleman. When Davy went on a long tour to the continent in 1813–15, his valet did not wish to go. Faraday was going as Davy's scientific assistant, and was asked to act as Davy's valet until a replacement could be found in Paris. Faraday was forced to fill the role of valet as well as assistant throughout the trip. Davy's wife, Jane Apreece, refused to treat Faraday as an equal (making him travel outside the coach, eat with the servants, etc.) and generally made Faraday so miserable that he contemplated returning to England alone and giving up science altogether. The trip did, however, give him access to the European scientific elite and a host of stimulating ideas.[1]

Faraday was a devout Christian. His Sandemanian denomination was an offshoot of the Church of Scotland. Well after his marriage, he served as Deacon and two terms as an Elder in the meeting house of his youth. His church was located at Paul's Alley in the Barbican. This meeting house relocated in 1862 to Barnsbury Grove, Islington. This North London location is where Faraday served the final two years of his second term as Elder prior to his resignation from that post.[14] [15]

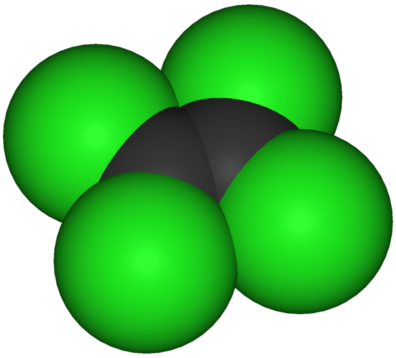
Faraday married Sarah Barnard (1800–1879) on 12 June 1821.[16] They had no children.[8] They met through their families at the Sandemanian church. He confessed his faith to the Sandemanian congregation the month after he married.

Scientific achievements

Chemistry


Michael Faraday in his laboratory. c1850s by artist Harriet Jane Moore who documented Faraday's life in watercolours.

Faraday's earliest chemical work was as an assistant to Humphry Davy. Faraday specifically studied chlorine, discovering two new compounds of chlorine and carbon. He also made the first rough experiments on the diffusion of gases, a phenomenon first pointed out by John Dalton, the physical importance of which was more fully brought to light by Thomas Graham and Joseph Loschmidt. He succeeded in liquefying several gases; he investigated the alloys of steel, and produced several new kinds of glass intended for optical purposes. A specimen of one of these heavy glasses afterwards became historically important as the substance in which Faraday detected the rotation of the plane of



The tetrachloroethylene molecule, first synthesised by Michael Faraday in 1821[17]

polarisation of light when the glass was placed in a magnetic field, and also as the first substance found to be repelled by the poles of a magnet. He also endeavoured, with some success, to make the general methods of chemistry, as distinguished from its results, the subject of special study and of popular exposition.

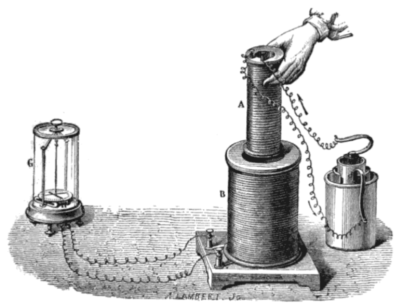
He invented an early form of what was to become the Bunsen burner, which is used almost universally in science laboratories as a convenient source of heat.[18] [19] Faraday worked extensively in the field of chemistry, discovering chemical substances such as benzene (which he called bicarburet of hydrogen), and liquefying gases such as chlorine. Liquification of gases helped establish that gases are simply the vapours of liquids possessing a very low boiling-point, and gave a more solid basis to conceptions of molecular aggregation. In 1820 Faraday reported on the first syntheses of compounds made from carbon and chlorine, C_2Cl_6 and C_2Cl_4, and published his results the following year.[20] [21] [22] Faraday also determined the composition of the chlorine clathrate hydrate, which had been discovered by Humphry Davy in 1810.[23] [24]

Faraday also discovered the laws of electrolysis and popularised terminology such as anode, cathode, electrode, and ion, terms largely created by William Whewell.

Faraday was the first to report what later came to be called metallic nanoparticles. In 1847 he discovered that the optical properties of gold colloids differed from those of the corresponding bulk metal. This was probably the first reported observation of the effects of quantum size, and might be considered to be the birth of nanoscience.[25]

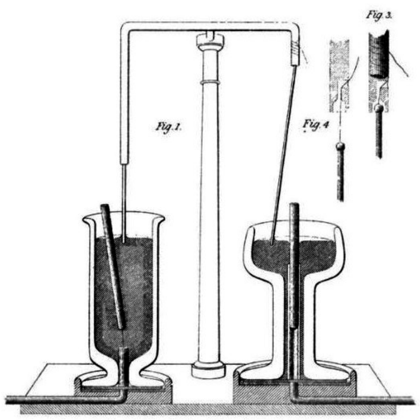
Electricity and magnetism

Faraday is best known for his work with electricity and magnetism. His first recorded experiment was the construction of a voltaic pile with seven halfpence pieces, stacked together with seven disks of sheet zinc, and six pieces of paper moistened with salt water. With this pile he decomposed sulphate of magnesia (first letter to Abbott, 12 July 1812).



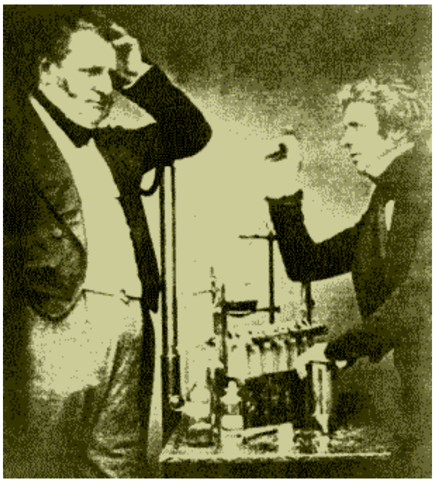
One of Faraday's 1831 experiments demonstrating induction. The liquid battery *(right)* sends an electric current through the small coil *(A)*. When it is moved in or out of the large coil *(B)*, its magnetic field induces a momentary voltage in the coil, which is detected by the galvanometer *(G)*.

In 1821, soon after the Danish physicist and chemist, Hans Christian Ørsted discovered the phenomenon of electromagnetism, Davy and British scientist William Hyde Wollaston tried but failed to design an electric motor.[2] Faraday, having discussed the problem with the two men, went on to build two devices to produce what he called electromagnetic rotation: a continuous circular motion from the circular magnetic force around a wire and a wire extending into a pool of mercury with a magnet placed inside that would rotate around the magnet if supplied with current from a chemical battery. The latter device is known as a homopolar motor. These experiments and inventions form the foundation of modern electromagnetic technology. In his excitement, Faraday published results without acknowledging his work with either Wollaston or Davy. The resulting controversy within the Royal Society strained his mentor relationship with Davy and may well have contributed to Faraday's assignment to other activities, thereby removing him from electromagnetic research for several years.[27] [28]



Electromagnetic rotation experiment of Faraday, *ca.* 1821[26]

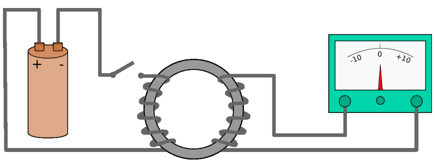
From his initial electromagnetic discovery in 1821, Faraday continued his laboratory work exploring properties of materials and developing the requisite experience. In 1824, Faraday briefly set up a circuit to study whether a magnetic field could regulate the flow of a current in an adjacent wire, but could find no such relationship.[29] This lab followed similar work with light and magnets three years earlier with identical results.[30] [31] During the next seven years, Faraday spent much of his time perfecting his recipe for optical quality (heavy) glass, boro-silicate of lead,[32] which he used in his future studies connecting light with magnetism.[33] In his spare time from this optics work, Faraday continued publishing his experimental work (some of which related to EM) and conducted foreign correspondence with scientists (also working on EM) he previously met on his journeys about Europe with Davy.[34] Two years after the death of Davy, in 1831, he began his great series of experiments in which he discovered electromagnetic induction. Joseph Henry likely discovered self-induction a few months earlier and both may have been anticipated by the work of Francesco Zantedeschi in Italy in 1829 and 1830.[35]



English chemists John Daniell (left) and Michael Faraday (right), credited as founders of electrochemistry today.

Faraday's breakthrough came when he wrapped two insulated coils of wire around an iron ring, and found that, upon passing a current through one coil, a momentary current was induced in the other coil.[2] This phenomenon is known as mutual induction. The iron ring-coil apparatus is still on display at the Royal Institution. In subsequent experiments, he found that, if he moved a magnet through a loop of wire, an electric current flowed in the wire. The current also flowed if the loop was moved over a stationary magnet. His demonstrations established that a changing magnetic field produces an electric field. This relation was modelled mathematically by James Clerk Maxwell as Faraday's law, which subsequently became one of the four Maxwell equations. These in turn have evolved into the generalisation known today as field theory.

Faraday later used the principle to construct the electric dynamo, the ancestor of modern power generators.



A diagram of Faraday's iron ring-coil apparatus

In 1839, he completed a series of experiments aimed at investigating the fundamental nature of electricity. Faraday used "static", batteries, and "animal electricity" to produce the phenomena of electrostatic attraction, electrolysis, magnetism, etc. He concluded that, contrary to scientific opinion of the time, the divisions between the various "kinds" of electricity were illusory. Faraday instead proposed that only a single "electricity" exists, and the changing values of quantity and intensity (current and voltage) would produce different groups of phenomena.[2]

Near the end of his career, Faraday proposed that electromagnetic forces extended into the empty space around the conductor. This idea was rejected by his fellow scientists, and Faraday did not live to see this idea eventually accepted. Faraday's concept of lines of flux emanating from charged bodies and magnets provided a way to visualise electric and magnetic fields. That mental model was crucial to the successful development of electromechanical devices that dominated engineering and industry for the remainder of the 19th century.

Diamagnetism

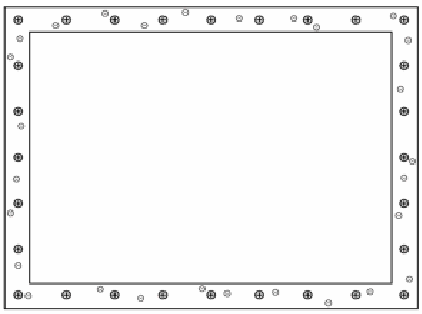
Michael Faraday holding a glass bar of the type he used in 1845 to show that magnetism can affect light in a dielectric material.[36]

In 1845, Faraday discovered that many materials exhibit a weak repulsion from a magnetic field, a phenomenon he named diamagnetism.

Faraday also found that the plane of polarisation of linearly polarised light can be rotated by the application of an external magnetic field aligned in the direction the light is moving. This is now termed the Faraday effect. He wrote in his notebook, "I have at last succeeded in *illuminating a magnetic curve* or *line of force* and in *magnetising a ray of light*".

Late in life (1862), Faraday used a spectroscope to search for a different alteration of light, the change of spectral lines by an applied magnetic field. However, the equipment available to him was insufficient for a definite determination of a spectral change. Pieter Zeeman later used an improved apparatus to study the same phenomenon, publishing his results in 1897 and receiving the 1902 Nobel Prize in Physics for his success. In both his 1897 paper[37] and his Nobel acceptance speech,[38] Zeeman referred to Faraday's work.

Faraday cage



An external electrical field causes the charges to rearrange, which cancels the field inside.

In his work on static electricity, Faraday's ice pail experiment demonstrated that the charge resided only on the exterior of a charged conductor, and exterior charge had no influence on anything enclosed within a conductor. This is because the exterior charges redistribute such that the interior fields due to them cancel. This shielding effect is used in what is now known as a Faraday cage.

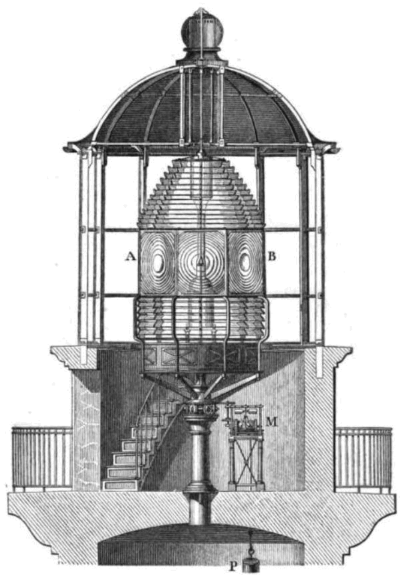
Faraday was an excellent experimentalist who conveyed his ideas in clear and simple language. However, his mathematical abilities did not extend as far as trigonometry or any but the simplest algebra. It was James Clerk Maxwell who took the work of Faraday, and others, and consolidated it with a set of equations that lie at the base of all modern theories of electromagnetic phenomena. On Faraday's uses of the lines of force, Maxwell wrote that they show Faraday "to have been in reality a mathematician of a very high order – one from whom the mathematicians of the future may derive valuable and fertile methods."[39]

Royal Institution and public service

Michael Faraday meets Father Thames, from *Punch* (21 July 1855)

Faraday was the first Fullerian Professor of Chemistry at the Royal Institution of Great Britain, a position to which he was appointed for life. His sponsor and mentor was John 'Mad Jack' Fuller, who created the position at the Royal Institution. Faraday was elected a member of the Royal Society in 1824,[8] appointed director of the laboratory in 1825; and in 1833 he was appointed Fullerian Professor of Chemistry in the institution for life, without the obligation to deliver lectures.

Beyond his scientific research into areas such as chemistry, electricity, and magnetism at the Royal Institution, Faraday undertook numerous, and often time-consuming, service projects for private enterprise and the British government. This work included investigations of explosions in coal mines, being an expert witness in court, and the preparation of high-quality optical glass. In 1846, together with Charles Lyell, he produced a lengthy and detailed report on a serious explosion in the colliery at Haswell County Durham, which killed 95 miners. Their report was a meticulous forensic investigation and indicated that coal dust contributed to the severity of the explosion. The report should have warned coal owners of the hazard of coal dust explosions, but the risk was ignored for over 60 years until the Senghenydd Colliery Disaster of 1913.



Lighthouse lantern room from mid 1800s

As a respected scientist in a nation with strong maritime interests, Faraday spent extensive amounts of time on projects such as the construction and operation of light houses and protecting the bottoms of ships from corrosion. His workshop still stands at Trinity Buoy Wharf above the Chain and Buoy Store, next to London's only lighthouse and a school named after him.

Faraday also was active in what would now be called environmental science, or engineering. He investigated industrial pollution at Swansea and was consulted on air pollution at the Royal Mint. In July 1855, Faraday wrote a letter to The Times on the subject of the foul condition of the River Thames, which resulted in an oft-reprinted cartoon in Punch. (See also The Great Stink.)

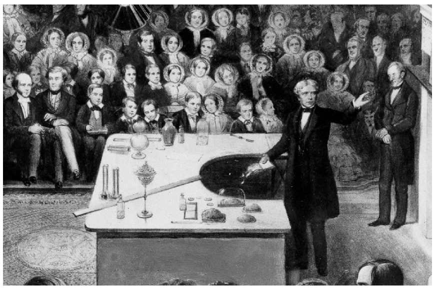
Faraday assisted with planning and judging of exhibits for the Great Exhibition of 1851 in London. He also advised the National Gallery on the cleaning and protection of its art collection, and served on the National Gallery Site Commission in 1857.

Education was another area of service for Faraday. He lectured on the topic in 1854 at the Royal Institution, and in 1862 he appeared before a Public Schools Commission to give his views on education in Great Britain. Faraday also weighed in, negatively, on the public's fascination with table-turning, mesmerism, and seances, chastising both the public and the nation's educational system.[40]

Faraday gave a successful series of lectures on the chemistry and physics of flames at the Royal Institution, entitled *The Chemical History of a Candle*. This was one of the earliest Christmas lectures for young people, which are still given each year. Between 1827 and 1860, Faraday gave the Christmas lectures a record nineteen times.

Later life

Faraday in old age.



Michael Faraday delivering a Christmas Lecture in 1856.

In June 1832, the University of Oxford granted Faraday a Doctor of Civil Law degree (honorary). During his lifetime, Faraday rejected a knighthood and twice refused to become President of the Royal Society. Faraday was elected a foreign member of the Royal Swedish Academy of Sciences in 1838, and was one of eight foreign members elected to the French Academy of Sciences in 1844.[41]

In 1848, as a result of representations by the Prince Consort, Michael Faraday was awarded a grace and favour house in Hampton Court in Middlesex, free of all expenses or upkeep. This was the Master Mason's House, later called Faraday House, and now No.37 Hampton Court Road. In 1858 Faraday retired to live there.[42]

When asked by the British government to advise on the production of chemical weapons for use in the Crimean War (1853–1856), Faraday refused to participate citing ethical reasons.[43]

Faraday died at his house at Hampton Court on 25 August 1867 aged 75 years and 11 months.[44] He had previously turned down burial in Westminster Abbey, but he has a memorial plaque there, near Isaac Newton's tomb. Faraday was interred in the dissenters' (non-Anglican) section of Highgate Cemetery. Hirshfeld maintains in his biography that Faraday suffered from mental breakdown due to his intellectual exertions so that he became debilitated by the end of his life and unable to conduct any meaningful research.

Commemorations

Michael Faraday, statue in Savoy Place, London. Sculptor John Henry Foley RA

Faraday School is located on Trinity Buoy Wharf where his workshop still stands above the Chain and Buoy Store, next to London's only lighthouse.

A statue of Faraday stands in Savoy Place, London, outside the Institution of Engineering and Technology. Also in London, the Michael Faraday Memorial, designed by brutalist architect Rodney Gordon and completed in 1961, is at the Elephant & Castle gyratory system, near Faraday's birthplace at Newington Butts.

Faraday Gardens is a small park in Walworth, London, not far from his birthplace at Newington Butts. This park lies within the local council ward of Faraday in the London Borough of Southwark.

A building at London South Bank University, which houses the institute's electrical engineering departments is named the Faraday Wing, due to its proximity to Faraday's birthplace in Newington Butts. A hall at Loughborough University was named after Faraday in 1960. Near the entrance to its dining hall is a bronze casting, which depicts the symbol of an electrical transformer, and inside there hangs a portrait, both in Faraday's honour. An eight-story building at the University of Edinburgh's science & engineering campus is named for Faraday, as is a recently built hall of accommodation at Brunel University, the main engineering building at Swansea University, and the instructional and experimental physics building at Northern Illinois University. The former UK Faraday Station [45] in Antarctica was named after him.

Streets named for Faraday can be found in many British cities (e.g., London, Fife, Swindon, Basingstoke, Nottingham, Whitby, Kirkby, Crawley, Newbury, Aylesbury and Stevenage) as well as in France (Paris), Germany (Hermsdorf), Canada (Quebec; Deep River, Ontario), and the United States (Reston, VA).

From 1991 until 2001, Faraday's picture featured on the reverse of Series E £20 banknotes issued by the Bank of England. He was shown conducting a lecture at the Royal Institution with the magneto-electric spark apparatus.[46]

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Carl Friedrich Gauss

Carl Friedrich Gauss	
 Carl Friedrich Gauss (1777–1855), painted by Christian Albrecht Jensen	
Born	30 April 1777 Braunschweig, Duchy of Brunswick-Wolfenbüttel, Holy Roman Empire
Died	23 February 1855 (aged 77) Göttingen, Kingdom of Hanover
Residence	Kingdom of Hanover
Nationality	German
Fields	Mathematics and Physics
Institutions	University of Göttingen
Alma mater	University of Helmstedt
Doctoral advisor	Johann Friedrich Pfaff
Other academic advisors	Johann Christian Martin Bartels
Doctoral students	Friedrich Bessel Christoph Gudermann Christian Ludwig Gerling Richard Dedekind Johann Encke Johann Listing Bernhard Riemann Christian Peters Moritz Cantor
Other notable students	Gotthold Eisenstein Gustav Kirchhoff Ernst Kummer Johann Dirichlet August Ferdinand Möbius Julius Weisbach L. C. Schnürlein
Known for	See full list
Influenced	Sophie Germain
Notable awards	Copley Medal (1838)

Signature

Johann Carl Friedrich Gauss (/ˈɡaʊs/; German: *Gauß* listen, Latin: *Carolus Fridericus Gauss*) (30 April 1777 – 23 February 1855) was a German mathematician and scientist who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geodesy, geophysics, electrostatics, astronomy and optics.

Sometimes referred to as the *Princeps mathematicorum*[1] (Latin, "the Prince of Mathematicians" or "the foremost of mathematicians") and "greatest mathematician since antiquity", Gauss had a remarkable influence in many fields of mathematics and science and is ranked as one of history's most influential mathematicians.[2] He referred to mathematics as "the queen of sciences".[3]

Early years (1777–1798)

Statue of Gauss at his birthplace, Braunschweig

Carl Friedrich Gauss was born on April 30, 1777 in Braunschweig, in the duchy of Braunschweig-Wolfenbüttel, now part of Lower Saxony, Germany, as the son of poor working-class parents.[4] Indeed, his mother was illiterate and never recorded the date of his birth, remembering only that he had been born on a Wednesday, eight days before the Feast of the Ascension, which itself occurs 40 days after Easter. Gauss would later solve this puzzle for his birthdate in the context of finding the date of Easter, deriving methods to compute the date in both past and future years.[5] He was christened and confirmed in a church near the school he attended as a child.[6]

Gauss was a child prodigy. There are many anecdotes pertaining to his precocity while a toddler, and he made his first ground-breaking mathematical discoveries while still a teenager. He completed *Disquisitiones Arithmeticae*, his magnum opus, in 1798 at the age of 21, though it was not published until 1801. This work was fundamental in consolidating number theory as a discipline and has shaped the field to the present day.

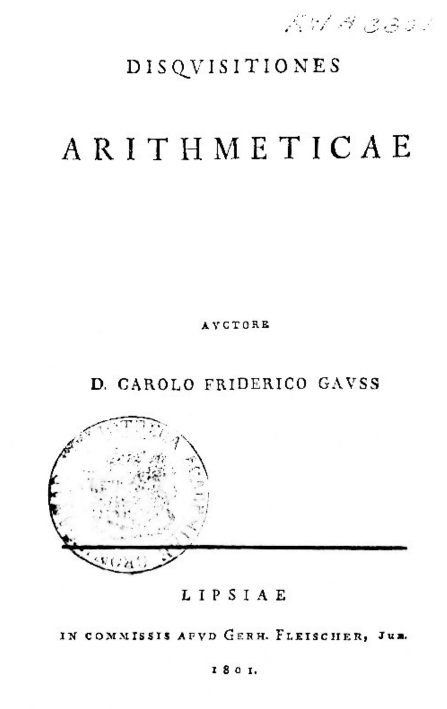
Gauss's intellectual abilities attracted the attention of the Duke of Braunschweig,[2] who sent him to the Collegium Carolinum (now Technische Universität Braunschweig), which he attended from 1792 to 1795, and to the University of Göttingen from 1795 to 1798. While in university, Gauss independently rediscovered several important theorems; his breakthrough occurred in 1796 when he was able to show that any regular polygon with a number of sides which is a Fermat prime (and, consequently, those polygons with any number of sides which is the product of distinct Fermat primes and a power of 2) can be constructed by compass and straightedge. This was a major discovery in an important field of mathematics; construction problems had occupied mathematicians since the days of the Ancient Greeks, and the discovery ultimately led Gauss to choose mathematics instead of philology as a career. Gauss was so pleased by this result that he requested that a regular heptadecagon be inscribed on his tombstone. The stonemason declined, stating that the difficult construction would essentially look like a circle.[7]

The year 1796 was most productive for both Gauss and number theory. He discovered a construction of the heptadecagon on March 30.[8] He invented modular arithmetic, greatly simplifying manipulations in number theory. He became the first to prove the quadratic reciprocity law on 8 April. This remarkably general law allows mathematicians to determine the solvability of any quadratic equation in modular arithmetic. The prime number theorem, conjectured on 31 May, gives a good understanding of how the prime numbers are distributed among the integers. Gauss also discovered that every positive integer is representable as a sum of at most three triangular numbers on 10 July and then jotted down in his diary the famous words, "ΕΥΡΗΚΑ! num = Δ + Δ + Δ". On October 1 he published a result on the number of solutions of polynomials with coefficients in finite fields, which ultimately led to the Weil conjectures 150 years later.

Middle years (1799–1830)

In his 1799 doctorate in absentia, *A new proof of the theorem that every integral rational algebraic function of one variable can be resolved into real factors of the first or second degree*, Gauss proved the fundamental theorem of algebra which states that every non-constant single-variable polynomial over the complex numbers has at least one root. Mathematicians including Jean le Rond d'Alembert had produced false proofs before him, and Gauss's dissertation contains a critique of d'Alembert's work. Ironically, by today's standard, Gauss's own attempt is not acceptable, owing to implicit use of the Jordan curve theorem. However, he subsequently produced three other proofs, the last one in 1849 being generally rigorous. His attempts clarified the concept of complex numbers considerably along the way.

Gauss also made important contributions to number theory with his 1801 book *Disquisitiones Arithmeticae* (Latin, Arithmetical Investigations), which, among things, introduced the symbol ≡ for congruence and used it in a clean presentation of modular arithmetic, had the first two proofs of the law of quadratic reciprocity, developed the theories of binary and ternary quadratic forms, stated the class number problem for them, and showed that a regular heptadecagon (17-sided polygon) can be constructed with straightedge and compass.



Title page of Gauss's *Disquisitiones Arithmeticae*

In that same year, Italian astronomer Giuseppe Piazzi discovered the dwarf planet Ceres. Piazzi had only been able to track Ceres for a few months, following it for three degrees across the night sky. Then it disappeared temporarily behind the glare of the Sun. Several months later, when Ceres should have reappeared, Piazzi could not locate it: the mathematical tools of the time were not able to extrapolate a position from such a scant amount of data—three degrees represent less than 1% of the total orbit.

Gauss, who was 23 at the time, heard about the problem and tackled it. After three months of intense work, he predicted a position for Ceres in December 1801—just about a year after its first sighting—and this turned out to be accurate within a half-degree when it was rediscovered by Franz Xaver von Zach on 31 December in Gotha, and one day later by Heinrich Olbers in Bremen.

Gauss's method involved determining a conic section in space, given one focus (the sun) and the conic's intersection with three given lines (lines of sight from the earth, which is itself moving on an ellipse, to the planet) and given the time it takes the planet to traverse the arcs determined by these lines (from which the lengths of the arcs can be calculated by Kepler's Second Law). This problem leads to an equation of the eighth degree, of which one solution, the Earth's orbit, is known. The solution sought is then separated from the remaining six based on physical conditions. In this work Gauss used comprehensive approximation methods which he created for that purpose.[9]

One such method was the fast Fourier transform. While this method is traditionally attributed to a 1965 paper by J. W. Cooley and J. W. Tukey, Gauss developed it as a trigonometric interpolation method. His paper, *Theoria Interpolationis Methodo Nova Tractata*, was only published posthumously in Volume 3 of his collected works. This paper predates the first presentation by Joseph Fourier on the subject in 1807.[10]

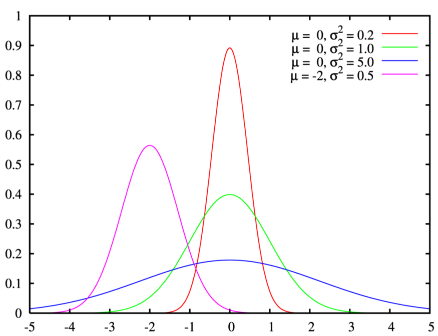
Zach noted that "without the intelligent work and calculations of Doctor Gauss we might not have found Ceres again". Though Gauss had been up to that point supported by the stipend from the Duke, he doubted the security of this arrangement, and also did not believe pure mathematics to be important enough to deserve support. Thus he sought a position in astronomy, and in 1807 was appointed Professor of Astronomy and Director of the astronomical observatory in Göttingen, a post he held for the remainder of his life.

The discovery of Ceres led Gauss to his work on a theory of the motion of planetoids disturbed by large planets, eventually published in 1809 as *Theoria motus corporum coelestium in sectionibus conicis solem ambientum* (theory of motion of the celestial bodies moving in conic sections around the sun). In the process, he so streamlined the cumbersome mathematics of 18th century orbital prediction that his work remains a cornerstone of astronomical computation. It introduced the Gaussian gravitational constant, and contained an influential treatment of the method of least squares, a procedure used in all sciences to this day to minimize the impact of measurement error. Gauss was able to prove the method under the assumption of normally distributed errors (see Gauss–Markov theorem; see also Gaussian). The method had been described earlier by Adrien-Marie Legendre in 1805, but Gauss claimed that he had been using it since 1795.

Gauss' portrait published in *Astronomische Nachrichten* 1828

In 1818 Gauss, putting his calculation skills to practical use, carried out a geodesic survey of the state of Hanover, linking up with previous Danish surveys. To aid in the survey, Gauss invented the heliotrope, an instrument that uses a mirror to reflect sunlight over great distances, to measure positions.

Gauss also claimed to have discovered the possibility of non-Euclidean geometries but never published it. This discovery was a major paradigm shift in mathematics, as it freed mathematicians from the mistaken belief that Euclid's axioms were the only way to make geometry consistent and non-contradictory. Research on these geometries led to, among other things, Einstein's theory of general relativity, which describes the universe as non-Euclidean. His friend Farkas Wolfgang Bolyai with whom Gauss had sworn "brotherhood and the banner of truth" as a student had tried in vain for many years to prove the parallel postulate from Euclid's other axioms of geometry. Bolyai's son, János Bolyai, discovered non-Euclidean geometry in 1829; his work was published in 1832. After seeing it, Gauss wrote to Farkas Bolyai: *"To praise it would amount to praising myself. For the entire content of the work... coincides almost exactly with my own meditations which have occupied my mind for the past thirty or thirty-five years."*



Four Gaussian distributions in statistics

This unproved statement put a strain on his relationship with János Bolyai (who thought that Gauss was "stealing" his idea), but it is now generally taken at face value. Letters by Gauss years before 1829 reveal him obscurely discussing the problem of parallel lines. Waldo Dunnington, a biographer of Gauss, argues in *Gauss, Titan of Science* that Gauss was in fact in full possession of non-Euclidian geometry long before it was published by János Bolyai, but that he refused to publish any of it because of his fear of controversy.

The survey of Hanover fueled Gauss's interest in differential geometry, a field of mathematics dealing with curves and surfaces. Among other things he came up with the notion of Gaussian curvature. This led in 1828 to an important theorem, the Theorema Egregium (*remarkable theorem* in Latin), establishing an important property of the notion of curvature. Informally, the theorem says that the curvature of a surface can be determined entirely by measuring angles and distances on the surface. That is, curvature does not depend on how the surface might be embedded in 3-dimensional space or 2-dimensional space.

In 1821, he was made a foreign member of the Royal Swedish Academy of Sciences.

Later years and death (1831–1855)

Daguerreotype of Gauss on his deathbed, 1855.

Grave of Gauss at Albanifriedhof in Göttingen, Germany.

In 1831 Gauss developed a fruitful collaboration with the physics professor Wilhelm Weber, leading to new knowledge in magnetism (including finding a representation for the unit of magnetism in terms of mass, length and time) and the discovery of Kirchhoff's circuit laws in electricity. It was during this time that he formulated his namesake law. They constructed the first electromechanical telegraph in 1833, which connected the observatory with the institute for physics in Göttingen. Gauss ordered a magnetic observatory to be built in the garden of the observatory, and with Weber founded the "Magnetischer Verein" (*magnetic club* in German), which supported measurements of earth's magnetic field in many regions of the world. He developed a method of measuring the horizontal intensity of the magnetic field which has been in use well into the second half of the 20th century and worked out the mathematical theory for separating the inner (core and crust) and outer (magnetospheric) sources of Earth's magnetic field.

In 1840, Gauss published his influential *Dioptrische Untersuchungen*,[11] in which he gave the first systematic analysis on the formation of images under a paraxial approximation (Gaussian optics).[12] Among his results, Gauss showed that under a paraxial approximation that an optical system can be characterized by its cardinal points[13] and he derived the Gaussian lens formula.[14]

In 1854, Gauss notably selected the topic for Bernhard Riemann's now famous Habilitationvortrag, *Über die Hypothesen, welche der Geometrie zu Grunde liegen*.[15] On the way home from Riemann's lecture, Weber reported that Gauss was full of praise and excitement.[11]

Gauss died in Göttingen, Hannover (now part of Lower Saxony, Germany) in 1855 and is interred in the cemetery Albanifriedhof there. Two individuals gave eulogies at his funeral, Gauss's son-in-law Heinrich Ewald and Wolfgang Sartorius von Waltershausen, who was Gauss's close friend and biographer. His brain was preserved and was studied by Rudolf Wagner who found its mass to be 1,492 grams and the cerebral area equal to 219,588 square millimeters[16] (340.362 square inches). Highly developed convolutions were also found, which in the early 20th century was suggested as the explanation of his genius.[2]

Religion

Bühler writes that, according to correspondence with Rudolf Wagner, Gauss did not appear to believe in a personal god. He further asserts that although Gauss firmly believed in the immortality of the soul and in some sort of life after death, it was not in a fashion that could be interpreted as Christian.[11]

According to Dunnington, Gauss's religion was based upon the search for truth. He believed in "the immortality of the spiritual individuality, in a personal permanence after death, in a last order of things, in an eternal, righteous, omniscient and omnipotent God". Gauss also upheld religious tolerance, believing it wrong to disturb others who were at peace with their own beliefs.[2]

Family

Gauss' daughter Therese (1816—1864)

Gauss's personal life was overshadowed by the early death of his first wife, Johanna Osthoff, in 1809, soon followed by the death of one child, Louis. Gauss plunged into a depression from which he never fully recovered. He married again, to Johanna's best friend named Friederica Wilhelmine Waldeck but commonly known as Minna. When his second wife died in 1831 after a long illness,[17] one of his daughters, Therese, took over the household and cared for Gauss until the end of his life. His mother lived in his house from 1817 until her death in 1839.[2]

Gauss had six children. With Johanna (1780–1809), his children were Joseph (1806–1873), Wilhelmina (1808–1846) and Louis (1809–1810). Of all of Gauss's children, Wilhelmina was said to have come closest to his talent, but she died young. With Minna Waldeck he also had three children: Eugene (1811–1896), Wilhelm (1813–1879) and Therese (1816–1864). Eugene shared a good measure of Gauss' talent in languages and computation.[18] Therese kept house for Gauss until his death, after which she married.

Gauss eventually had conflicts with his sons. He did not want any of his sons to enter mathematics or science for "fear of lowering the family name".[18] Gauss wanted Eugene to become a lawyer, but Eugene wanted to study languages. They had an argument over a party Eugene held, which Gauss refused to pay for. The son left in anger and, in about 1832, emigrated to the United States, where he was quite successful. Wilhelm also settled in Missouri, starting as a farmer and later becoming wealthy in the shoe business in St. Louis. It took many years for Eugene's success to counteract his reputation among Gauss's friends and colleagues. See also the letter from Robert Gauss to Felix Klein on 3 September 1912.

Personality

Gauss was an ardent perfectionist and a hard worker. He was never a prolific writer, refusing to publish work which he did not consider complete and above criticism. This was in keeping with his personal motto *pauca sed matura* ("few, but ripe"). His personal diaries indicate that he had made several important mathematical discoveries years or decades before his contemporaries published them. Mathematical historian Eric Temple Bell estimated that, had Gauss published all of his discoveries in a timely manner, he would have advanced mathematics by fifty years.[19]

Though he did take in a few students, Gauss was known to dislike teaching. It is said that he attended only a single scientific conference, which was in Berlin in 1828. However, several of his students became influential mathematicians, among them Richard Dedekind, Bernhard Riemann, and Friedrich Bessel. Before she died, Sophie

Germain was recommended by Gauss to receive her honorary degree.

Gauss usually declined to present the intuition behind his often very elegant proofs—he preferred them to appear "out of thin air" and erased all traces of how he discovered them. This is justified, if unsatisfactorily, by Gauss in his "Disquisitiones Arithmeticae", where he states that all analysis (i.e., the paths one travelled to reach the solution of a problem) must be suppressed for sake of brevity.

Gauss supported monarchy and opposed Napoleon, whom he saw as an outgrowth of revolution.

Mythology

There are several stories of his early genius. According to one, his gifts became very apparent at the age of three when he corrected, mentally and without fault in his calculations, an error his father had made on paper while calculating finances.

Another famous story has it that in primary school after the young Gauss misbehaved, his teacher, J.G. Büttner, gave him a task : add a list of integers in arithmetic progression; as the story is most often told, these were the numbers from 1 to 100. The young Gauss reputedly produced the correct answer within seconds, to the astonishment of his teacher and his assistant Martin Bartels.

Gauss's presumed method was to realize that pairwise addition of terms from opposite ends of the list yielded identical intermediate sums: $1 + 100 = 101$, $2 + 99 = 101$, $3 + 98 = 101$, and so on, for a total sum of $50 \times 101 = 5050$. However, the details of the story are at best uncertain (see [20] for discussion of the original Wolfgang Sartorius von Waltershausen source and the changes in other versions); some authors, such as Joseph Rotman in his book *A first course in Abstract Algebra*, question whether it ever happened.

According to Isaac Asimov, Gauss was once interrupted in the middle of a problem and told that his wife was dying. He is purported to have said, "Tell her to wait a moment till I'm done."[21] This anecdote is briefly discussed in G. Waldo Dunnington's *Gauss, Titan of Science* where it is suggested that it is an apocryphal story.

Commemorations

German 10-Deutsche Mark banknote (1993; discontinued) featuring Gauss

From 1989 through 2001, Gauss's portrait, a normal distribution curve and some prominent Göttingen buildings were featured on the German ten-mark banknote. The reverse featured the heliotrope and a triangulation approach for Hannover. Germany has also issued three postage stamps honoring Gauss. One (no. 725) appeared in 1955 on the hundredth anniversary of his death; two others, nos. 1246 and 1811, in 1977, the 200th anniversary of his birth.

Daniel Kehlmann's 2005 novel *Die Vermessung der Welt*, translated into English as *Measuring the World* (2006), explores Gauss's life and work through a lens of historical fiction, contrasting them with those of the German explorer Alexander von Humboldt.

In 2007 a bust of Gauss was placed in the Walhalla temple.[22]

Things named in honor of Gauss include:

- The CGS unit for magnetic field was named gauss in his honour,
- The crater Gauss on the Moon,[23]
- Asteroid 1001 Gaussia,
- The ship *Gauss*, used in the Gauss expedition to the Antarctic,
- Gaussberg, an extinct volcano discovered by the above mentioned expedition,

- Gauss Tower, an observation tower in Dransfeld, Germany,
- In Canadian junior high schools, an annual national mathematics competition (Gauss Mathematics Competition) administered by the Centre for Education in Mathematics and Computing is named in honour of Gauss,
- In University of California, Santa Cruz, in Crown College, a dormitory building is named after him,
- The Gauss Haus, an NMR center at the University of Utah,

Gauss (aged about 26) on East German stamp produced in 1977. Next to him: heptadecagon, compass and straightedge.

- The Carl-Friedrich-Gauß School for Mathematics, Computer Science, Business Administration, Economics, and Social Sciences of University of Braunschweig,
- The Gauss Building - University of Idaho (College of Engineering).

In 1929 the Polish mathematician Marian Rejewski, who would solve the German Enigma cipher machine in December 1932, began studying actuarial statistics at Göttingen. At the request of his Poznań University professor, Zdzisław Krygowski, on arriving at Göttingen Rejewski laid flowers on Gauss's grave.[24]

Writings

- 1799: Doctoral dissertation on the Fundamental theorem of algebra, with the title: *Demonstratio nova theorematis omnem functionem algebraicam rationalem integram unius variabilis in factores reales primi vel secundi gradus resolvi posse* ("New proof of the theorem that every integral algebraic function of one variable can be resolved into real factors (i.e., polynomials) of the first or second degree")
- 1801: *Disquisitiones Arithmeticae* [25]. German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second edition)*. New York: Chelsea. 1965. ISBN 0-8284-0191-8, pp. 1–453. English translation by Arthur A. Clarke *Disquisitiones Arithemeticae (Second, corrected edition)*. New York: Springer. 1986. ISBN 0387962549.
- 1808: *Theorematis arithmetici demonstratio nova*. Göttingen: Comment. Soc. regiae sci, Göttingen XVI. German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second edition)*. New York: Chelsea. 1965. ISBN 0-8284-0191-8, pp. 457–462 [Introduces Gauss's lemma, uses it in the third proof of quadratic reciprocity]
- 1809: *Theoria Motus Corporum Coelestium in sectionibus conicis solem ambientium* [26] (Theorie der Bewegung der Himmelskörper, die die Sonne in Kegelschnitten umkreisen), English translation by C. H. Davis, reprinted 1963, Dover, New York.
- 1811: *Summatio serierun quarundam singularium*. Göttingen: Comment. Soc. regiae sci, Göttingen. German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second edition)*. New York: Chelsea. 1965. ISBN 0-8284-0191-8, pp. 463–495 [Determination of the sign of the quadratic Gauss sum, uses this to give the fourth proof of quadratic reciprocity]
- 1812: *Disquisitiones Generales Circa Seriem Infinitam* $1 + \frac{\alpha\beta}{\gamma.1} + \text{etc.}$
- 1818: *Theorematis fundamentallis in doctrina de residuis quadraticis demonstrationes et amplicationes novae*. Göttingen: Comment. Soc. regiae sci, Göttingen. German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second edition)*. New York: Chelsea. 1965. ISBN 0-8284-0191-8, pp. 496–510 [Fifth and sixth proofs of quadratic reciprocity]

- 1821, 1823 und 1826: *Theoria combinationis observationum erroribus minimis obnoxiae*. Drei Abhandlungen betreffend die Wahrscheinlichkeitsrechnung als Grundlage des Gauß'schen Fehlerfortpflanzungsgesetzes. English translation by G. W. Stewart, 1987, Society for Industrial Mathematics.
- 1827: *Disquisitiones generales circa superficies curvas* [27], Commentationes Societatis Regiae Scientiarum Gottingesis Recentiores. Volume **VI**, pp. 99–146. "General Investigations of Curved Surfaces [28]" (published 1965) Raven Press, New York, translated by A.M.Hiltebeitel and J.C.Morehead.
- 1828: *Theoria residuorum biquadraticorum, Commentatio prima*. Göttingen: Comment. Soc. regiae sci, Göttingen 6. German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second edition)*. New York: Chelsea. 1965. ISBN 0-8284-0191-8, pp. 511–533 [Elementary facts about biquadratic residues, proves one of the supplements of the law of biquadratic reciprocity (the biquadratic character of 2)]
- 1832: *Theoria residuorum biquadraticorum, Commentatio secunda*. Göttingen: Comment. Soc. regiae sci, Göttingen 7. German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second edition)*. New York: Chelsea. 1965. ISBN 0-8284-0191-8, pp. 534–586 [Introduces the Gaussian integers, states (without proof) the law of biquadratic reciprocity, proves the supplementary law for $1 + i$]
- 1843/44: *Untersuchungen über Gegenstände der Höheren Geodäsie. Erste Abhandlung* [29], Abhandlungen der Königlichen Gesellschaft der Wissenschaften in Göttingen. Zweiter Band [30], pp. 3–46
- 1846/47: *Untersuchungen über Gegenstände der Höheren Geodäsie. Zweite Abhandlung* [31], Abhandlungen der Königlichen Gesellschaft der Wissenschaften in Göttingen. Dritter Band [32], pp. 3–44
- *Mathematisches Tagebuch 1796–1814*, Ostwaldts Klassiker, Harri Deutsch Verlag 2005, mit Anmerkungen von Neumamn, ISBN 978-3-8171-3402-1 (English translation with annotations by Jeremy Gray: Expositiones Math. 1984)
- Gauss' collective works are online here [33] This includes German translations of Latin texts and commentaries by various authorities

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[2] Dunnington, G. Waldo. (May, 1927). " The Sesquicentennial of the Birth of Gauss (http://www.mathsong.com/cfgauss/Dunnington/1927/)". *Scientific Monthly* XXIV: 402–414. Retrieved on 29 June 2005. Comprehensive biographical article.

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External links

- Carl Friedrich Gauss (http://planetmath.org/?op=getobj&from=objects&id=5594) on PlanetMath
- Complete works (http://www-gdz.sub.uni-goettingen.de/cgi-bin/digbib.cgi?PPN235957348)
- Works by or about Carl Friedrich Gauss (http://worldcat.org/identities/lccn-n79-38533) in libraries (WorldCat catalog)
- Gauss and his children (http://www.gausschildren.org)
- Gauss biography (http://www.corrosion-doctors.org/Biographies/GaussBio.htm)
- Carl Friedrich Gauss (http://genealogy.math.ndsu.nodak.edu/id.php?id=18231) at the Mathematics Genealogy Project.
- Carl Friedrich Gauss (http://fermatslasttheorem.blogspot.com/2005/06/carl-friedrich-gauss.html), Biography at Fermat's Last Theorem Blog.
- Gauss: mathematician of the millennium (http://www.idsia.ch/~juergen/gauss.html), by Jürgen Schmidhuber

- English translation of Waltershausen's 1862 biography (http://books.google.com/books?id=yh0PAAAAIAAJ)
- Gauss (http://www.gauss.info) general website on Gauss
- MNRAS **16** (1856) 80 (http://adsabs.harvard.edu//full/seri/MNRAS/0016//0000080.000.html) Obituary
- Carl Friedrich Gauss on the 10 Deutsche Mark banknote (http://www-personal.umich.edu/~jbourj/money1.htm)
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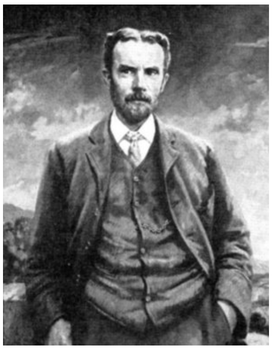
- Carl Friedrich Gauss at Wikiquote

Oliver Heaviside

Oliver Heaviside	
 Portrait by Francis Edwin Hodge	
Born	18 May 1850 Camden Town, London, England
Died	3 February 1925 (aged 74) Torquay, Devon, England
Residence	England
Nationality	English
Fields	Electrical engineering, mathematics and physics
Institutions	Great Northern Telegraph Company
Known for	Heaviside cover-up method Kennelly–Heaviside layer Reactance Heaviside step function Differential operators Vector analysis Heaviside condition
Notable awards	Faraday Medal
Notes Famous quote: *Shall I refuse my dinner because I do not fully understand the process of digestion?*	

Oliver Heaviside(/ˈɒlɪvərˈhɛvisaɪd/ (18 May 1850 – 3 February 1925) was a self-taught English electrical engineer, mathematician, and physicist who adapted complex numbers to the study of electrical circuits, invented mathematical techniques to the solution of differential equations (later found to be equivalent to Laplace transforms), reformulated Maxwell's field equations in terms of electric and magnetic forces and energy flux, and independently co-formulated vector analysis. Although at odds with the scientific establishment for most of his life, Heaviside changed the face of mathematics and science for years to come.

Biography

Early years

Heaviside was born at 55 Kings Street[1] (now Plender Street) in London's Camden Town. He was short and red-headed, and suffered from scarlet fever when young, which left him with a hearing impairment. He was a good student (e.g. placed fifth out of five hundred students in 1865). Heaviside's uncle Sir Charles Wheatstone (1802–1875) was the original co-inventor of the telegraph in the mid 1830s, and was an internationally celebrated expert in telegraphy and electromagnetism. Wheatstone was married to Heaviside's aunt in London and took a strong interest in his nephew's education.[2]

Heaviside left school at age 16 to study at home in the subjects of telegraphy and electromagnetism. He continued fulltime study at home until age 18. Then – in the only paid employment he ever had[2] – he took a job as a telegraph operator with the Great Northern Telegraph Company working first in Denmark and then in Newcastle-upon-Tyne, and was soon made a chief operator. It is likely that his uncle Sir Charles was instrumental in getting Heaviside the telegraph operator position.[3] Heaviside continued to study while working, and at age 21 and 22 he published some research related to electric circuits and telegraphy. In 1874 at age 24 he quit his job and returned to studying fulltime on his own at his parents' home in London. He remained single throughout his life.

In 1873 Heaviside had encountered James Clerk Maxwell's newly published, and today famous, two-volume *Treatise on Electricity and Magnetism*. In his old age Heaviside recalled:

> “I remember my first look at the great treatise of Maxwell's when I was a young man... I saw that it was great, greater and greatest, with prodigious possibilities in its power... I was determined to master the book and set to work. I was very ignorant. I had no knowledge of mathematical analysis (having learned only school algebra and trigonometry which I had largely forgotten) and thus my work was laid out for me. It took me several years before I could understand as much as I possibly could. Then I set Maxwell aside and followed my own course. And I progressed much more quickly... It will be understood that I preach the gospel according to my interpretation of Maxwell.[4] ”

Doing fulltime research from home, he helped develop transmission line theory (also known as the "*telegrapher's equations*"). Heaviside showed mathematically that uniformly distributed inductance in a telegraph line would diminish both attenuation and distortion, and that, if the inductance were great enough and the insulation resistance not too high, the circuit would be distortionless while currents of all frequencies would have equal speeds of propagation. Heaviside's equations helped further the implementation of the telegraph.

Middle years

In 1880, Heaviside researched the skin effect in telegraph transmission lines. That same year he patented, in England, the coaxial cable. In 1884 he recast Maxwell's mathematical analysis from its original cumbersome form (they had already been recast as quaternions) to its modern vector terminology, thereby reducing twelve of the original twenty equations in twenty unknowns down to the four differential equations in two unknowns we now know as Maxwell's equations. The four re-formulated Maxwell's equations describe the nature of static and moving electric charges and magnetic dipoles, and the relationship between the two, namely electromagnetic induction.

Between 1880 and 1887, Heaviside developed the operational calculus (involving the *D* notation for the differential operator, which he is credited with creating), a method of solving differential equations by transforming them into ordinary algebraic equations which caused a great deal of controversy when first introduced, owing to the lack of rigour in his derivation of it. He famously said, "Mathematics is an experimental science, and definitions do not come first, but later on." He was replying to criticism over his use of operators that were not clearly defined. On another occasion he stated somewhat more defensively, "I do not refuse my dinner simply because I do not understand the process of digestion."

In 1887, Heaviside proposed that induction coils (inductors) should be added to telephone and telegraph lines to increase their self-induction and correct the distortion which they suffered. For political reasons, this was not done. The importance of Heaviside's work remained undiscovered for some time after publication in The Electrician, and so its rights lay in the public domain. AT&T later employed one of its own scientists, George A. Campbell, and an external investigator Michael I. Pupin to determine whether Heaviside's work was incomplete or incorrect. Campbell and Pupin extended Heaviside's work, and AT&T filed for patents covering not only their research, but also the technical method of constructing the coils previously invented by Heaviside. AT&T later offered Heaviside money in exchange for his rights; it is possible that the Bell engineers' respect for Heaviside influenced this offer. However, Heaviside refused the offer, declining to accept any money unless the company were to give him full recognition.

Heaviside was chronically poor, making his refusal of the offer even more striking.[5]

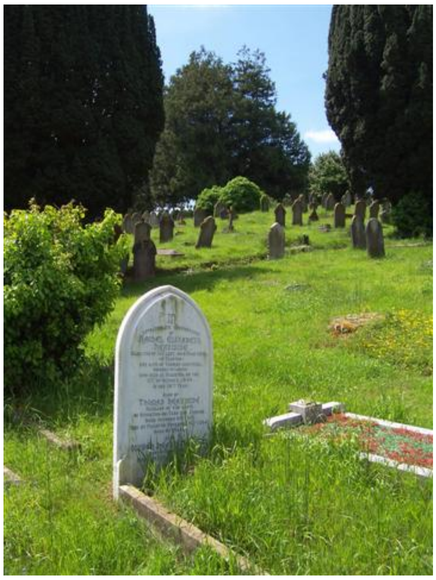
In two papers of 1888 and 1889, Heaviside calculated the deformations of electric and magnetic fields surrounding a moving charge, as well as the effects of it entering a denser medium. This included a prediction of what is now known as Cherenkov radiation, and inspired his friend George FitzGerald to suggest what now is known as the Lorentz-Fitzgerald contraction.

In the late 1880s and early 1890s, Heaviside worked on the concept of electromagnetic mass. Heaviside treated this as material mass, capable of producing the same effects. Wilhelm Wien later verified Heaviside's expression (for low velocities).

In 1891 the British Royal Society recognized Heaviside's contributions to the mathematical description of electromagnetic phenomena by naming him a Fellow of the Royal Society, and the following year devoting more than fifty pages of the *Philosophical Transactions* of the Society to his vector methods and electromagnetic theory. In 1905 Heaviside was given an honorary doctorate by the University of Göttingen.

Later years

In 1902, Heaviside proposed the existence of the Kennelly-Heaviside Layer of the ionosphere which bears his name. Heaviside's proposal included means by which radio signals are transmitted around the Earth's curvature. The existence of the ionosphere was confirmed in 1923. The predictions by Heaviside, combined with Planck's radiation theory, probably discouraged further attempts to detect radio waves from the Sun and other astronomical objects. For whatever reason, there seem to have been no attempts for 30 years, until Jansky's development of radio astronomy in 1932.


Heaviside's grave in Paignton cemetery

In later years his behavior became quite eccentric. Though he had been an active cyclist in his youth, his health seriously declined in his sixth decade. During this time Heaviside would sign letters with the initials "*W.O.R.M.*" after his name. Heaviside also reportedly started painting his fingernails pink and had granite blocks moved into his house for furniture.[6] In 1922, he became the first recipient of the Faraday Medal, which was established that year.

Heaviside died at Torquay in Devon, and is buried in Paignton cemetery. Most of his recognition was gained posthumously.

Innovations and discoveries

Heaviside did much to develop and advocate vector methods and the vector calculus. Maxwell's formulation of electromagnetism consisted of 20 equations in 20 variables. Heaviside employed the curl and divergence operators of the vector calculus to reformulate 12 of these 20 equations into four equations in four variables (**B**, **E**, **J**, and ρ), the form by which they have been known ever since (see Maxwell's equations). He invented the Heaviside step function and employed it to model the current in an electric circuit. He invented the operator method for solving linear differential equations, which resembles current Laplace transform methods (see inverse Laplace transform, also known as the "Bromwich integral"). The UK mathematician Thomas John I'Anson Bromwich later devised a rigorous mathematical justification for Heaviside's operator method.

Heaviside advanced the idea that the Earth's uppermost atmosphere contained an ionized layer known as the ionosphere; in this regard, he predicted the existence of what later was dubbed the Kennelly-Heaviside Layer. He developed the transmission line theory (also known as the "telegrapher's equations"). He also independently co-discovered the Poynting vector.

Electromagnetic terms

Heaviside coined the following terms of art in electromagnetic theory:

- admittance (December 1887);
- conductance (September 1885);
- electret for the electric analogue of a permanent magnet, or, in other words, any substance that exhibits a quasi-permanent electric polarization (e.g. ferroelectric);
- impedance (July 1886);
- inductance (February 1886);
- permeability (September 1885);
- permittance (later susceptance; June 1887);
- reluctance (May 1888).

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[2] See History of Wireless (http://books.google.com/books?id=NBLEAA6QKYkC&pg=PA230&lpg=PA230&dq="wheatstone+took+a+strong+interest+in+his+nephews"), a book by Tapan K Sarkar *et al.*

[3] See (http://books.google.com/books?id=NBLEAA6QKYkC&pg=PA230&lpg=PA230&dq="wheatstone+took+a+strong+interest+in+his+nephews")

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- Many books by Heaviside available online (http://www.archive.org/search.php?query=creator:Heaviside, Oliver) at The Internet Archive

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Joseph Henry

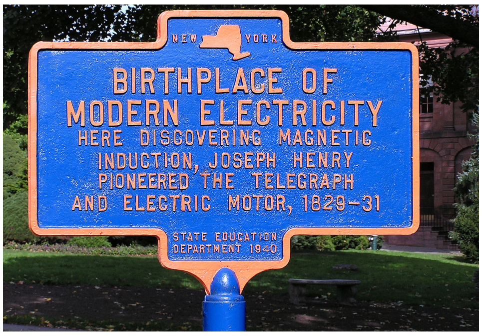
Joseph Henry	
	
Born	December 17, 1797 Albany, New York, USA
Died	May 13, 1878 (aged 80) Washington, D. C., USA
Nationality	United States
Fields	Physics
Institutions	The Albany Academy Princeton University Smithsonian Institution
Alma mater	The Albany Academy
Known for	Electromagnetic induction
Influences	Michael Faraday
Influenced	Charles Grafton Page

Joseph Henry (December 17, 1797 – May 13, 1878) was an American scientist who served as the first Secretary of the Smithsonian Institution, as well as a founding member of the National Institute for the Promotion of Science, a precursor of the Smithsonian Institution.[1] During his lifetime, he was highly regarded. While building electromagnets, Henry discovered the electromagnetic phenomenon of self-inductance. He also discovered mutual inductance independently of Michael Faraday, though Faraday was the first to publish his results.[2] [3] The SI unit of inductance, the henry, is named in his honor. Henry's work on the electromagnetic relay was the basis of the electrical telegraph, invented by Samuel Morse and Charles Wheatstone separately.

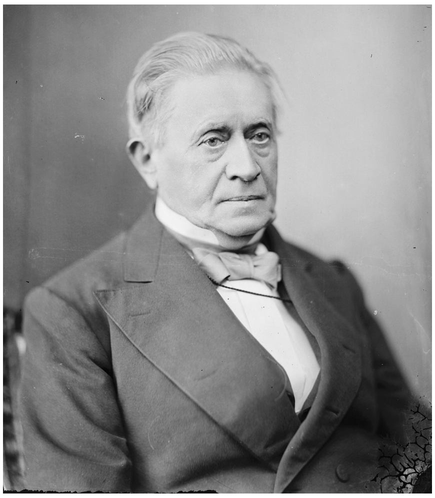
Biography

Henry was born in Albany, New York to Scottish immigrants Ann Alexander Henry and William Henry. His parents were poor, and Henry's father died while he was still young. For the rest of his childhood, Henry lived with his grandmother in Galway, New York. He attended a school which would later be named the "Joseph Henry Elementary School" in his honor. After school, he worked at a general store, and at the age of thirteen became an apprentice watchmaker and silversmith. Joseph's first love was theater and he came close to becoming a professional actor. His interest in science was sparked at the age of sixteen by a book of lectures on scientific topics titled *Popular Lectures on Experimental Philosophy*. In 1819 he entered The Albany Academy, where he was given free tuition. He was so poor, even with free tuition, that he had to support himself with teaching and private tutoring positions. He intended to go into the field of medicine, but in 1824 he was appointed an assistant engineer for the survey of the State road being constructed between the Hudson River and Lake Erie. From then on, he was inspired to a career in either civil or mechanical engineering.

Henry excelled at his studies (so much so, that he would often be helping his teachers teach science) that in 1826 he was appointed Professor of Mathematics and Natural Philosophy at The Albany Academy by Principal T. Romeyn Beck. Some of his most important research was conducted in this new position. His curiosity about terrestrial magnetism led him to experiment with magnetism in general. He was the first to coil insulated wire tightly around an iron core in order to make a more powerful electromagnet, improving on William Sturgeon's electromagnet which used loosely coiled uninsulated wire. Using this technique, he built the strongest electromagnet at the time for Yale. He also showed that, when making an electromagnet using just two electrodes attached to a battery, it is best to wind several coils of wire in parallel, but when using a set-up with multiple batteries, there should be only one single long coil. The latter made the telegraph feasible.



Historical marker in Academy Park (Albany, New York) commemorating Henry's work with electricity.


Joseph Henry, taken between 1865 and 1878, possibly by Mathew Brady.

Using his newly-developed electromagnetic principle, Henry in 1831 created one of the first machines to use electromagnetism for motion. This was the earliest ancestor of modern DC motor. It did not make use of rotating motion, but was merely an electromagnet perched on a pole, rocking back and forth. The rocking motion was caused by one of the two leads on both ends of the magnet rocker touching one of the two battery cells, causing a polarity change, and rocking the opposite direction until the other two leads hit the other battery.

This apparatus allowed Henry to recognize the property of self inductance. British scientist Michael Faraday also recognized this property around the same time; since Faraday published his results first, he became the officially recognized discoverer of the phenomenon.

In 1848 Henry worked in conjunction with Professor Stephen Alexander to determine the relative temperatures for different parts of the solar disk. They used a thermopile to determine that sunspots were cooler than the surrounding regions.[4] [5] [6] [7] This work was shown to the astronomer Angelo Secchi who extended it, but with some question as to whether Henry was given proper credit for his earlier work.[8]

Influences in aeronautics

Prof. Henry was introduced to Prof. Thaddeus Lowe, a balloonist from New Hampshire who had taken interest in the phenomenon of lighter-than-air gases, and exploits into meteorology, in particular, the high winds which we call the Jet stream today. It was Lowe's intent to make a transatlantic crossing by utilizing an enormous gas-inflated aerostat. Henry took a great interest in Lowe's endeavors, promoting him among some of the more prominent scientists and institutions of the day.

In June 1860, Lowe had made a successful test flight with his gigantic balloon, first named the *City of New York* and later renamed *The Great Western*, flying from Philadelphia to Medford, New York. Lowe would not be able to attempt a transatlantic flight until late Spring of the 1861, so Henry convinced him to take his balloon to a point more West and fly the balloon back to the eastern seaboard, an exercise that would keep his investors interested.

Lowe took several smaller balloons to Cincinnati, Ohio in March 1861. On 19 April, he launched on a fateful flight that landed him in Confederate South Carolina. With the Southern States seceding from the union, and the onset of civil war, Lowe abandoned further attempts at a transatlantic crossing and, with Henry's endorsement, went to Washington to offer his services as an aeronaut to the Federal government. Henry submitted a letter to Secretary of War Simon Cameron which carried Henry's endorsement:

Hon. SIMON CAMERON:

DEAR SIR: In accordance with your request made to me orally on the morning of the 6th of June, I have examined the apparatus and witnessed the balloon experiments of Mr. Lowe, and have come to the following conclusions:

1st. The balloon prepared by Mr. Lowe, inflated with ordinary street gas, will retain its charge for several days.

2d. In an inflated condition it can be towed by a few men along an ordinary road, or over fields, in ordinarily calm weather, from the places where it is galled [i.e. swelled or inflated] to another, twenty or more miles distant.

3d. It can be let up into the air by means of a rope in a calm day to a height sufficient to observe the country for twenty miles around and more, according to the degree of clearness of the atmosphere. The ascent may also be made at night and the camp lights of the enemy observed.

4th. From experiments made here for the first time it is conclusively proved that telegrams can be sent with ease and certainty between the balloon and the quarters of the commanding officer.

5th. I feel assured, although I have not witnessed the experiment, that when the surface wind is from the east, as it was for several days last week, an observer in the balloon can be made to float nearly to the enemy's camp (as it is now situated to the west of us), or even to float over it, and then return eastward by rising to a higher elevation. This assumption is based on the fact that the upper strata of wind in this latitude is always flowing eastward. Mr. Lowe informs me, and I do not doubt his statement, that he will on any day which is favorable make an excursion of the kind above mentioned.

6th. From all the facts I have observed and the information I have gathered I am sure that important information may be obtained in regard to the topography of the country and to the position and movements of an enemy by means of the balloon now, and that Mr. Lowe is well qualified to render service in this way by the balloon now in his possession.

7th. The balloon which Mr. Lowe now has in Washington can only be inflated in a city where street gas is to be obtained. If an exploration is required at a point too distant for the transportation of the inflated balloon, an additional apparatus for the generation of hydrogen gas will be required. The necessity of generating the gas renders the use of the balloon more expensive, but this, where important results are required, is of comparatively small importance.

For these preliminary experiments, as you may recollect, a sum not to exceed $200 or $250 was to be appropriated, and in accordance with this Mr. Lowe has presented me with the inclosed statement of items, which I think are reasonable, since nothing is charged for labor and time of the aeronaut.

I have the honor to remain, very respectfully, your obedient servant,

JOSEPH HENRY,

Secretary Smithsonian Institution.

On Henry's recommendation Lowe went on to form the Union Army Balloon Corps and served two years with the Army of the Potomac as a Civil War Aeronaut.

Influences in room acoustics

Over 150 years ago, Henry identified the room acoustics phenomena we now call direct sound, early reflections, and reverberation. He demonstrated the early sound integration period and laid the groundwork for further fundamental research on early reflections that was not followed up until the work at Göttingen University in the 1950–1960s. He brought a robust scientific approach to the subject of acoustics.

Henry devised a simple experiment to demonstrate the integration of direct and early sound. A listener, standing in an open space 100 feet from a wall, claps his hands and hears an echo. He gradually approaches the wall, clapping, until no echo is perceived, at a distance of 30 feet—the "Henry Distance"—equating to an early sound integration time of 60 ms.[9]

Later years

Henry's grave, Oak Hill Cemetery, Washington, D.C.

As a famous scientist and director of the Smithsonian Institution, Henry received visits from other scientists and inventors who sought his advice. Henry was patient, kindly, self-controlled, and gently humorous.[10] One such visitor was Alexander Graham Bell, who on 1 March 1875 carried a letter of introduction to Henry. Henry showed an interest in seeing Bell's experimental apparatus, and Bell returned the following day. After the demonstration, Bell mentioned his untested theory on how to transmit human speech electrically by means of a "harp apparatus" which would have several steel reeds tuned to different frequencies to cover the voice spectrum. Henry said Bell had "the germ of a great invention". Henry advised Bell not to publish his ideas until he had perfected the invention. When Bell objected that he lacked the necessary knowledge, Henry firmly advised: "Get it!"

On 25 June 1876, Bell's experimental telephone (using a different design) was demonstrated at the Centennial Exhibition in Philadelphia where Henry was one of the judges for electrical exhibits. On 13 January 1877, Bell demonstrated his instruments to Henry at the Smithsonian Institution and Henry invited Bell to demonstrate them again that night at the Washington Philosophical Society. Henry praised "the value and astonishing character of Mr. Bell's discovery and invention."[11]

Henry died on 13 May 1878, and was buried in Oak Hill Cemetery in the Georgetown section of northwest Washington, D.C.

Legacy

Henry was a member of the Lighthouse Board from 1852 until his death. He was appointed chairman in 1871 and served in that position the remainder of his life. He was the only civilian to serve as chairman. The United States Coast Guard honored Henry for his work on lighthouses and fog signal acoustics by naming a cutter after him. The *Joseph Henry*, usually referred to as the *Joe Henry*, was launched in 1880 and was active until 1904.[12]

In 1915 Henry was inducted into the Hall of Fame for Great Americans in the Bronx, New York.

At Princeton, the Joseph Henry Laboratories [13] and the Joseph Henry House are named for him.

Curriculum vitae

- 1826 – Professor of Mathematics and Natural Philosophy at The Albany Academy, New York.
- 1832 – Professor at Princeton.
- 1835 – Invented the electromechanical relay.
- 1846 – First secretary of the Smithsonian Institution until 1878
- 1848 – Edited Ephraim G. Squier and Edwin H. Davis' *Ancient Monuments of the Mississippi Valley*, the Institution's first publication.
- 1852 – Appointed to the Lighthouse Board
- 1871 – Appointed chairman of the Lighthouse Board

Statue of Henry before Smithsonian Institution

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[10] *Alexander Graham Bell and the Conquest of Solitude*, Robert V. Bruce, pages 139-140
[11] *Alexander Graham Bell and the Conquest of Solitude*, Robert V. Bruce, page 214
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[13] http://www.princeton.edu/physics/about-us/history/memorable-members/joseph-henry/

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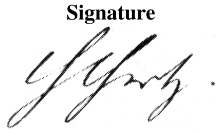
Heinrich Hertz

Heinrich Rudolf Hertz	
	
Born	February 22, 1857 Hamburg, Germany
Died	January 1, 1894 (aged 36) Bonn, Germany
Residence	Germany
Nationality	German
Fields	Physics Electronic Engineering
Institutions	University of Kiel University of Karlsruhe University of Bonn
Alma mater	University of Munich University of Berlin
Doctoral advisor	Hermann von Helmholtz
Known for	Electromagnetic radiation Photoelectric effect
Signature	

Heinrich Rudolf Hertz (February 22, 1857 – January 1, 1894) was a German physicist who clarified and expanded the electromagnetic theory of light that had been put forth by Maxwell. He was the first to satisfactorily demonstrate the existence of electromagnetic waves by building an apparatus to produce and detect radio waves.

Biography

Early years

Hertz was born in Hamburg, Germany, into a prosperous and cultured Hanseatic family. His father, Gustav Ferdinand Hertz, was a writer and later a senator. His mother was the former Anna Elisabeth Pfefferkorn. His paternal grandfather David Wolff Hertz (1757-1822), fourth son of Benjamin Wolff Hertz, moved to Hamburg in 1793 where he made his living as a jeweller. He and his wife Schöne Hertz (1760-1834) were buried in the former Jewish cemetery in Ottensen. Their first son Wolff Hertz (1790-1859), was chairman of the Jewish community. His brother Hertz Hertz (1797-1862) was a respected businessman. He was married to Betty Oppenheim, the daughter of the banker Salomon Oppenheim, from Cologne. Hertz Hertz converted from Judaism to Christianity and took the name Heinrich David Hertz.[1]

While studying at the Gelehrtenschule des Johanneums in Hamburg, he showed an aptitude for sciences as well as languages, learning Arabic and Sanskrit. He studied sciences and engineering in the German cities of Dresden, Munich and Berlin, where he studied under Gustav R. Kirchhoff and Hermann von Helmholtz.

In 1880, Hertz obtained his PhD from the University of Berlin; and remained for post-doctoral study under Hermann von Helmholtz.

In 1883, Hertz took a post as a lecturer in theoretical physics at the University of Kiel.

In 1885, Hertz became a full professor at the University of Karlsruhe where he discovered electromagnetic waves.

The most dramatic prediction of Maxwell's theory of electromagnetism, published in 1865, was the existence of electromagnetic waves moving at the speed of light, and the conclusion that light itself was just such a wave. This challenged experimentalists to generate and detect electromagnetic radiation using some form of electrical apparatus.

The first clearly successful attempt was made by Heinrich Hertz in 1886. For his radio wave transmitter he used a high voltage induction coil, a condenser (capacitor, Leyden jar) and a spark gap - whose poles on either side are formed by spheres of 2 cm radius - to cause a spark discharge between the spark gap's poles oscillating at a frequency determined by the values of the capacitor and the induction coil.

To prove there really was radiation emitted, it had to be detected. Hertz used a piece of copper wire, 1 mm thick, bent into a circle of a diameter of 7.5 cm, with a small brass sphere on one end, and the other end of the wire was pointed, with the point near the sphere. He bought a screw mechanism so that the point could be moved very close to the sphere in a controlled fashion. This "receiver" was designed so that current oscillating back and forth in the wire would have a natural period close to that of the "transmitter" described above. The presence of oscillating charge in the receiver would be signaled by sparks across the (tiny) gap between the point and the sphere (typically, this gap was hundredths of a millimeter).

In more advanced experiments, Hertz measured the velocity of electromagnetic radiation and found it to be the same as the light's velocity. He also showed that the nature of radio waves' reflection and refraction was the same as those of light, and established beyond any doubt that light is a form of electromagnetic radiation obeying the Maxwell equations.

Hertz's experiments would soon trigger the invention of the wireless telegraph, radio, and later television. In recognition of his work, the unit of frequency - one cycle per second - is named the "hertz".

Meteorology

He always had a deep interest in meteorology probably derived from his contacts with Wilhelm von Bezold (who was Hertz's professor in a laboratory course at the Munich Polytechnic in the summer of 1878). Hertz, however, did not contribute much to the field himself except some early articles as an assistant to Helmholtz in Berlin, including research on the evaporation of liquids, a new kind of hygrometer, and a graphical means of determining the properties of moist air when subjected to adiabatic changes.[2]

Contact mechanics

Memorial of Heinrich Hertz on the campus of the Karlsruhe Institute of Technology

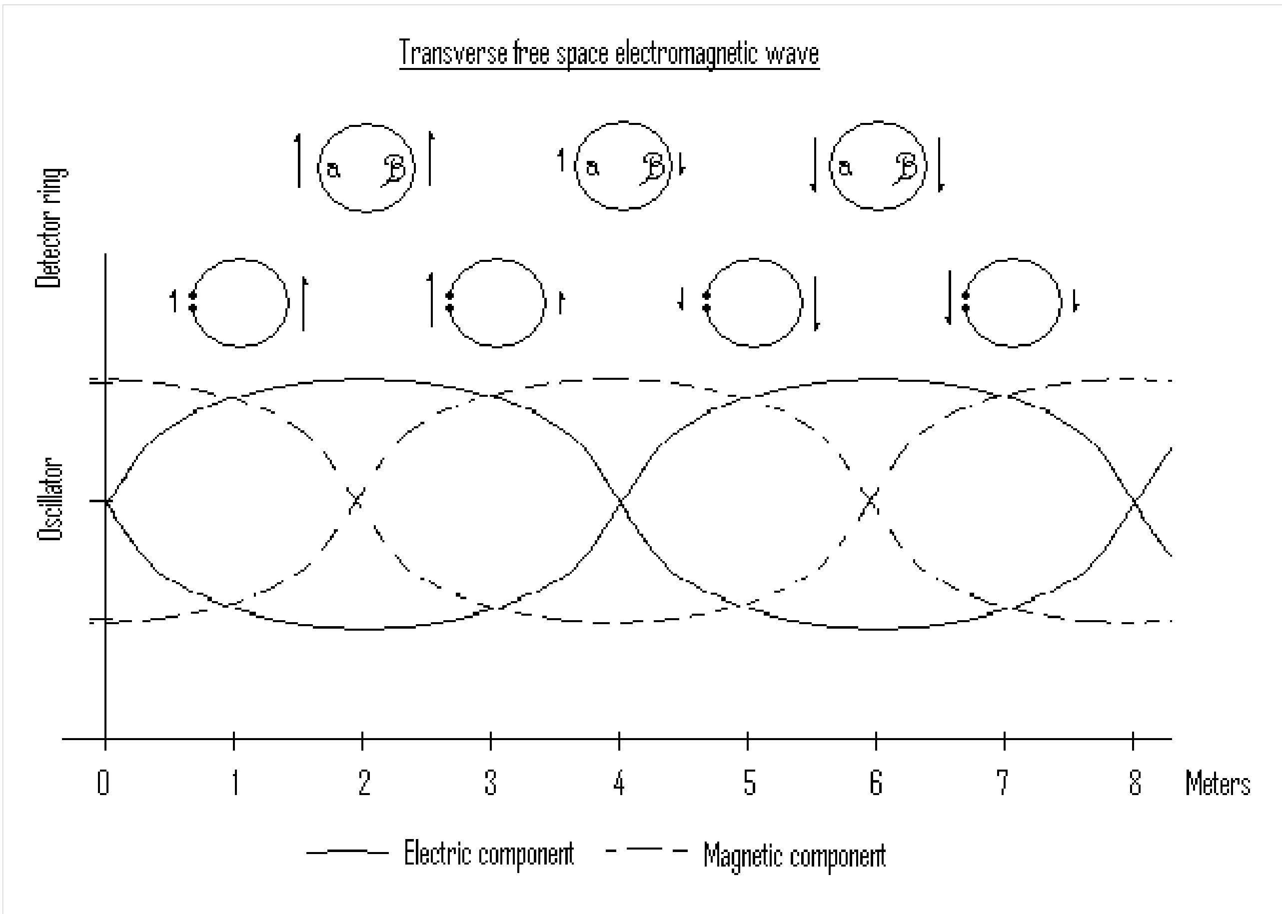
In 1886–1889, Hertz published two articles on what was to become known as the field of contact mechanics. Hertz is well known for his contributions to the field of electrodynamics (*see below*); however, most papers that look into the fundamental nature of contact cite his two papers as a source for some important ideas. Joseph Valentin Boussinesq published some critically important observations on Hertz's work, nevertheless establishing this work on contact mechanics to be of immense importance. His work basically summarises how two axi-symmetric objects placed in contact will behave under loading, he obtained results based upon the classical theory of elasticity and continuum mechanics. The most significant failure of his theory was the neglect of any nature of adhesion between the two solids, which proves to be important as the materials composing the solids start to assume high elasticity. It was natural to neglect adhesion in that age as there were no experimental methods of testing for it.

To develop his theory Hertz used his observation of elliptical Newton's rings formed upon placing a glass sphere upon a lens as the basis of assuming that the pressure exerted by the sphere follows an elliptical distribution. He used the formation of Newton's rings again while validating his theory with experiments in calculating the displacement which the sphere has into the lens. K. L. Johnson, K. Kendall and A. D. Roberts (JKR) used this theory as a basis while calculating the theoretical displacement or *indentation depth* in the presence of adhesion in their landmark article "Surface energy and contact of elastic solids" published in 1971 in the Proceedings of the Royal Society (A324, 1558, 301-313). Hertz's theory is recovered from their formulation if the adhesion of the materials is assumed to be zero. Similar to this theory, however using different assumptions, B. V. Derjaguin, V. M. Muller and Y. P. Toporov published another theory in 1975, which came to be known as the DMT theory in the research community, which also recovered Hertz's formulations under the assumption of zero adhesion. This DMT theory proved to be rather premature and needed several revisions before it came to be accepted as another material contact theory in addition to the JKR theory. Both the DMT and the JKR theories form the basis of contact mechanics upon which all transition contact models are based and used in material parameter prediction in Nanoindentation and Atomic Force Microscopy. So Hertz's research from his days as a lecturer, preceding his great work on electromagnetism, which he himself considered with his characteristic soberness to be trivial, has come down to the age of nanotechnology.

Electromagnetic research

In 1886, Hertz developed the **Hertz antenna** receiver. This is a set of terminals which is not electrically grounded for its operation. He also developed a transmitting type of dipole antenna, which was a center-fed driven element for transmitting UHF radio waves. These antennas are the simplest practical antennas from a theoretical point of view.

In 1887, Hertz experimented with radio waves in his laboratory. These actions followed Michelson's 1881 experiment (precursor to the 1887 Michelson-Morley experiment) which did not detect the existence of aether drift, Hertz altered the Maxwell's equations to take this view into account for electromagnetism. Hertz used a Ruhmkorff coil-driven spark gap and one meter wire pair as a radiator. Capacity spheres were present at the ends for circuit resonance adjustments. His receiver, a precursor to the dipole antenna, was a simple half-wave dipole antenna for shortwaves. Hertz published his work in a book titled: *Electric waves: being researches on the propagation of electric action with finite velocity through space*.[3]

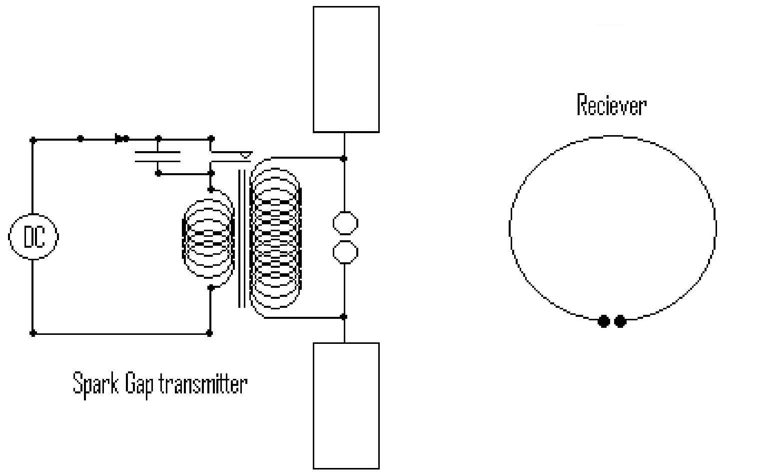


Through experimentation, he proved that transverse free space electromagnetic waves can travel over some distance. This had been predicted by James Clerk Maxwell and Michael Faraday. With his apparatus configuration, the electric and magnetic fields would radiate away from the wires as transverse waves. Hertz had positioned the oscillator about 12 meters from a zinc reflecting plate to produce standing waves. Each wave was about 4 meters. Using the ring detector, he recorded how the magnitude and wave's component direction vary. Hertz measured Maxwell's waves and demonstrated that the velocity of radio waves was equal to the velocity of light. The electric field intensity and polarity was also measured by Hertz. (Hertz, 1887, 1888).

The Hertzian cone was first described by Hertz as a type of wave-front propagation through various media. His experiments expanded the field of electromagnetic transmission and his apparatus was developed further by others in the radio. Hertz also found that radio waves could be transmitted through different types of materials, and were reflected by others, leading in the distant future to radar.

Hertz helped establish the photoelectric effect (which was later explained by Albert Einstein) when he noticed that a charged object loses its charge more readily when illuminated by ultraviolet light. In 1887, he made observations of the photoelectric effect and of the production and reception of electromagnetic (EM) waves, published in the journal Annalen der Physik. His receiver consisted of a coil with a spark gap, whereupon a spark would be seen upon detection of EM waves. He placed the apparatus in a darkened box to see the spark better. He observed that the maximum spark length was reduced when in the box. A glass panel placed between the source of EM waves and the receiver absorbed ultraviolet radiation that assisted the electrons in jumping across the gap.

When removed, the spark length would increase. He observed no decrease in spark length when he substituted quartz for glass, as quartz does not absorb UV radiation. Hertz concluded his months of investigation and reported the results obtained. He did not further pursue investigation of this effect, nor did he make any attempt at explaining how the observed phenomenon was brought about.



1887 experimental setup of Hertz's apparatus.

Hertz did not realize the practical importance of his experiments. He stated that,

> "*It's of no use whatsoever*[...] *this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.*" [4]

Asked about the ramifications of his discoveries, Hertz replied,

> "*Nothing, I guess*." [4]

His discoveries would later be more fully understood by others and be part of the new "wireless age". In bulk, Hertz' experiments explain reflection, refraction, polarization, interference, and velocity of electric waves.

In 1892, Hertz began experimenting and demonstrated that cathode rays could penetrate very thin metal foil (such as aluminium). Philipp Lenard, a student of Heinrich Hertz, further researched this "ray effect". He developed a version of the cathode tube and studied the penetration by X-rays of various materials. Philipp Lenard, though, did not realize that he was producing X-rays. Hermann von Helmholtz formulated mathematical equations for X-rays. He postulated a dispersion theory before Röntgen made his discovery and announcement. It was formed on the basis of the electromagnetic theory of light (*Wiedmann's Annalen*, Vol. XLVIII). However, he did not work with actual X-rays.

Death at age 36

In 1892, an infection was diagnosed (after a bout of severe migraines) and Hertz underwent some operations to correct the illness. He died of Wegener's granulomatosis at the age of 36 in Bonn, Germany in 1894, and was buried in Ohlsdorf, Hamburg at the Jewish cemetery.[5]

Hertz's wife, Elizabeth Hertz (maiden name: Elizabeth Doll), did not remarry. Heinrich Hertz left two daughters, Joanna and Mathilde. Subsequently, all three women left Germany in the 1930s to England, after the rise of Adolf Hitler. Charles Susskind interviewed Mathilde Hertz in the 1960s and he later published a book on Heinrich Hertz. Heinrich Hertz's daughters never married and he does not have any descendants, according to the book by Susskind.

Legacy and honors

His nephew Gustav Ludwig Hertz was a Nobel Prize winner, and Gustav's son Carl Hellmuth Hertz invented medical ultrasonography.

The SI unit *hertz* (Hz) was established in his honor by the IEC in 1930 for frequency, a measurement of the number of times that a repeated event occurs per second (also called "cycles per sec" (cps)). It was adopted by the CGPM (Conférence générale des poids et mesures) in 1964.

In 1969 (East Germany), there was cast a Heinrich Hertz memorial medal. The IEEE Heinrich Hertz Medal, established in 1987, is "*for outstanding achievements in Hertzian waves* [...] *presented annually to an individual for achievements which are theoretical or experimental in nature*".

Heinrich Hertz

A crater that lies on the far side of the Moon, just behind the eastern limb, is named in his honor. The Hertz market for radioelectronics products in Nizhny Novgorod, Russia, is named after him. The Heinrich-Hertz-Turm radio telecommunication tower in Hamburg is named after the city's famous son.

Hertz is honored by Japan with a membership in the Order of the Sacred Treasure, which has multiple layers of honor for prominent people, including scientists.[6]

Heinrich Hertz was honored by a number of countries around the world in their postage issues and in Post World War II times has appeared on various German stamp issues as well.

Nazi revisionism

Although Hertz would not have considered himself Jewish, his "Jewish" portrait was removed by the Nazis from its prominent position of honor in Hamburg's City Hall (*Rathaus*) because of his partly "Jewish ancestry." Hertz was a Lutheran; and although his father's family had been Jewish,[7] his father had converted to Catholicism before marrying. The painting has since been returned to public display.[8]

Notes

[1] http://www1.uni-hamburg.de/rz3a035//bundesstrasse1.html

[2] Mulligan, J. F., and H. G. Hertz, "On the energy balance of the Earth," *American Journal of Physics*, Vol. 65, pp. 36-45.

[3] *Electric waves: being researches on the propagation of electric action with finite velocity through space* (http://books.google.com/books?id=8GkOAAAAIAAJ&pg=PR3#v=onepage&q&f=false), download here

[4] Institute of Chemistry, Hebrew University of Jerusalem: Hertz biography, digitized photographs (http://chem.ch.huji.ac.il/history/hertz.htm)

[5] For a photograph of his gravesite, see "Heinrich Rudolf Hertz" (http://phisicist.info/hertz.html). . Retrieved 2008-12-02.

[6] L'Harmattan: List of recipients of Japanese Order of the Sacred Treasure (in French) (http://www.editions-harmattan.fr/index.asp?navig=catalogue&obj=article&no=8245)

[7] Koertge, Noretta. (2007). *Dictionary of Scientific Biography,* Vol. 6, p. 340.

[8] Robertson, Struan: Hertz biography (http://www1.uni-hamburg.de/rz3a035//hertz.html)

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Rudolf Kohlrausch

Rudolf Hermann Arndt Kohlrausch (November 6, 1809, Göttingen - March 8, 1858, Erlangen) was a German physicist.

Rudolf Kohlrausch (1809-1858)

Biography

He was a native of Göttingen, the son of educator Heinrich Friedrich Theodor Kohlrausch. He was successively teacher of mathematics and physics at Lüneburg, Rinteln, Cassell, and Marburg, and a professor at the Universities of Marburg and Erlangen.

Research

In 1854 Kohlrausch introduced the relaxation phenomena, and used the stretched exponential function to explain relaxation effects of a discharging Leyden jar. In 1856 with Wilhelm Weber (1804–1891) he demonstrated that the ratio of electrostatic to electromagnetic units produced a number that matched the value of the then known speed of light. This finding led to Maxwell's conjecture that light is an electromagnetic wave. Also, the first usage of the letter "c" to denote the speed of light was in an 1856 paper by Kohlrausch and Weber.

Family

He was the father of physicist Friedrich Kohlrausch.

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[2] http://physicsworld.com/cws/article/print/18921

Heinrich Lenz


Heinrich Lenz

Heinrich Friedrich Emil Lenz (Russian: **Эмилий Христианович Ленц**) (February 12, 1804 – February 10, 1865) was a Russian physicist of Baltic German ethnicity. He is most noted for formulating Lenz's law in electrodynamics in 1833.

Lenz was born in Dorpat (now *Tartu*), the Governorate of Livonia, in the Russian Empire at that time. After completing his secondary education in 1820, Lenz studied chemistry and physics at the University of Dorpat. He traveled with the navigator Otto von Kotzebue on his third expedition around the world from 1823 to 1826. On the voyage Lenz studied climatic conditions and the physical properties of seawater. The results have been published in "Memoirs of the St. Petersburg Academy of Sciences" (1831).

After the voyage, Lenz began working at the University of St. Petersburg, Russia, where he later served as the Dean of Mathematics and Physics from 1840 to 1863 and was Rector from 1863 until his death in 1865. Lenz also taught at the Petrischule in 1830 and 1831, and at the Mikhailovskaya Artillery Academy.

Lenz had begun studying electromagnetism in 1831. Besides the law named in his honor, Lenz also independently discovered Joule's law in 1842; to honor his efforts on the problem, it is also given the name the "Joule–Lenz law," named also for James Prescott Joule.

Lenz eagerly participated in development of the electroplating technology, invented by his friend and colleague Moritz von Jacobi. In 1839 Lenz produced several medallions using electrotyping. Along with the electrotyped relief produced by Jacobi the same year, these were the first instances of galvanoplastic sculpture.[1]

Lenz died in Rome, Italy, after suffering from a stroke.

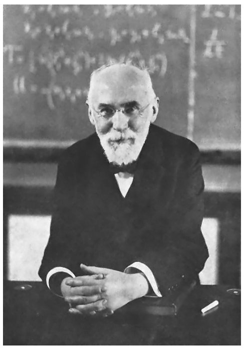
References

[1] History of electroplating in the 19th century Russia (http://www.galteh.ru/article_galvanotehnika.html) (Russian)

External links

- Page on Lenz from a list of famous electroscientists (http://chem.ch.huji.ac.il/~eugeniik/history/lenz.html)
- Biography of Lenz (http://cse.unl.ecdu/~jtooker/Files/Lenz.pdf)

Hendrik Lorentz

Hendrik Antoon Lorentz	
	
Born	18 July 1853 Arnhem, Netherlands
Died	4 February 1928 (aged 74) Haarlem, Netherlands
Nationality	Netherlands
Fields	Physics
Alma mater	University of Leiden
Doctoral advisor	Pieter Rijke
Doctoral students	Geertruida L. de Haas-Lorentz Adriaan Fokker Leonard Ornstein
Known for	Theory of EM radiation Lorentz force Lorentz contraction
Notable awards	Nobel Prize for Physics (1902) Rumford Medal (1908) Franklin Medal (1917) Copley Medal (1918)

Hendrik Antoon Lorentz (18 July 1853 – 4 February 1928) was a Dutch physicist who shared the 1902 Nobel Prize in Physics with Pieter Zeeman for the discovery and theoretical explanation of the Zeeman effect. He also derived the transformation equations subsequently used by Albert Einstein to describe space and time.

Biography

Early life

Hendrik Lorentz was born in Arnhem, Gelderland (The Netherlands), the son of Gerrit Frederik Lorentz (1822 – 1893), a well-off nurseryman, and Geertruida van Ginkel (1826 – 1861). In 1862, after his mother's death, his father married Luberta Hupkes. From 1866-1869 he attended the newly established high school in Arnhem, and in 1870 he passed the exams in classical languages which were then required for admission to University.

Lorentz studied physics and mathematics at the University of Leiden, where he was strongly influenced by the teaching of astronomy professor Frederik Kaiser; it was his influence that led him to become a physicist. After earning a bachelor's degree, he returned to Arnhem in 1872 to teach high school classes in mathematics, but he continued his studies in Leiden in addition to his teaching position. In 1875 Lorentz earned a doctoral degree under Pieter Rijke on a thesis entitled *"Over de theorie der terugkaatsing en breking van het licht"* (On the theory of reflection and refraction of light), in which he refined the electromagnetic theory of James Clerk Maxwell.
In 1881 Hendrik married Aletta Catharina Kaiser, niece of Frederik Kaiser. She was the daughter of Johann Wilhelm Kaiser, director of the Amsterdam's Engraving School and professor of Fine Arts, and designer of the first Dutch postage stamps (1852). Later Kaiser was the Director of the National Gallery of Amsterdam. Hendrik and Aletta's eldest daughter Geertruida Luberta Lorentz was to become a physicist as well.

Career

Portrait by Jan Veth

Professor in Leiden

In 1878, only 24 years of age, Hendrik Antoon Lorentz was appointed to the newly established chair in theoretical physics at the University of Leiden. On January 25, 1878, he delivered his inaugural lecture on *"De moleculaire theoriën in de natuurkunde"* (The molecular theories in physics).

During the first twenty years in Leiden, Lorentz was primarily interested in the theory of electromagnetism to explain the relationship of electricity, magnetism, and light. After that, he extended his research to a much wider area while still focusing on theoretical physics. From his publications, it appears that Lorentz made contributions to mechanics, thermodynamics, hydrodynamics, kinetic theories, solid state theory, light, and propagation. His most important contributions were in the area of electromagnetism, the electron theory, and relativity.

Lorentz theorized that the atoms might consist of charged particles and suggested that the oscillations of these charged particles were the source of light. When a colleague and former student of Lorentz, Pieter Zeeman, discovered the Zeeman effect in 1896, Lorentz supplied its theoretical interpretation. The experimental and theoretical work was honored with the Nobel prize in physics in 1902. Lorentz' name is now associated with the Lorentz-Lorenz formula, the Lorentz force, the Lorentzian distribution, and the Lorentz transformation.

Electrodynamics and relativity

In 1895, with the attempt to explain the Michelson-Morley experiment, Lorentz proposed that moving bodies contract in the direction of motion (see length contraction; George FitzGerald had already arrived at this conclusion, see FitzGerald-Lorentz Contraction). Lorentz worked on describing electromagnetic phenomena (the propagation of light) in reference frames that moved relative to each other. He discovered that the transition from one to another reference frame could be simplified by using a new time variable which he called *local time*. The local time depended on the universal time and the location under consideration. Lorentz's publications (of 1895 and 1899) made use of the term local time without giving a detailed interpretation of its physical relevance. In 1900, Henri Poincaré called Lorentz's local time a "wonderful invention" and illustrated it by showing that clocks in moving frames are synchronized by exchanging light signals that are assumed to travel at the same speed against and with the motion of the frame.

In 1899, and again in his paper "Electromagnetic phenomena in a system moving with any velocity smaller than that of light" (1904), Lorentz added time dilation to his transformations and published what Poincaré in 1905 named Lorentz transformations. It was apparently unknown to Lorentz that Joseph Larmor had used identical transformations to describe orbiting electrons in 1897. Larmor's and Lorentz's equations look somewhat unfamiliar, but they are algebraically equivalent to those presented by Poincaré and Einstein in 1905.[1] Lorentz's 1904 paper includes the covariant formulation of electrodynamics, in which electrodynamic phenomena in different reference frames are described by identical equations with well defined transformation properties. The paper clearly recognizes the significance of this formulation, namely that the outcomes of electrodynamic experiments do not depend on the relative motion of the reference frame. The 1904 paper includes a detailed discussion of the increase of the inertial mass of rapidly moving objects. In 1905, Einstein would use many of the concepts, mathematical tools and results discussed to write his paper entitled "On the Electrodynamics of Moving Bodies",[2] known today as the theory of special relativity. Because Lorentz laid the fundamentals for the work by Einstein, this theory was called the *Lorentz-Einstein theory* originally.

The increase of mass was the first prediction of special relativity to be tested, but the early (1901–1903) experiments by Kaufmann appeared to show a slightly different mass increase; this led Lorentz to the famous remark that he was "at the end of his Latin."[3] The confirmation of his prediction had to wait until 1908. In 1909, Lorentz published "Theory of Electrons" based on a series of lectures in Mathematical Physics he gave at Columbia University.[4]

Albert Einstein and Hendrik Antoon Lorentz, photographed by Ehrenfest in front of his home in Leiden in 1921. Source: Museum Boerhaave, Leiden

Assessments

Poincaré (1902) said of Lorentz's theory of electrodynamics:

> *The most satisfactory theory is that of Lorentz; it is unquestionably the theory that best explains the known facts, the one that throws into relief the greatest number of known relations ... it is due to Lorentz that the results of Fizeau on the optics of moving bodies, the laws of normal and abnormal dispersion and of absorption are connected with each other ... Look at the ease with which the new Zeeman phenomenon found its place, and even aided the classification of Faraday's magnetic rotation, which had defied all Maxwell's efforts.*

Paul Langevin (1911) said of Lorentz:

It is the great merit of H. A. Lorentz to have seen that the fundamental equations of electromagnetism admit a group of transformations which enables them to have the same form when one passes from one frame of reference to another; this new transformation has the most profound implications for the transformations of space and time

Lorentz and Emil Wiechert (Göttingen) had an interesting correspondence on the topics of electromagnetism and the theory of relativity, and Lorentz explained his ideas in letters to Wiechert. The correspondence between Lorentz and Wiechert has been published by Wilfried Schröder (Arch. ex. hist. Sci, 1984).

Lorentz was chairman of the first Solvay Conference held in Brussels in the autumn of 1911. Shortly after the conference, Poincaré wrote an essay on quantum physics which gives an indication of Lorentz's status at the time:

... at every moment [the twenty physicists from different countries] could be heard talking of the [quantum mechanics] which they contrasted with the old mechanics. Now what was the old mechanics? Was it that of Newton, the one which still reigned uncontested at the close of the nineteenth century? No, it was the mechanics of Lorentz, the one dealing with the principle of relativity; the one which, hardly five years ago, seemed to be the height of boldness.

Albert Einstein (1953) wrote of Lorentz:

For me personally he meant more than all the others I have met on my life's journey.[5]

While Lorentz is mostly known for fundamental theoretical work, he also had an interest in practical applications. In the years 1918-1926, at the request of the Dutch government, Lorentz headed a committee to calculate some of the effects of the proposed Afsluitdijk (Closure Dike) flood control dam on other seaworks in the Netherlands. Hydraulic engineering was mainly an empirical science at that time, but the disturbance of the tidal flow caused by the Afsluitdijk was so unprecedented that the empirical rules could not be trusted. Lorentz proposed to start from the basic hydrodynamic equations of motion and solve the problem numerically. This was feasible for a "human computer", because of the quasi-one-dimensional nature of the water flow in the Waddenzee. The Afsluitdijk was completed in 1933 and the predictions of Lorentz and his committee turned out to be remarkably accurate.[6] One of the two sets of locks in the Afsluitdijk was named after him.

Personal life

In 1912, Lorentz retired early to become director of research at Teylers Museum in Haarlem, although he remained external professor at Leiden and gave weekly lectures there. Paul Ehrenfest succeeded him in his chair at the University of Leiden, founding the Institute for Theoretical Physics which would become known as the Lorentz Institute. In addition to the Nobel prize, Lorentz received a great many honours for his outstanding work. He was elected a Fellow of the Royal Society in 1905. The Society awarded him their Rumford Medal in 1908 and their Copley Medal in 1918.

Lorentz died in Haarlem, Netherlands. The respect in which he was held in the Netherlands is apparent from O. W. Richardson's description of his funeral:

The funeral took place at Haarlem at noon on Friday, February 10. At the stroke of twelve the State telegraph and telephone services of Holland were suspended for three minutes as a revered tribute to the greatest man Holland has produced in our time. It was attended by many colleagues and distinguished physicists from foreign countries. The President, Sir Ernest Rutherford, represented the Royal Society and made an appreciative oration by the graveside.[7]

Unique 1928 film footage of the funeral procession with a lead carriage followed by ten mourners, followed by a carriage with the coffin, followed in turn by at least four more carriages, passing by a crowd at the Grote Markt, Haarlem from the Zijlstraat to the Smedesstraat, and then back again through the Grote Houtstraat towards the Barteljorisstraat, on the way to the "Algemene Begraafplaats" at the Kleverlaan (northern Haarlem cemetery) has been digitized on Youtube.[8]

Legacy

Richardson describes Lorentz as:

> *[A] man of remarkable intellectual powers ... . Although steeped in his own investigation of the moment, he always seemed to have in his immediate grasp its ramifications into every corner of the universe. ... The singular clearness of his writings provides a striking reflection of his wonderful powers in this respect. .... He possessed and successfully employed the mental vivacity which is necessary to follow the interplay of discussion, the insight which is required to extract those statements which illuminate the real difficulties, and the wisdom to lead the discussion among fruitful channels, and he did this so skillfully that the process was hardly perceptible.*[7]

M. J. Klein (1967) wrote of Lorentz's reputation in the 1920s:

> *For many years physicists had always been eager "to hear what Lorentz will say about it" when a new theory was advanced, and, even at seventy-two, he did not disappoint them.*[9]

- Nobel Prize for Physics (1902)
- Rumford Medal (1908)
- Copley Medal (1918)

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[8] funeral procession (http://www.youtube.com/watch?v=H2VtrJD0xJk) Hendrik Lorentz

[9] Przibram 1967

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External links

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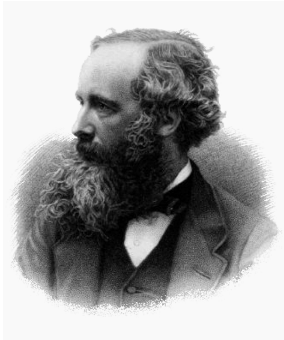
James Clerk Maxwell

James Clerk Maxwell



James Clerk Maxwell (1831–1879)

Born	13 June 1831 Edinburgh, UK
Died	5 November 1879 (aged 48) Cambridge, UK
Citizenship	United Kingdom
Nationality	British/Scottish
Fields	Physics and mathematics
Institutions	Marischal College, Aberdeen King's College London University of Cambridge
Alma mater	University of Edinburgh University of Cambridge
Academic advisors	William Hopkins
Notable students	George Chrystal
Known for	Maxwell's equations Maxwell distribution Maxwell's demon Maxwell's discs Maxwell speed distribution Maxwell's theorem Maxwell material Generalized Maxwell model Displacement current
Notable awards	Smith's Prize (1854) Adams Prize (1857) Rumford Medal (1860) Keith Prize (1869-71)

Signature

James Clerk Maxwell of Glenlair[1] FRS FRSE (13 June 1831 – 5 November 1879) was a Scottish[2] physicist and mathematician. His most prominent achievement was formulating classical electromagnetic theory. This united all previously unrelated observations, experiments and equations of electricity, magnetism and optics into a consistent theory.[3] Maxwell's equations demonstrated that electricity, magnetism and light are all manifestations of the same phenomenon, namely the electromagnetic field. Subsequently, all other classic laws or equations of these disciplines became simplified cases of Maxwell's equations. Maxwell's achievements concerning electromagnetism have been called the "second great unification in physics",[4] after the first one realised by Isaac Newton.

Maxwell demonstrated that electric and magnetic fields travel through space in the form of waves, and at the constant speed of light. In 1865 Maxwell published *A Dynamical Theory of the Electromagnetic Field*. It was with this that he first proposed that light was in fact undulations in the same medium that is the cause of electric and magnetic phenomena.[5] His work in producing a unified model of electromagnetism is one of the greatest advances in physics.

Maxwell also helped develop the Maxwell–Boltzmann distribution, which is a statistical means of describing aspects of the kinetic theory of gases. These two discoveries helped usher in the era of modern physics, laying the foundation for such fields as special relativity and quantum mechanics.

Maxwell is also known for presenting the first durable colour photograph in 1861 and for his foundational work on the rigidity of rod-and-joint frameworks like those in many bridges.

Maxwell is considered by many physicists to be the 19th-century scientist who had the greatest influence on 20th-century physics. His contributions to the science are considered by many to be of the same magnitude as those of Isaac Newton and Albert Einstein.[6] In the millennium poll—a survey of the 100 most prominent physicists—Maxwell was voted the third greatest physicist of all time, behind only Newton and Einstein.[7] On the centennial of Maxwell's birthday, Einstein himself described Maxwell's work as the "most profound and the most fruitful that physics has experienced since the time of Newton."[8] Einstein kept a photograph of Maxwell on his study wall, alongside pictures of Michael Faraday and Newton.[9]

Life

Early life, 1831–39

James Clerk Maxwell was born 13 June 1831 at 14 India Street, Edinburgh, to John Clerk, an advocate, and Frances Cay.[10] Maxwell's father was a man of comfortable means, of the Clerk family of Penicuik, Midlothian, holders of the baronetcy of Clerk of Penicuik; his brother being the 6th Baronet.[11] James was the first cousin of notable 19th century artist Jemima Blackburn.

He had been born **John Clerk**,[12] adding the surname Maxwell to his own after he inherited a country estate in Middlebie, Kirkcudbrightshire from connections to the Maxwell family, themselves members of the peerage.[10]

Maxwell's parents did not meet and marry until they were well into their thirties,[13] which was unusual for the time; moreover, his mother was nearly 40 years old when James was born. They had had one earlier child, a daughter, Elizabeth, who died in infancy.[14] They named their only surviving child James, a name that had sufficed not only for his grandfather, but also many of his other ancestors.

When Maxwell was young his family moved to Glenlair House, which his parents had built on the 1500 acre (6.1 km^2) Middlebie estate.[15] All indications suggest that Maxwell had maintained an unquenchable curiosity from an early age.[16] By the age of three, everything that moved, shone, or made a noise drew the question: "what's the go o' that?".[17] In a passage added to a letter from his father to his sister-in-law Jane Cay in 1834, his mother described this innate sense of inquisitiveness:

> "He is a very happy man, and has improved much since the weather got moderate; he has great work with doors, locks, keys, etc., and "show me how it doos" is never out of his mouth. He also investigates the hidden course of streams and bell-wires, the way the water gets from the pond through the wall..."[18]

Education, 1839–47

Recognising the potential of the young boy, his mother Frances took responsibility for James' early education, which in the Victorian era was largely the job of the woman of the house.[19] She was however taken ill with abdominal cancer, and after an unsuccessful operation, died in December 1839 when Maxwell was only eight. James' education was then overseen by John Maxwell and his sister-in-law Jane, both of whom played pivotal roles in the life of Maxwell.[19] His formal schooling began unsuccessfully under the guidance of a sixteen-year-old hired tutor. Little is known about the young man John Maxwell hired to instruct his son, except that he treated the younger boy harshly, chiding him for being slow and wayward.[19] John Maxwell dismissed the tutor in November 1841, and after considerable thought, sent James to the prestigious Edinburgh Academy.[20] He lodged during term times at the house of his aunt Isabella. During this time his passion for drawing was encouraged by his older cousin Jemima, who was herself a talented artist.[21]

Edinburgh Academy, where Maxwell was schooled

The ten-year-old Maxwell, having been raised in isolation on his father's countryside estate, did not fit in well at school.[22] The first year had been full, obliging him to join the second year with classmates a year his senior.[22] His mannerisms and Galloway accent struck the other boys as rustic, and his having arrived on his first day of school wearing a pair of homemade shoes and a tunic, earned him the unkind nickname of "Daftie".[23] Maxwell, however, never seemed to have resented the epithet, bearing it without complaint for many years.[24] Social isolation at the Academy ended when he met Lewis Campbell and Peter Guthrie Tait, two boys of a similar age who were to become notable scholars later in life. They would remain lifetime friends.[10]

Maxwell was fascinated by geometry at an early age, rediscovering the regular polyhedron before any formal instruction.[21] Much of his talent however, went overlooked, and despite winning the school's scripture biography prize in his second year his academic work remained unnoticed[21] until, at the age of 13, he won the school's mathematical medal and first prize for both English and poetry.[25]

Maxwell wrote his first scientific paper at the age of 14. In it he described a mechanical means of drawing mathematical curves with a piece of twine, and the properties of ellipses, Cartesian ovals, and related curves with more than two foci. His work, *Oval Curves*, was presented to the Royal Society of Edinburgh by James Forbes, who was a professor of natural philosophy at Edinburgh University.[10] [26] Maxwell was deemed too young for the work presented.[27] The work was not entirely original, since Descartes had also examined the properties of such multifocal curves in the seventeenth century, but Maxwell had simplified their construction.[27]

Edinburgh University, 1847–50

Edinburgh University

Maxwell left the Academy in 1847 at the age of 16 and began attending classes at the University of Edinburgh.[28] Having had the opportunity to attend the University of Cambridge after his first term Maxwell instead decided to complete the full course of his undergraduate studies at Edinburgh. The academic staff of Edinburgh University included some highly regarded names, and Maxwell's first year tutors included Sir William Hamilton, who lectured him on logic and metaphysics, Philip Kelland on mathematics, and James Forbes on natural philosophy.[10] Maxwell, however, did not find his classes at Edinburgh University very demanding,[29] and was therefore able to immerse himself in private study during free time at the university, and particularly when back home at Glenlair.[30] There he would experiment with improvised chemical, electric, and magnetic apparatuses, but his chief concerns regarded the properties of polarized light.[31] He constructed shaped blocks of gelatine, subjected them to various stresses, and with a pair of polarizing prisms given to him by the famous scientist William Nicol he would view the coloured fringes which had developed within the jelly.[32] Through this practice Maxwell discovered photoelasticity, which is a means of determining the stress distribution within physical structures.[33]

Maxwell contributed two papers for the Transactions of the Royal Society of Edinburgh at the age of 18. One of these, *On the equilibrium of elastic solids*, laid the foundation for an important discovery later in his life, which was the temporary double refraction produced in viscous liquids by shear stress.[34] His other paper was titled *Rolling curves*, and just as with the paper *Oval Curves* that he had written at the Edinburgh Academy, Maxwell was again considered too young to stand at the rostrum and present it himself. The paper was delivered to the Royal Society by his tutor Kelland instead.[35]

Cambridge University, 1850–56

A young Maxwell at Trinity College, Cambridge. He is holding one of his colour wheels.

In October 1850, already an accomplished mathematician, Maxwell left Scotland for Cambridge University.[36] He initially attended Peterhouse, but before the end of his first term transferred to Trinity College, where he believed it would be easier to obtain a fellowship.[37] At Trinity, he was elected to the elite secret society known as the Cambridge Apostles.[38] In November 1851, Maxwell studied under William Hopkins, whose success in nurturing mathematical genius had earned him the nickname of "senior wrangler-maker".[39] A considerable part of Maxwell's translation of his equations regarding electromagnetism was accomplished during his time at Trinity.

In 1854, Maxwell graduated from Trinity with a degree in mathematics. He scored second highest in the final examination, coming behind Edward Routh, and thereby earning himself the title of Second Wrangler. He was later declared equal with Routh, however, in the more exacting ordeal of the Smith's Prize examination.[40] Immediately after earning his degree, Maxwell read a novel paper to the Cambridge Philosophical Society entitled *On the transformation of surfaces by bending*.[41] This is one of the few purely mathematical papers he had written, and it demonstrated Maxwell's growing stature as a mathematician.[42] Maxwell decided to remain at Trinity after graduating and applied for a fellowship, which was a process that he could expect to take a couple of years.[43]

Buoyed by his success as a research student, he would be free, aside from some tutoring and examining duties, to pursue scientific interests at his own leisure.[43]

The nature and perception of colour was one such interest, and had begun at Edinburgh University while he was a student of Forbes.[44] Maxwell took the coloured spinning tops invented by Forbes, and was able to demonstrate that white light would result from a mixture of red, green and blue light.[44] His paper, *Experiments on colour*, laid out the principles of colour combination, and was presented to the Royal Society of Edinburgh in March 1855.[45] Fortunately for Maxwell this time it would be he himself who delivered his lecture.[45]

Maxwell was made a fellow of Trinity on 10 October 1855, sooner than was the norm,[45] and was asked to prepare lectures on hydrostatics and optics, and to set examination papers.[46] However, the following February he was urged by Forbes to apply for the newly vacant Chair of Natural Philosophy at Marischal College, Aberdeen.[47] His father assisted him in the task of preparing the necessary references, but he would die on 2 April, at Glenlair before either knew the result of Maxwell's candidacy.[47] Maxwell nevertheless accepted the professorship at Aberdeen, leaving Cambridge in November 1856.[46]

Aberdeen University, 1856–60

The 25-year-old Maxwell was a decade and a half younger than any other professor at Marischal, but engaged himself with his new responsibilities as head of department, devising the syllabus and preparing lectures.[48] He committed himself to lecturing 15 hours a week, including a weekly *pro bono* lecture to the local working men's college.[48] He lived in Aberdeen during the six months of the academic year, and spent the summers at Glenlair, which he had inherited from his father.

James and Katherine Maxwell, 1869.

His mind was focused on a problem that had eluded scientists for two hundred years: the nature of Saturn's rings. It was unknown how they could remain stable without breaking up, drifting away or crashing into Saturn. The problem took on a particular resonance at this time as St John's College, Cambridge had chosen it as the topic for the 1857 Adams Prize.[49] Maxwell devoted two years to studying the problem, proving that a regular solid ring could not be stable, and a fluid ring would be forced by wave action to break up into blobs. Since neither was observed, Maxwell concluded that the rings must comprise numerous small particles he called "brick-bats", each independently orbiting Saturn.[49] Maxwell was awarded the £130 Adams Prize in 1859 for his essay *On the stability of Saturn's rings*; he was the only entrant to have made enough headway to submit an entry.[50] His work was so detailed and convincing that when George Biddell Airy read it he commented "It is one of the most remarkable applications of mathematics to physics that I have ever seen."[51] It was considered the final word on the issue until direct observations by the *Voyager* flybys of the 1980s confirmed Maxwell's prediction. Maxwell would also go on to disprove mathematically the nebular hypothesis (which stated that the solar system formed through the progressive condensation of a purely gaseous nebula), forcing the theory to account for additional portions of small solid particles.

In 1857 Maxwell befriended the Reverend Daniel Dewar, who was the Principal of Marischal, and through him met Dewar's daughter, Katherine Mary Dewar. They were engaged in February 1858 and married in Aberdeen on 2 June 1859. Seven years Maxwell's senior, comparatively little is known of Katherine although it is known that she helped in his lab and worked on experiments in viscosity.[52] Maxwell's biographer and friend Campbell adopted an uncharacteristic reticence on the subject of Katherine, though describing their married life as "one of unexampled devotion".[53]

In 1860, Marischal College merged with the neighbouring King's College to form the University of Aberdeen. There was no room for two professors of Natural Philosophy, and Maxwell, despite his scientific reputation, found himself laid off. He was unsuccessful in applying for Forbes' recently vacated chair at Edinburgh, the post instead going to Tait. Maxwell was granted the Chair of Natural Philosophy at King's College London instead.[54] After recovering from a near-fatal bout of smallpox in the summer of 1860, Maxwell headed south to London with his wife Katherine.[55]

King's College London, 1860–65

Maxwell's time at King's was probably the most productive of his career. He was awarded the Royal Society's Rumford Medal in 1860 for his work on colour, and was later elected to the Society in 1861.[56] This period of his life would see him display the world's first light-fast colour photograph, further develop his ideas on the viscosity of gases, and propose a system of defining physical quantities—now known as dimensional analysis. Maxwell would often attend lectures at the Royal Institution, where he came into regular contact with Michael Faraday. The relationship between the two men could not be described as close, as Faraday was 40 years Maxwell's senior and showed signs of senility. They nevertheless maintained a strong respect for each other's talents.[57]

This time is especially known for the advances Maxwell made in the fields of electricity and magnetism. He had examined the nature of both electric and magnetic fields in his two-part paper *On physical lines of force*, published in 1861, in which he had provided a conceptual model for electromagnetic induction, consisting of tiny spinning cells of magnetic flux. Two more parts later added to the paper were published in early 1862. In the first of these he discussed the nature of electrostatics and displacement current. The final part dealt with the rotation of the plane of polarization of light in a magnetic field, a phenomenon discovered by Faraday and now known as the Faraday effect.[58]

Later years

In 1865, Maxwell resigned the chair at King's College London and returned to Glenlair with Katherine.

He wrote a textbook entitled *Theory of Heat* (1871), and an elementary treatise, *Matter and Motion* (1876). Maxwell was also the first to make explicit use of dimensional analysis, in 1871.

In 1871, he became the first Cavendish Professor of Physics at Cambridge. Maxwell was put in charge of the development of the Cavendish Laboratory. He supervised every step in the progress of the building and of the purchase of the very valuable collection of apparatus paid for by its generous founder, the 7th Duke of Devonshire (chancellor of the university, and one of its most distinguished alumni). One of Maxwell's last great contributions to science was the editing (with copious original notes) of the electrical researches of Henry Cavendish, from which it appeared that Cavendish researched, amongst other things, such questions as the mean density of the earth and the composition of water.

Maxwell's gravestone at Parton

He died in Cambridge of abdominal cancer on 5 November 1879 at the age of 48.[28] His mother had died at the same age of the same type of cancer. Maxwell is buried at Parton Kirk, near Castle Douglas in Galloway, Scotland. The extended biography *The Life of James Clerk Maxwell*, by his former schoolfellow and lifelong friend Professor Lewis Campbell, was published in 1882. His collected works, including the series of articles on the properties of matter, such as "Atom", "Attraction", "Capillary action", "Diffusion", "Ether", etc., were issued in two volumes by the Cambridge University Press in 1890.

Personality

As a great lover of Scottish poetry, Maxwell memorised poems and wrote his own.[59] The best known is *Rigid Body Sings*, closely based on *Comin' Through the Rye* by Robert Burns, which he apparently used to sing while accompanying himself on a guitar. It has the opening lines[60]

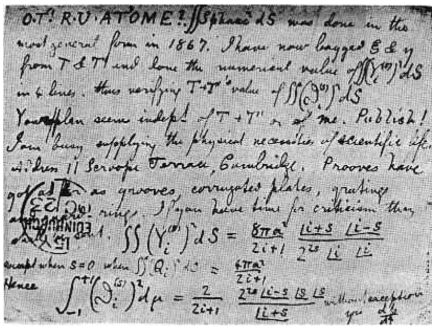
> Gin a body meet a body
> Flyin' through the air.
> Gin a body hit a body,
> Will it fly? And where?

A collection of his poems was published by his friend Lewis Campbell in 1882. Many appreciations of Maxwell remark upon his remarkable intellectual qualities being matched by social awkwardness.

Ivan Tolstoy, author of one of Maxwell's biographies, has noted the frequency with which scientists writing short biographies of Maxwell omit the subject of his Christianity. He was an evangelical Presbyterian, and in his later years became an Elder of the Church of Scotland.[61] Maxwell's religious beliefs and related activities have been the focus of several peer-reviewed and well-referenced papers.[62] [63] [64] [65] Attending both Church of Scotland (his father's denomination) and Episcopalian (his mother's denomination) services as a child, Maxwell later underwent an evangelical conversion in April 1853, which committed him to an anti-positivist position.[64]

Contributions

Electromagnetism


O.T.! R.U. ATOME? ∬ [illegible] dS was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(s)})^2 dS$ in 4 lines, thus verifying T+T' value of $\iint (\vartheta_i^{(s)})^2 dS$. Your plan seems indep^t of T+T' or of me. Publish! I am busy supplying the physical necessities of scientific life. Address 11 Scroope Terrace, Cambridge. [illegible] as grooves, corrugated plates, gratings, rings. If you have time for criticism then [illegible]

$$\iint (Y_i^{(s)})^2 dS = \frac{8\pi a^2}{2i+1}\,\frac{\lfloor i+s \;\lfloor i-s}{2^{2s}\;\lfloor i\;\lfloor i}$$

except when s=0 when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

Hence

$$\int_{-1}^{+1} (\vartheta_i^{(s)})^2 d\mu = \frac{2}{2i+1}\,\frac{2^{2s}\lfloor i-s\;\lfloor s\;\lfloor s}{\lfloor i+s}$$ without exception [illegible]

A postcard from Maxwell to Peter Tait.

Maxwell had studied and commented on the field of electricity and magnetism as early as 1855/6 when "On Faraday's lines of force" was read to the Cambridge Philosophical Society. The paper presented a simplified model of Faraday's work, and how the two phenomena were related. He reduced all of the current knowledge into a linked set of differential equations with 20 equations in 20 variables. This work was later published as "On physical lines of force" in March 1861.[66]

Around 1862, while lecturing at King's College, Maxwell calculated that the speed of propagation of an electromagnetic field is approximately that of the speed of light. He considered this to be more than just a coincidence, and commented "We can scarcely avoid the conclusion that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."[51]

Working on the problem further, Maxwell showed that the equations predict the existence of waves of oscillating electric and magnetic fields that travel through empty space at a speed that could be predicted from simple electrical experiments; using the data available at the time, Maxwell obtained a velocity of 310,740,000 m/s. In his 1864 paper "A dynamical theory of the electromagnetic field", Maxwell wrote, "The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws".[5]

His famous equations, in their modern form of four partial differential equations, first appeared in fully developed form in his textbook *A Treatise on Electricity and Magnetism* in 1873. Most of this work was done by Maxwell at Glenlair during the period between holding his London post and his taking up the Cavendish chair.[51] Maxwell expressed electromagnetism in the algebra of quaternions and made the electromagnetic potential the centerpiece of his theory. In 1881 Oliver Heaviside replaced Maxwell’s electromagnetic potential field by ‘force fields’ as the centerpiece of electromagnetic theory. Heaviside reduced the complexity of Maxwell’s theory down to four differential equations, known now collectively as Maxwell's Laws or Maxwell's equations. According to Heaviside, the electromagnetic potential field was arbitrary and needed to be "murdered".[67] However, the use of scalar and vector potentials is now standard in the solution of Maxwell's equations.[68]

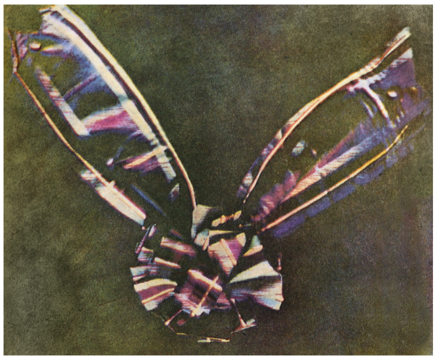
A few years later there was a great debate between Heaviside and Peter Guthrie Tait about the relative merits of vector analysis and quaternions. The result was the realization that there was no need for the greater physical insights provided by quaternions if the theory was purely local, and vector analysis became commonplace.[69]

Maxwell was proven correct, and his quantitative connection between light and electromagnetism is considered one of the great accomplishments of 19th century mathematical physics.

Maxwell also introduced the concept of the *electromagnetic field* in comparison to force lines that Faraday discovered. By understanding the propagation of electromagnetism as a field emitted by active particles, Maxwell could advance his work on light. At that time, Maxwell believed that the propagation of light required a medium for the waves, dubbed the luminiferous aether. Over time, the existence of such a medium, permeating all space and yet apparently undetectable by mechanical means, proved more and more difficult to reconcile with experiments such as the Michelson–Morley experiment. Moreover, it seemed to require an absolute frame of reference in which the equations were valid, with the distasteful result that the equations changed form for a moving observer. These difficulties inspired Albert Einstein to formulate the theory of special relativity, and in the process Einstein dispensed with the requirement of a luminiferous aether.

Colour analysis

Maxwell contributed to the field of optics and the study of colour vision, creating the foundation for practical colour photography.



The first permanent colour photograph, taken by James Clerk Maxwell in 1861.

From 1855 to 1872, he published at intervals a series of valuable investigations concerning the perception of colour, colour-blindness and colour theory, for the earlier of which the Royal Society awarded him the Rumford Medal. The instruments which he devised for these investigations were simple and convenient to use. For example, Maxwell's discs were used to compare a variable mixture of three primary colours with a sample colour by observing the spinning "colour top."

In the course of his 1855 paper on the perception of colour, Maxwell proposed that if three black-and-white photographs of a scene were taken through red, green and violet filters, and transparent prints of the images were projected onto a screen using three projectors equipped with similar filters, when superimposed on the screen the result would be perceived by the human eye as a complete reproduction of all the colours in the scene.[70]

During an 1861 Royal Institution lecture on colour theory, Maxwell presented the world's first demonstration of colour photography by this principle of three-colour analysis and synthesis, the basis of nearly all subsequent photochemical and electronic methods of colour photography. Thomas Sutton, inventor of the single-lens reflex camera, did the actual picture-taking. He photographed a tartan ribbon three times, through red, green and blue filters. He also made a fourth exposure through a yellow filter, but according to Maxwell's account this was not used in the demonstration. Because Sutton's photographic plates were in fact insensitive to red and barely sensitive to green, the results of this pioneering experiment were far from perfect. It was remarked in the published account of the lecture that "if the red and green images had been as fully photographed as the blue," it "would have been a truly-coloured image of the riband. By finding photographic materials more sensitive to the less refrangible rays, the representation of the colours of objects might be greatly improved."[56] [71] [72]

Researchers in 1961 concluded that the seemingly impossible partial success of the red-filtered exposure was due to ultraviolet light. Some red dyes strongly reflect it, the red filter used does not entirely block it, and Sutton's plates were sensitive to it.[73]

The demonstration was not of a print or transparency containing tangible colouring matter, but of colour which was photographically recorded from nature and reproduced by the same additive colour synthesis principle now used by all common types of colour video displays. Maxwell's purpose was not to present a method of colour photography, but to illustrate the basis of human colour perception and to show that the correct additive primaries are not red, yellow and blue, as was then taught, but red, green and blue.

The three photographic plates now reside in a small museum at 14 India Street, Edinburgh, the house where Maxwell was born.

Kinetic theory and thermodynamics

Maxwell also investigated the kinetic theory of gases. Originating with Daniel Bernoulli, this theory was advanced by the successive labours of John Herapath, John James Waterston, James Joule, and particularly Rudolf Clausius, to such an extent as to put its general accuracy beyond a doubt; but it received enormous development from Maxwell, who in this field appeared as an experimenter (on the laws of gaseous friction) as well as a mathematician.

In 1866, he formulated statistically, independently of Ludwig Boltzmann, the Maxwell–Boltzmann kinetic theory of gases. His formula, called the Maxwell distribution, gives the fraction of gas molecules moving at a specified velocity at any given temperature. In the kinetic theory, temperatures and heat involve only molecular movement. This approach generalized the previously established laws of thermodynamics and explained existing observations and experiments in a better way than had been achieved previously. Maxwell's work on thermodynamics led him to devise the *Gedankenexperiment* (thought experiment) that came to be known as Maxwell's demon.

In 1871, he established Maxwell's thermodynamic relations, which are statements of equality among the second derivatives of the thermodynamic potentials with respect to different thermodynamic variables. In 1874, he constructed a plaster thermodynamic visualisation as a way of exploring phase transitions, based on the American scientist Josiah Willard Gibbs's graphical thermodynamics papers.

Control theory

Maxwell published a famous paper "On governors" in the *Proceedings of Royal Society*, vol. 16 (1867–1868). This paper is quite frequently considered a classical paper of the early days of control theory. Here governors refer to the governor or the centrifugal governor used in steam engines.

Legacy

Maxwell was ranked 91st on the BBC poll of the 100 Greatest Britons. His name is honoured in a number of ways:

Maxwell statue in Edinburgh

- The maxwell (Mx), a compound derived CGS unit measuring magnetic flux.
- Maxwell Montes, a mountain range on Venus, one of only three features on the planet that are not given female names.
- The Maxwell Gap in the Rings of Saturn.
- The James Clerk Maxwell Telescope, the largest submillimetre-wavelength astronomical telescope in the world, with a diameter of 15 metres.
- The 1977 James Clerk Maxwell Building of the University of Edinburgh, housing the schools of mathematics, physics and meteorology.
- The James Clerk Maxwell building at the Waterloo campus of King's College London, in commemoration of his time as Professor of Natural Philosophy at King's from 1860 to 1865. The university also has a chair in Physics named after him, and a society for undergraduate physicists.
- The £4 million James Clerk Maxwell Centre of the Edinburgh Academy was opened in 2006 to mark his 175th anniversary.
- James Clerk Maxwell Road in Cambridge, which runs beside the Cavendish Laboratory.
- The University of Salford's main building is named after him.
- Maxwell bridge, a bridge circuit involving resistors, a capacitor and an inductor
- A statue on Edinburgh's George Street[74]
- A street in Aberdeen's Kincorth area is named after him [75]
- Thomas Pynchon, an American novelist, alludes to and explains Maxwell's demon in The Crying of Lot 49.
- P J Moore, keyboard player with The Blue Nile is developing a theatre piece based on the life of J.C.M.

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External links

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- The James Clerk Maxwell Foundation (http://www.clerkmaxwellfoundation.org/) Including a virtual tour of the museum.
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- James Clerk Maxwell's grave (http://local.upmystreet.com/picture-of-james-clerk-maxwell-grave-id-366375.html)

Albert Abraham Michelson

Albert Abraham Michelson	
	
Born	December 19, 1852 Strzelno, Kingdom of Prussia
Died	May 9, 1931 (aged 78) Pasadena, California
Nationality	United States
Fields	Physics
Institutions	Case Western Reserve University Clark University University of Chicago
Alma mater	United States Naval Academy University of Berlin
Doctoral advisor	Hermann Helmholtz
Doctoral students	Robert Millikan
Known for	Speed of light Michelson-Morley experiment
Notable awards	Nobel Prize for Physics (1907) Copley Medal (1907) Henry Draper Medal (1916)
Signature 	

Albert Abraham Michelson (December 19, 1852 – May 9, 1931) was an American physicist known for his work on the measurement of the speed of light and especially for the Michelson-Morley experiment. In 1907 he received the Nobel Prize in Physics. He became the first American to receive the Nobel Prize in sciences.

Biography

Michelson was born in Strzelno, Provinz Posen in the Kingdom of Prussia (now Poland) into a Jewish family.[1] He moved to the US with his parents in 1855, at the age of two. He grew up in the mining towns of Murphy's Camp, California and Virginia City, Nevada, where his father was a merchant. He spent his high school years in San Francisco in the home of his aunt, Henriette Levy (née Michelson), who was the mother of author Harriet Lane Levy.[2]

President Ulysses S. Grant awarded Michelson a special appointment to the U.S. Naval Academy in 1869[3] . During his four years as a midshipman at the Academy[4] , Michelson excelled in optics, heat, climatology and drawing. After graduating in 1873 and two years at sea, he returned to the Naval Academy in 1875 to become an instructor in physics and chemistry until 1879. In 1879, he was posted to the Nautical Almanac Office, Washington (part of the United States Naval Observatory[5] [6] [7]), to work with Simon Newcomb. In the following year he obtained leave of absence to continue his studies in Europe. He visited the Universities of Berlin and Heidelberg, and the Collège de France and École Polytechnique in Paris.

Michelson was fascinated with the sciences, and the problem of measuring the speed of light in particular. While at Annapolis, he conducted his first experiments of the speed of light, as part of a class demonstration in 1877. His Annapolis experiment was refined, and in 1879, he measured the speed of light in air to be 299,864±51 kilometres per second, and estimated the speed of light in vacuum as 299,940 km/s, or 186,380 mps[8] [9] [10] . After two years of studies in Europe, he resigned from the Navy in 1881. In 1883 he accepted a position as professor of physics at the Case School of Applied Science in Cleveland, Ohio and concentrated on developing an improved interferometer. In 1887 he and Edward Morley carried out the famous Michelson-Morley experiment which seemed to rule out the existence of the aether. He later moved on to use astronomical interferometers in the measurement of stellar diameters and in measuring the separations of binary stars.

In 1889 Michelson became a professor at Clark University at Worcester, Massachusetts and in 1892 was appointed professor and the first head of the department of physics at the newly organized University of Chicago.

In 1899, he married Edna Stanton. They raised one son and three daughters.

In 1907, Michelson had the honor of being the first American to receive a Nobel Prize in Physics "for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid". He also won the Copley Medal in 1907, the Henry Draper Medal in 1916 and the Gold Medal of the Royal Astronomical Society in 1923. A crater on the Moon is named after him.

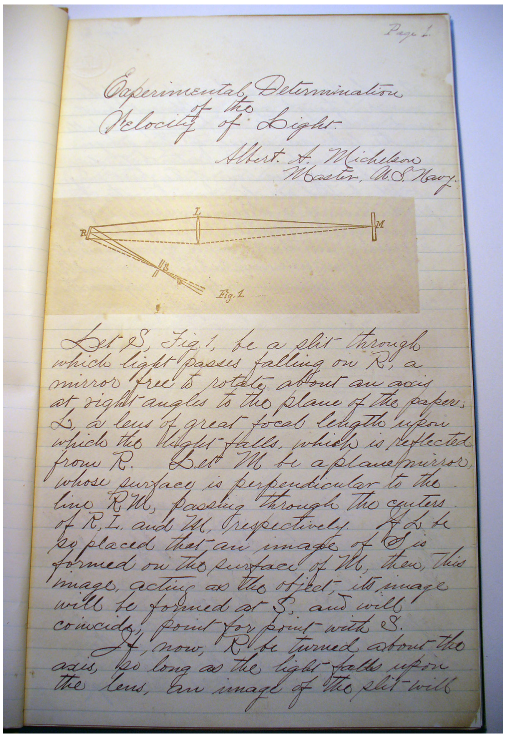
Michelson died in Pasadena, California at the age of 78. The University of Chicago Residence Halls remembered Michelson and his achievements by dedicating 'Michelson House' in his honor. Case Western Reserve has dedicated a Michelson House to him, and the chemistry academic building at the United States Naval Academy also bears his name. Clark University named a theatre after him.[11] Michelson Laboratory at Naval Air Weapons Station China Lake in Ridgecrest, California is named for him. There is a display in the publicly accessible area of the Lab which includes facsimiles of Michelson's Nobel Prize medal, the prize document, and examples of his diffraction gratings.

Speed of light

Early measurements

As early as 1877, while still serving as an officer in the United States Navy, Michelson started planning a refinement of the rotating-mirror method of Léon Foucault for measuring the speed of light, using improved optics and a longer baseline. He conducted some preliminary measurements using largely improvised equipment in 1878, about the same time that his work came to the attention of Simon Newcomb, director of the Nautical Almanac Office who was already advanced in planning his own study. Michelson published his result of 299,910±50 km/s in 1879 before joining Newcomb in Washington DC to assist with his measurements there. Thus began a long professional collaboration and friendship between the two.

Simon Newcomb, with his more adequately funded project, obtained a value of 299,860±30 km/s, just at the extreme edge of consistency with Michelson's. Michelson continued to "refine" his method and in 1883 published a measurement of 299,853±60 km/s, rather closer to that of his mentor.


Page 1.

Experimental Determination of the Velocity of Light.

Albert A. Michelson
Master, U.S. Navy.

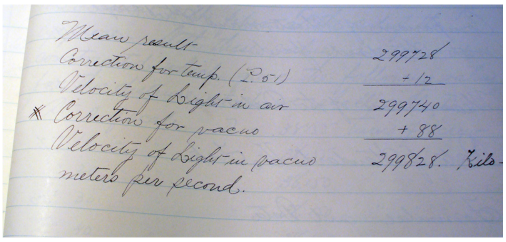
Let S, Fig. 1, be a slit through which light passes falling on R, a mirror free to rotate about an axis at right angles to the plane of the paper; L, a lens of great focal length upon which the light falls, which is reflected from R. Let M be a plane mirror, whose surface is perpendicular to the line RM, passing through the centers of R, L, and M, respectively. If L be so placed that an image of S is formed on the surface of M, then, this image, acting as the object, its image will be formed at S, and will coincide, point for point with S.

If, now, R be turned about the axis, so long as the light falls upon the lens, an image of the slit will

Page one of Michelson's *Experimental Determination of the Velocity of Light*

Mount Wilson and Lookout Mountain

In 1906, a novel electrical method was used by E. B. Rosa and N. E. Dorsey of the National Bureau of Standards to obtain a value for the speed of light of 299,781±10 km/s. Though this result has subsequently been shown to be severely biased by the poor electrical standards in use at the time, it seems to have set a fashion for rather lower measured values.


Mean result	299728
Correction for temp. (P. 51)	+ 12
Velocity of Light in air	299740
* Correction for vacuo	+ 88
Velocity of Light in vacuo	299828. Kilo-
meters per second.	

Concluding page of Michelson's *Experimental Determination of the Velocity of Light*

From 1920, Michelson started planning a *definitive* measurement from the Mount Wilson Observatory, using a baseline to Lookout Mountain, a prominent bump on the south ridge of Mount San Antonio (*Old Baldy*), some 22 miles distant.

In 1922, the U.S. Coast and Geodetic Survey began two years of painstaking measurement of the baseline using the recently available invar tapes. With the baseline length established in 1924, measurements were carried out over the next two years to obtain the published value of 299,796±4 km/s.[12]

Famous as the measurement is, it was beset by problems, not least of which was the haze created by the smoke from forest fires which blurred the mirror image. It is also probable that the intensively detailed work of the geodetic survey, with an estimated error of less than one part in 1 million, was compromised by a shift in the baseline arising from the Santa Barbara earthquake of June 29, 1925, which was an estimated magnitude of 6.3 on the Richter scale.

The now-famous Michelson-Morley Experiment also influenced the affirmation attempts of peer Albert Einstein's theory of general relativity and special relativity, using similar optical instrumentation. These instruments and related collaborations included the participations of fellow physicists, Dayton Miller, Hendrick Lorentz, and Robert Shankland.

Michelson, Pease & Pearson

The period after 1927 marked the advent of new measurements of the speed of light using novel electro-optic devices, all substantially lower than Michelson's 1926 value.

Michelson sought another measurement, but this time in an evacuated tube to avoid difficulties in interpreting the image owing to atmospheric effects. In 1930, he began a collaboration with Francis G. Pease and Fred Pearson to perform a measurement in a 1.6 km tube at Pasadena, California. Michelson died with only 36 of the 233 measurement series completed and the experiment was subsequently beset by geological instability and condensation problems before the result of 299,774±11 km/s, consistent with the prevailing electro-optic values, was published posthumously in 1935.

Interferometry

In 1887 he collaborated with colleague Edward Williams Morley of Western Reserve College, now part of Case Western Reserve University, in the Michelson-Morley experiment. Their experiment for the expected motion of the Earth relative to the aether, the hypothetical medium in which light was supposed to travel, resulted in a null result. Surprised, Michelson repeated the experiment with greater and greater precision over the next years, but continued to find no ability to measure the aether. The Michelson-Morley results were immensely influential in the physics community, leading Hendrik Lorentz to devise his now-famous Lorentz contraction equations as a means of explaining the null result.

There has been some historical controversy over whether Albert Einstein was aware of the Michelson-Morley results when he developed his theory of special relativity, which pronounced the aether to be "superfluous". Regardless of Einstein's specific knowledge, the experiment is today considered the canonical experiment in regards to showing the lack of a detectable aether.[13] [14]

Astronomical interferometry

From 1920 and into 1921 Michelson and Francis G. Pease became the first individuals to measure the diameter of a star other than the Sun. They used an astronomical interferometer at the Mount Wilson Observatory to measure the diameter of the super-giant star Betelgeuse. A periscope arrangement was used to obtain a densified pupil in the interferometer, a method later investigated in detail by Antoine Émile Henry Labeyrie for use in "Hypertelescopes". The measurement of stellar diameters and the separations of binary stars took up an increasing amount of Michelson's life after this.

A century later, the specific interferometer instrumentation design produced by Albert Michelson has become the principle means to conduct astronomical interferometry. The "Michelson Interferometer" design is found on modern operational observatories such as VLTI, CHARA -- and the U.S. Navy's NPOI.

Michelson in popular culture

In an episode of the television series *Bonanza* (*Look to the Stars*, broadcast March 18, 1962), Ben Cartwright (Lorne Greene) helps the 16-year-old Albert Abraham Michelson (portrayed by 25-year-old Douglas Lambert (1936–1986)) obtain an appointment to the U.S. Naval Academy, despite the opposition of the bigoted town schoolteacher (played by William Schallert). *Bonanza* was set in and around Virginia City, Nevada, where Michelson lived with his parents prior to leaving for the Naval Academy. In a voice-over at the end of the episode, Greene mentions Michelson's 1907 Nobel Prize.

The home in which Michelson lived as a child in Murphys Camp, California is now a tasting room for Twisted Oak Winery.

New Beast Theater Works in collaboration with High Concept Laboratories produced a "semi-opera" about Michelson, his obsessive work style and its effect on his family life which ran from February 11 to February 26, 2011 in Chicago at The Building Stage. Michelson was portrayed by Jon Stutzman. The play was directed by Davaid Maral with music composed by Joshua Dumas.

Honors and awards

- Nobel Prize for Physics (1907)
- Rumford Prize (1888)
- Matteucci Medal (1903)
- Copley Medal (1907)
- Elliott Cresson Medal (1912)
- Henry Draper Medal from the National Academy of Sciences (1916)[15]
- Gold Medal of the Royal Astronomical Society (1923)
- Franklin Medal (1923)

A monument at United States Naval Academy marks the path of Michelson's experiments measuring the speed of light.

Michaelsen was a member of the Royal Society, the National Academy of Sciences, the American Physical Society and the American Association for the Advancement of Science.

The Computer Measurement Group gives an annual A. A. Michelson award.

Notes

[1] http://www.aip.org/history/gap/Michelson/Michelson.html

[2] Levy, *920 O'Farrell Street*, 47.

[3] http://www.usna.edu/LibExhibits/collections/michelson/

[4] http://en.wikipedia.org/wiki/List_of_United_States_Naval_Academy_alumni#Nobel_laureates

[5] http://adsabs.harvard.edu/abs/2002JAHH....5..165S

[6] http://www.usno.navy.mil/USNO/about-us/usno-command-history

[7] http://eprints.jcu.edu.au/4957/1/4957_Shankland%26Orchiston_2002.pdf

[8] http://www.raman-scattering.eu/raman/texts/009_menu_vitesse.php

[9] http://www.opticsinfobase.org/on/abstract.cfm?uri=on-4-4-14

[10] http://sas.uwaterloo.ca/~rwoldfor/papers/sci-method/paperrev/node6.html

[11] http://www.clarku.edu/departments/clarkarts/facilities/littlecenter.cfm

[12] Garner, C. L., Captain (retired) (April 1949). "A Geodetic Measurement of Unusually High Accuracy" (http://www.pvaa.us/nightwatch/GeodeticMeasurementOfUnusuallyHighAccuracy.pdf). Coast and Geodetic Survey. pp. 68–74. . Retrieved August 13, 2009.

[13] Note that while Einstein's 1905 paper *On the Electrodynamics of Moving Bodies* appears to reference the experiment on first glance—"together with the unsuccessful attempts to discover any motion of the earth relatively to the 'light medium,' suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest"—it has been shown that Einstein was referring to a different category of experiments here.

[14] Holton, Gerald, "Einstein, Michelson, and the 'Crucial' Experiment", *Isis*, Vol. 60, No. 2 (Summer, 1969), pp. 133–197

[15] "Henry Draper Medal" (http://www.nasonline.org/site/PageServer?pagename=AWARDS_draper). National Academy of Sciences. . Retrieved 19 February 2011.

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- Michelson's Life and Works from the American Institute of Physics (http://www.aip.org/history/gap/Michelson/Michelson.html)
- U. S. Naval Academy and The Navy (http://www.usna.edu/LibExhibits/Michelson/Michelson_navy.html)
- USNA Guide to the Albert A. Michelson Collection, 1803-1989 (http://www.usna.edu/library/sca/findingaids/michelson/index.html)
- From USNA to Nobel: Albert A. Michelson's Life and Contributions (http://www.usna.com/Document.Doc?id=617)
- Michelson House at the University of Chicago (http://michelson-house.uchicago.edu)
- Michelson's Nobel Prize Biography (http://nobelprize.org/nobel_prizes/physics/laureates/1907/michelson-bio.html)
- Works by Albert Abraham Michelson (http://www.gutenberg.org/author/Albert_A._Michelson) at Project Gutenberg
- *Experimental Determination of the Velocity of Light* (http://www.gutenberg.net/browse/BIBREC/BR11753.HTM)
- IMDB: Bonanza episode *Look to the Stars* (http://www.imdb.com/title/tt0529603/)
- Norman Maclean: *"Billiards Is a Good Game": Gamesmanship and America's First Nobel Prize Scientist*; reprinted in Lapham's Quarterly (http://www.laphamsquarterly.org/voices-in-time/norman-maclean-learns-from-the-master.php?page=all)
- The U.S. Naval Academy Observatory Programs and Times Gone By: A Tale of Two Domes (http://www.europa.com/~telscope/usna.doc)
- NAWS China Lake (http://www.navair.navy.mil/nawcwd/nawcwd/news/2008/2008-02_michelson.htm)
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Edward Morley

Edward Williams Morley	
	
Born	January 29, 1838 Newark, New Jersey
Died	February 24, 1923 (aged 85) West Hartford, Connecticut
Institutions	Western Reserve College
Alma mater	Williams College
Known for	Michelson-Morley experiment
Notable awards	Elliott Cresson Medal (1912) Davy Medal (1907) Willard Gibbs Medal (1899)

Edward Williams Morley (January 29, 1838 – February 24, 1923) was an American scientist famous for the Michelson–Morley experiment.

Biography

Morley was born in Newark, New Jersey to Anna Clarissa Treat and the Reverend Sardis Brewster Morley. Both parents were of early colonial ancestry and of purely British origin. Morely grew up in West Hartford, Connecticut. During his childhood, he suffered much from ill health and was therefore educated by his father at home until the age of nineteen.[1]

In 1857, Morley entered Williams College at Williamstown, Massachusetts, his father's alma mater. He graduated in 1860 as A.B., and in 1863 received his master's degree. Around 1960 he gradually shifted his attention from chemistry, which fascinated him since he was child, to optics and astronomy. In 1860–61 he mounted a transit instrument, constructed a chronograph, and made the first accurate determination of the latitude of the college observatory. This determination was the subject of his first published paper, which was read before the American Association for the Advancement of Science in 1866.[1]

Upon advise of his parents, Morley entered Andover Theological Seminary in 1861 and completed the course in 1864. It was here, probably, that he acquired a good working knowledge of Hebrew. From 1866 to 1868 he was a teacher in a private school, and later, in 1868, he was called to preach in a small country parish in Ohio. At about the same time, he was appointed professor of chemistry in Western Reserve College (then situated at Hudson, Ohio and later moved to Cleveland and renamed Case Western Reserve University), where he remained until his retirement in 1906. This appointment was the turning point in his career. In 1873 he also became professor of chemistry in Cleveland Medical College, but resigned this chair in 1888 to have more time for research. Just before moving to Hudson he married Miss Imbella A. Birdsall.[1]

During his residence in Cleveland, Morley assembled one of the best private collections of chemical periodicals in America. He even included Russian journals and learned enough of the Russian language to use them. After his retirement from teaching, the university purchased his library and relocated to the chemical laboratory named after him. In 1906, Morley moved to West Hartford, Connecticut, where he built a small house and a laboratory for his personal studies of rocks and minerals.[1]

Morley was not a voluminous writer and published only 55 articles. He outlived his wife by only a few months, and died, following a surgical operation, in the Hartford Hospital in 1923.[1]

Research

Optics and astronomy

Morley's most-significant work came in the field of physics and optics. In this, Morley collaborated with and assisted the physicist Albert A. Michelson for several years around 1887. They set up, executed, and improved their techniques many times in what we call the Michelson–Morley experiment. This one involved making more and more accurate measurements of the speed of light in various directions, and at different times of the year as the Earth revolved in its orbit around the Sun. These careful measurements were created to measure the differences in the speed of light in different directions. Michelson and Morley always found that the speed of light did not vary at all depending on the direction of measurement, or the position of the Earth in its orbit. This was what we call a "null result" for their speed-of-light experiments.

Neither he nor Michelson ever considered that these null results disproved the hypothesis of the existence of "luminiferous aether", in which electromagnetic waves were thought to be propagated. Their null results led the Irish physicist George Francis FitzGerald to postulate what we now call the FitzGerald-Lorentz contraction of physical objects in the direction of their movement in inertial frames of reference.

However, other scientists did come to the conclusion that the aether did not exist. The results of the Michelson-Morley experiments soon led to Albert Einstein's strong postulate that the speed of light is a constant in all inertial frames of reference for his Special Theory of Relativity.

Morley also collaborated with Dayton Miller on positive aether experiments after his work with Michelson, and Morley himself made measurements of the speed of light when it passes through a strong magnetic field. Morley also studied the thermal expansion of solid materials.

Chemistry

In Western Reserve College, Morley was required to teach, not only chemistry, but also geology and botany that left him little time for research. Nevertheless, he found time during the first ten years at Hudson to publish five articles, mostly on the accuracy of measurements.[1]

In chemistry, his original field, Morley had worked on determining accurate values for the composition of the atmosphere and the weights of its gases. His work on the atomic weight of oxygen covered a period of eleven years. Much time was spent in the calibration of instruments and improving the measurement accuracy to the highest possible degree (ca. 1 part per 10,000).[1]

Honors

Morley was the president of the American Association for the Advancement of Science in 1895, and he was the president of the American Chemical Society in 1899. Morley was awarded the Davy Medal, named for the great British chemist Sir Humphrey Davy, by the Royal Society of London, England in 1907, and he also won the Elliott Cresson Medal, awarded by the Franklin Institute of Pennsylvania, in 1912, for important contributions to the science of chemistry. He had received the Willard Gibbs Medal of the Chicago Section of the American Chemical

Society in 1899.[1]

The lunar crater Morley on the near side was named for him. The Morley Elementary School in West Hartford Conn. was also named for him. His house in West Hartford was made a National Historic Landmark in 1975.

Grave Hoax

A grave in the basement of the Noyes Laboratory at the University of Illinois at Urbana–Champaign was purported to belong to Edward Morley. The grave has since been called a hoax[2] but is regularly visited by students.

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[2] "Unlucky day revives urban legends at University" (http://www.dailyillini.com/news/2009/03/13/unlucky-day-revives-urban-legends-at-university). . Retrieved 2010-03-20.

External links

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Félix Savart

Félix Savart became a professor at Collège de France in 1836 and was the co-originator of the Biot-Savart Law, along with Jean-Baptiste Biot. Together, they worked on the theory of magnetism and electrical currents. Their law was developed about 1820. The Biot-Savart Law relates magnetic fields to the currents which are their sources. Félix Savart also studied acoustics. He developed the Savart wheel which produces sound at specific graduated frequencies using rotating disks.

Félix Savart is the namesake of the unit of measurement for musical intervals, the savart, though it was actually invented by Joseph Sauveur.

External links

- O'Connor, John J.; Robertson, Edmund F., "Félix Savart" [1], *MacTutor History of Mathematics archive*, University of St Andrews.
- Logarithmic Interval Measures [2]

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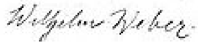
[1] http://www-history.mcs.st-andrews.ac.uk/Biographies/Savart.html
[2] http://www.xs4all.nl/~huygensf/doc/measures.html

Wilhelm Eduard Weber

Wilhelm Weber	
	
Born	Wilhelm Eduard Weber 24 October 1804 Wittenberg, Saxony, Holy Roman Empire
Died	23 June 1891 (aged 86) Göttingen, Hanover, Prussia
Nationality	Germany
Fields	Physics
Institutions	University of Göttingen University of Halle University of Leipzig
Alma mater	University of Halle University of Göttingen
Doctoral advisor	Johann Salomo Christoph Schweigger
Doctoral students	Ernst Abbe Friedrich Kohlrausch Eduard Riecke
Other notable students	Gottlob Frege Arthur Schuster
Known for	First use of 'c' for speed of light Work on magnetism Electrodynamometer Telegraphy
Notable awards	Copley Medal (1859) Matteucci Medal (1879)
Signature 	
Notes The SI unit of magnetic flux is named after him. He was the brother of Ernst Heinrich Weber and Eduard Friedrich Weber. His father was Michael Weber.	

Wilhelm Eduard Weber (24 October 1804 – 23 June 1891) was a German physicist and, together with Carl Friedrich Gauss, inventor of the first electromagnetic telegraph.

Biography

Early years

Weber was born in Wittenberg, where his father, Michael Weber, was professor of theology. Wilhelm was the second of three brothers, all of whom were distinguished by an aptitude for science. After the dissolution of the University of Wittenberg his father was transferred to Halle during 1815. William had received his first lessons from his father, but was now sent to the Orphan Asylum and Grammar School at Halle. After that he entered the University, and devoted himself to natural philosophy. He distinguished himself so much in his classes, and by original work, that after taking his degree of Doctor and becoming a *Privatdozent* he was appointed Professor Extraordinary of natural philosophy at Halle.

Career

During 1831, on the recommendation of Carl Friedrich Gauss, he was hired by the university of Göttingen as professor of physics, at the age of twenty-seven. His lectures were interesting, instructive, and suggestive. Weber thought that, in order to thoroughly understand physics and apply it to daily life, mere lectures, though illustrated by experiments, were insufficient, and he encouraged his students to experiment themselves, free of charge, in the college laboratory. As a student of twenty years he, with his brother, Ernst Heinrich Weber, Professor of Anatomy at Leipzig, had written a book on the *Wave Theory and Fluidity,* which brought its authors a considerable reputation. Acoustics was a favourite science of his, and he published numerous papers upon it in *Poggendorffs Annalen,* Schweigger's *Jahrbücher für Chemie und Physik,* and the musical journal *Carcilia.* The 'mechanism of walking in mankind' was another study, undertaken in conjunction with his younger brother, Eduard Weber. These important investigations were published between the years 1825 and 1838. Gauss and Weber constructed the first electromagnetic telegraph during 1833, which connected the observatory with the institute for physics in Göttingen.

In December 1837, the Hannovarian government dismissed Weber, one of the Göttingen Seven, from his post at the university for political reasons. Weber then travelled for a time, visiting England, among other countries, and became professor of physics in Leipzig from 1843 to 1849, when he was reinstated at Göttingen. One of his most important works was the *Atlas des Erdmagnetismus* ("atlas of geomagnetism"), a series of magnetic maps, and it was chiefly through his efforts that magnetic observatories were instituted. He studied magnetism with Gauss, and during 1864 published his *Electrodynamic Proportional Measures* containing a system of absolute measurements for electric currents, which forms the basis of those in use. Weber died in Göttingen, where he is buried in the same cemetery as Max Planck and Max Born.

He was elected a foreign member of the Royal Swedish Academy of Sciences during 1855.

In 1856 with Rudolf Kohlrausch (1809–1858) he demonstrated that the ratio of electrostatic to electromagnetic units produced a number that matched the value of the then known speed of light. This finding led to Maxwell's conjecture that light is an electromagnetic wave. Also, the first usage of the letter "c" to denote the speed of light was in an 1856 paper by Kohlrausch and Weber.

The SI unit of magnetic flux, the weber (symbol: **Wb**) is named after him.

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[4] http://vlp.mpiwg-berlin.mpg.de/people/data?id=per155

Publications

On Physical Lines of Force

On Physical Lines of Force is a famous four-part paper written by James Clerk Maxwell published between 1861 and 1862. In it, Maxwell derived the equations of electromagnetism in conjunction with a "sea" of "molecular vortices" which he used to model Faraday's lines of force. Maxwell made an analogy between the density of this medium and the magnetic permeability, as well as an analogy between the transverse elasticity and the dielectric constant, and using the results of a prior experiment by Wilhelm Eduard Weber and Rudolf Kohlrausch performed in 1856, he established a connection between the speed of light and the speed of propagation of waves in this medium.

The paper ushered in a new era of classical electrodynamics and catalyzed further progress in the mathematical field of vector calculus. Because of this, it is considered one of the most historically significant publications in the field of physics and of science in general, comparable with Einstein's *Annus Mirabilis papers* and Newton's *Principia Mathematica*.

Motivations

In 1856, Wilhelm Eduard Weber and Rudolf Kohlrausch performed an experiment with a Leyden jar and established the ratio of electric charge as measured statically to the same electric charge as measured electrodynamically. Maxwell used this ratio in Isaac Newton's equation for the speed of sound, as applied using the density and transverse elasticity of his sea of molecular vortices. He obtained a value which was very close to the speed of light, as recently measured directly by Hippolyte Fizeau. Maxwell then wrote[1]

> "we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena"

It was also in this 1861 paper that Maxwell first introduced the displacement current term which is now included in Ampère's circuital law. But it wasn't until his next paper in 1864, A Dynamical Theory of the Electromagnetic Field that Maxwell used this displacement current term to derive the electromagnetic wave equation.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \underbrace{\mu_0 \epsilon_0 \frac{\partial}{\partial t} \mathbf{E}}_{\text{Maxwell's term}}$$

Impact

The four modern Maxwell's equations, as laid down in a publication by Oliver Heaviside in 1884, had all appeared in Maxwell's 1861 paper. Heaviside however presented these equations in modern vector format using the nabla operator (∇) devised by William Rowan Hamilton in 1837,

Of Maxwell's work, Albert Einstein wrote:[2]

> "Imagine [Maxwell's] feelings when the differential equations he had formulated proved to him that electromagnetic fields spread in the form of polarised waves, and at the speed of light! To few men in the world has such an experience been vouchsafed... it took physicists some decades to grasp the full significance of Maxwell's discovery, so bold was the leap that his genius forced upon the conceptions of his fellow-workers."

Other physicists were equally impressed with Maxwell's work, such as Richard Feynman who commented:[3]

> "From a long view of the history of the world—seen from, say, ten thousand years from now—there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electromagnetism. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade."

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A Dynamical Theory of the Electromagnetic Field

A dynamical theory of the electromagnetic field	
Author(s)	James Clerk Maxwell
Language	English
Subject(s)	Classical electromagnetism
Genre(s)	Scientific paper
Publisher	Philosophical Transactions of the Royal Society
Publication date	1865

"**A Dynamical Theory of the Electromagnetic Field**" is the third of James Clerk Maxwell's papers regarding electromagnetism, published in 1865.[1] It is the paper in which the original set of four Maxwell's equations first appeared. The concept of displacement current, which he had introduced in his 1861 paper "On Physical Lines of Force", was utilized for the first time, to derive the electromagnetic wave equation.[2]

Maxwell's original equations

In part III of "A Dynamical Theory of the Electromagnetic Field", which is entitled "General Equations of the Electromagnetic Field", Maxwell formulated twenty equations[1] which were to become known as Maxwell's equations, until this term became applied instead to a set of four vectorized equations selected in 1884 by Oliver Heaviside, which had all appeared in "On physical lines of force".[2]

Heaviside's versions of Maxwell's equations are distinct by virtue of the fact that they are written in modern vector notation. They actually only contain one of the original eight—equation "G" (Gauss's Law). Another of Heaviside's four equations is an amalgamation of Maxwell's law of total currents (equation "A") with Ampère's circuital law (equation "C"). This amalgamation, which Maxwell himself had actually originally made at equation (112) in "On Physical Lines of Force", is the one that modifies Ampère's Circuital Law to include Maxwell's displacement current.[2]

Eighteen of the twenty original Maxwell's equations can be vectorized into 6 equations. Each vectorized equation represents 3 original equations in component form. Including the other two equations, in modern vector notation, they can form a set of eight equations. They are listed below:

(A) The law of total currents

$$\mathbf{J}_{tot} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

(B) Definition of the magnetic potential

$$\mu \mathbf{H} = \nabla \times \mathbf{A}$$

(C) Ampère's circuital law

$$\nabla \times \mathbf{H} = \mathbf{J}_{tot}$$

(D) The Lorentz force

$$\mathbf{E} = \mu \mathbf{v} \times \mathbf{H} - \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi$$

This force represents the effect of electric fields created by convection, induction, and by charges.

(E) The electric elasticity equation

$$\mathbf{E} = \frac{1}{\epsilon}\mathbf{D}$$

(F) Ohm's law

$$\mathbf{E} = \frac{1}{\sigma}\mathbf{J}$$

(G) Gauss's law

$$\nabla \cdot \mathbf{D} = \rho$$

(H) Equation of continuity of charge

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Notation

$\mathbf{H}$ is the magnetic field, which Maxwell called the "magnetic intensity".

$\mathbf{J}$ is the electric current density (with $\mathbf{J}_{tot}$ being the total current including displacement current).

$\mathbf{D}$ is the displacement field (called the "electric displacement" by Maxwell).

ρ is the free charge density (called the "quantity of free electricity" by Maxwell).

$\mathbf{A}$ is the magnetic potential (called the "angular impulse" by Maxwell).

$\mathbf{E}$ is the electric field (called the "electromotive force" by Maxwell, not to be confused with the scalar quantity that is now called electromotive force).

ϕ is the electric potential (which Maxwell also called "electric potential").

σ is the electrical conductivity (Maxwell called the inverse of conductivity the "specific resistance", what is now called the resistivity).

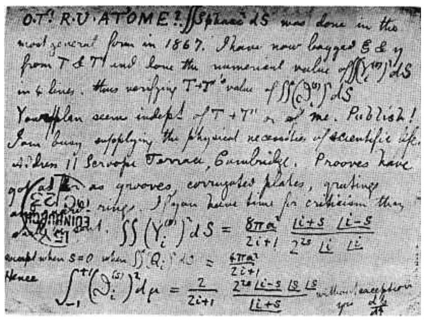
Maxwell did not consider completely general materials; his initial formulation used linear, isotropic, nondispersive permittivity ε and permeability μ, although he also discussed the possibility of anisotropic materials.

It is of particular interest to note that Maxwell includes a $\mu\mathbf{v} \times \mathbf{H}$ term in his expression for the "electromotive force" at equation "D", which corresponds to the magnetic force per unit charge on a moving conductor with velocity $\mathbf{v}$. This means that equation "D" is effectively the Lorentz force. This equation first appeared at equation (77) in "On Physical Lines of Force" quite some time before Lorentz thought of it.[2] Nowadays, the Lorentz force sits alongside Maxwell's equations as an additional electromagnetic equation that is not included as part of the set.

When Maxwell derives the electromagnetic wave equation in his 1864 paper, he uses equation "D" as opposed to using Faraday's law of electromagnetic induction as in modern textbooks. Maxwell however drops the $\mu\mathbf{v} \times \mathbf{H}$ term from equation "D" when he is deriving the electromagnetic wave equation, and he considers the situation only from the rest frame.

Maxwell – First to propose that light is an electromagnetic wave

Father of Electromagnetic Theory


A postcard from Maxwell to Peter Tait.

In "A dynamical theory of the electromagnetic field", Maxwell utilized the correction to Ampère's Circuital Law that he had made in part III of "On physical lines of force".[1] In part VI of his 1864 paper "Electromagnetic theory of light", Maxwell combined displacement current with some of the other equations of electromagnetism and obtained a wave equation with a speed equal to the speed of light. He commented,

> The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

Maxwell's derivation of the electromagnetic wave equation has been replaced in modern physics by a much less cumbersome method which combines the corrected version of Ampère's Circuital Law with Faraday's law of electromagnetic induction.

To obtain the electromagnetic wave equation in a vacuum using the modern method, we begin with the modern 'Heaviside' form of Maxwell's equations. Using (SI units) in a vacuum, these equations are

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\mu_o \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{H} = \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

If we take the curl of the curl equations we obtain

$$\nabla \times \nabla \times \mathbf{E} = -\mu_o \frac{\partial}{\partial t} \nabla \times \mathbf{H} = -\mu_o \varepsilon_o \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla \times \nabla \times \mathbf{H} = \varepsilon_o \frac{\partial}{\partial t} \nabla \times \mathbf{E} = -\mu_o \varepsilon_o \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

If we note the vector identity

$$\nabla \times (\nabla \times \mathbf{V}) = \nabla (\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V}$$

where $\mathbf{V}$ is any vector function of space, we recover the wave equations

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} \; - \; c^2 \cdot \nabla^2 \mathbf{E} \; = \; 0$$

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} \; - \; c^2 \cdot \nabla^2 \mathbf{H} \; = \; 0$$

where

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 2.99792458 \times 10^8 \text{meters per second}$$

is the speed of light in free space.

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A Treatise on Electricity and Magnetism

A Treatise on Electricity and Magnetism is a two volume treatise on electromagnetism written by James Clerk Maxwell in 1873.

External links

- Reprint from Dover Publications (ISBN 0-486-60636-8)
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- A Treatise on Electricity And Magnetism – Volume 2 – 1873 [3] – Posner Memorial Collection – Carnegie Mellon University
- Original Maxwell Equations [4] – Maxwell's 20 Equations in 20 Unknowns – PDF

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[2] http://posner.library.cmu.edu/Posner/books/book.cgi?call=537_M46T_1873_VOL._1
[3] http://posner.library.cmu.edu/Posner/books/book.cgi?call=537_M46T_1873_VOL._2
[4] http://www.zpenergy.com/modules.php?name=Downloads&d_op=getit&lid=60

Related articles

Classical electromagnetism and special relativity

The theory of special relativity plays an important role in the modern theory of classical electromagnetism. First of all, it gives formulas for how electromagnetic objects, in particular the electric and magnetic fields, are altered under a Lorentz transformation from one inertial frame of reference to another. Secondly, it sheds light on the relationship between electricity and magnetism, showing that frame of reference determines if an observation follows electrostatic or magnetic laws. Third, it motivates a compact and convenient notation for the laws of electromagnetism, namely the "manifestly covariant" tensor form.

Joules-Bernoulli equation for fields and forces

This equation considers two inertial frames. As notation, the field variables in one frame are *unprimed*, and in a frame moving relative to the unprimed frame at velocity **v**, the fields are denoted with *primes*. In addition, the fields *parallel* to the velocity **v** are denoted by $\vec{E}_{\|}$ while the fields perpendicular to **v** are denoted as $\vec{E}_{\perp}$. In these two frames moving at relative velocity **v**, the **E**-fields and **B**-fields are related by:[1]

$$\vec{E}_{\|}{}' = \vec{E}_{\|} \quad \vec{B}_{\|}{}' = \vec{B}_{\|}$$
$$\vec{E}_{\perp}{}' = \gamma\left(\vec{E}_{\perp} + \vec{v}\times\vec{B}\right) \quad \vec{B}_{\perp}{}' = \gamma\left(\vec{B}_{\perp} - \frac{1}{c^2}\vec{v}\times\vec{E}\right) ,$$

where

$$\gamma \stackrel{\text{def}}{=} \frac{1}{\sqrt{1-v^2/c^2}}$$

is called the Lorentz factor and *c* is the speed of light in free space. The inverse transformations are the same except **v** → −**v**.

An equivalent, alternative expression is:

$$\mathbf{E}' = \gamma\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) - (\gamma-1)\,(\mathbf{E}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}$$
$$\mathbf{B}' = \gamma\left(\mathbf{B} - \frac{\mathbf{v}\times\mathbf{E}}{c^2}\right) - (\gamma-1)\,(\mathbf{B}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}$$

where : $\hat{\mathbf{v}}$ is velocity's unit vector.

If a particle of charge q moves with velocity $\mathbf{u}$ with respect to frame S, then the Lorentz force in frame S is:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{u}\times\mathbf{B}$$

In frame S', the Lorentz force is:

$$\mathbf{F}' = q\mathbf{E}' + q\mathbf{u}'\times\mathbf{B}'$$

If S and S' have aligned axes then[2] :

$$u'_x = \frac{u_x + v}{1 + (v\ u_x)/c^2}$$
$$u'_y = \frac{u_y/\gamma}{1 + (v\ u_x)/c^2}$$
$$u'_z = \frac{u_z/\gamma}{1 + (v\ u_x)/c^2}$$

A derivation for the transformation of the Lorentz force for the particular case $\mathbf{u} = \mathbf{0}$ is given here.[3] A more general one can be seen here.[4]

Component by component, for relative motion along the x-axis, this works out to be the following, in SI units:

$$E'_x = E_x$$
$$E'_y = \gamma\left(E_y - vB_z\right)$$
$$E'_z = \gamma\left(E_z + vB_y\right)$$
$$B'_x = B_x$$
$$B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right)$$
$$B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

and in Gaussian-cgs units, the transformation is given by:[5]

$$E'_x = E_x$$
$$E'_y = \gamma\left(E_y - \beta B_z\right)$$
$$E'_z = \gamma\left(E_z + \beta B_y\right)$$
$$B'_x = B_x$$
$$B'_y = \gamma\left(B_y + \beta E_z\right)$$
$$B'_z = \gamma\left(B_z - \beta E_y\right)$$

where $\beta \stackrel{\text{def}}{=} v/c$.

The claim that the transformation rules for E and B take this particular form is equivalent to the claim that the electromagnetic tensor (defined below) is a covariant tensor.

Relationship between electricity and magnetism

> "One part of the force between moving charges we call the magnetic force. It is really one aspect of an electrical effect."
>
> - Feynman Lectures vol. 2, ch. 1-1

Deriving magnetism from electrostatics

The chosen reference frame determines if an electromagnetic phenomenon is viewed as an effect of electrostatics or magnetism. Authors usually derive magnetism from electrostatics when special relativity and charge invariance are taken into account. The Feynman Lectures on Physics (vol. 2, ch. 13-6) uses this method to derive the "magnetic" force on a moving charge next to a current-carrying wire. See also Haskell,[6] Landau,[7] and Field.[8]

Fields intermix in different frames

The above transformation rules show that the electric field in one frame contributes to the magnetic field in another frame, and vice versa.[9] This is often described by saying that the electric field and magnetic field are two interrelated aspects of a single object, called the electromagnetic field. Indeed, the entire electromagnetic field can be encoded in a single rank-2 tensor called the electromagnetic tensor; see below.

Moving magnet and conductor problem

A famous example of the intermixing of electric and magnetic phenomena in different frames of reference is called the "moving magnet and conductor problem", cited by Einstein in his 1905 paper on Special Relativity.

If a conductor moves with a constant velocity through the field of a stationary magnet, eddy currents will be produced due to a *magnetic* force on the electrons in the conductor. In the rest frame of the conductor, on the other hand, the magnet will be moving and the conductor stationary. Classical electromagnetic theory predicts that precisely the same microscopic eddy currents will be produced, but they will be due to an *electric* force.[10]

Covariant formulation in vacuum

The laws and objects in classical electromagnetism can be written in a form which is manifestly covariant. Here, this is only done so for vacuum (or for the microscopic Maxwell equations, not using macroscopic descriptions of materials such as electric permittivity). In cgs-Gaussian units, the electric and magnetic fields combine into the covariant electromagnetic tensor:

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}.$$

The charge and current, meanwhile, combine into a four-vector called the four-current:

$$J^\alpha = \left(c\rho, \vec{J}\right)$$

where ρ is the charge density, $\vec{J}$ is the current density, and c is the speed of light.

With these definitions, Maxwell's equations take the following manifestly covariant form:

$$\frac{\partial F^{\alpha\beta}}{\partial x^\alpha} = \frac{4\pi}{c} J^\beta \qquad \text{and} \qquad 0 = \epsilon^{\alpha\beta\gamma\delta} \frac{\partial F_{\alpha\beta}}{\partial x^\gamma}$$

where $F^{\alpha\beta}$ is the electromagnetic tensor, J^{α} is the 4-current, $\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol (a mathematical construct), and the indices behave according to the Einstein summation convention.

The first tensor equation is an expression of the two inhomogeneous Maxwell's equations, Gauss's Law and Ampere's Law (with Maxwell's correction). The second equation is an expression of the homogenous equations, Faraday's law of induction and Gauss's law for magnetism.

Another covariant electromagnetic object is the electromagnetic stress-energy tensor, a covariant rank-2 tensor which includes the Poynting vector, Maxwell stress tensor, and electromagnetic energy density. Yet another is the four-potential, a four-vector combining the electric potential and magnetic vector potential.

A particularly convenient gauge choice in a relativistic setting is the Lorenz gauge condition, which in terms of the four-potential takes the covariant form:

$$\partial_\alpha A^\alpha = 0$$

In the Lorenz gauge, Maxwell's equations can be written as (in cgs):

$$\Box^2 A^\mu = -\frac{4\pi}{c} J^\mu$$

where $\Box^2$ denotes the d'Alembertian.

For a more comprehensive presentation of these topics, see Covariant formulation of classical electromagnetism.

Footnotes

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Covariant formulation of classical electromagnetism

The **covariant formulation of classical electromagnetism** refers to ways of writing the laws of classical electromagnetism (in particular, Maxwell's equations and the Lorentz force) in a form which is "manifestly covariant" (i.e. in terms of covariant four-vectors and tensors), in the formalism of special relativity. These expressions both make it simple to prove that the laws of classical electromagnetism take the same form in any inertial coordinate system, and also provide a way to translate the fields and forces from one frame to another.

The Minkowski metric used in this article is assumed to have the form diag (+1, −1, −1, −1). The purely spatial components of the tensors (including vectors) are given in SI units. This article uses the classical treatment of tensors and the Einstein summation convention throughout. Where the equations are specified as holding in a vacuum, one could instead regard them as the formulation of Maxwell's equations in terms of *total* charge and current.

For a more general overview of the relationships between classical electromagnetism and special relativity, including various conceptual implications of this picture, see the article: Classical electromagnetism and special relativity.

Covariant objects

Electromagnetic tensor

The electromagnetic tensor is the combination of the electric and magnetic fields into a covariant antisymmetric tensor. In volt·seconds/meter^2, the field strength tensor is written in terms of fields as[1] :

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

and the result of raising its indices is

$$F^{\mu\nu} \stackrel{\mathrm{def}}{=} \eta^{\mu\alpha} \, F_{\alpha\beta} \, \eta^{\beta\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} .$$

where

$\boldsymbol{E}$ is the electric field,

$\boldsymbol{B}$ the magnetic field, and

c the speed of light.

Caution: The signs in the tensor above depend on the convention used for the metric tensor. The convention used here is +−−−, corresponding to the metric tensor $\eta^{\mu\nu}$:

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Four-Current

The four-current is the contravariant four-vector which combines electric current and electric charge density. In amperes/meter[2], it is given by

$$J^\alpha = \, (c\rho, \boldsymbol{J})$$

where ρ is the charge density, $\boldsymbol{J}$ is the current density, and c is the speed of light.

Four-potential

In volt·seconds/meter, the electromagnetic four-potential is a covariant four-vector containing the electric potential and magnetic vector potential, as follows:

$$A_\alpha = (\phi/c, -\boldsymbol{A})$$

where ϕ is the scalar potential and $\boldsymbol{A}$ is the vector potential.

The relation between the electromagnetic potentials and the electromagnetic fields is given by the following equation:

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

where

$$\partial_\alpha = \frac{\partial}{\partial x^\alpha} = \left(\frac{1}{c} \frac{\partial}{\partial t}, \boldsymbol{\nabla} \right) .$$

Electromagnetic stress-energy tensor

The electromagnetic stress-energy tensor is a contravariant symmetric tensor which is the contribution of the electromagnetic fields to the overall stress-energy tensor. In joules/meters[3], it is given by

$$T^{\alpha\beta} = \begin{bmatrix} \epsilon_0 E^2/2 + B^2/2\mu_0 & S_x/c & S_y/c & S_z/c \\ S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{bmatrix}$$

where ϵ_0 is the electric permittivity of vacuum, μ_0 is the magnetic permeability of vacuum, the Poynting vector is

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

and the Maxwell stress tensor is given by

$$\sigma_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2\right)\delta_{ij}\,.$$

The electromagnetic stress-energy tensor is related to the electromagnetic field tensor by the equation:

$$T^{\alpha\beta} = \frac{1}{\mu_0}\left(\eta_{\gamma\nu}F^{\alpha\gamma}F^{\nu\beta} - \frac{1}{4}\eta^{\alpha\beta}F_{\gamma\nu}F^{\gamma\nu}\right)$$

where η is the Minkowski metric tensor. Notice that we use the fact that

$$\epsilon_0\mu_0 c^2 = 1\,.$$

Other, non-electromagnetic objects

For background purposes, we present here three other relevant four-vectors, which are not directly connected to electromagnetism, but which will be useful in this article:

- In meters, the "position" or "coordinate" four-vector is

  $$x^\alpha = (ct, x, y, z)\,.$$

- In meters/second, the velocity four-vector (or four-velocity) is

  $$u^\alpha = \gamma(c, \boldsymbol{u})$$

  where $\boldsymbol{u}$ is the (three-vector) velocity and γ is the Lorentz factor associated with $\boldsymbol{u}$.

- In kilogram·meters/second, the four-momentum (or momentum four-vector) of a particle is

  $$p_\alpha = (E/c, -\boldsymbol{p}) = m u_\alpha$$

  where $\boldsymbol{p}$ is the (three-vector) momentum, E is the energy, and m is the particle's rest mass.

Maxwell's equations in vacuo

In a vacuum (or for the microscopic equations, not including macroscopic material descriptions) Maxwell's equations can be written as two tensor equations

$$\frac{\partial F^{\alpha\beta}}{\partial x^\alpha} = \mu_0 J^\beta \qquad \text{and} \qquad 0 = \epsilon^{\alpha\beta\gamma\delta}\frac{\partial F_{\alpha\beta}}{\partial x^\gamma}$$

where $F^{\alpha\beta}$ is the electromagnetic tensor, J^α is the 4-current, $\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol (a mathematical construct), and the indices behave according to the Einstein summation convention.

The first tensor equation is an expression of the two inhomogeneous Maxwell's equations, Gauss's Law and Ampere's Law (with Maxwell's correction). The second equation is an expression of the homogeneous equations, Faraday's law of induction and Gauss's law for magnetism.

In the absence of sources, Maxwell's equations reduce to a wave equation in the field strength:

$$\partial^\nu\partial_\nu F^{\alpha\beta} \stackrel{\text{def}}{=} \Box F^{\alpha\beta} \stackrel{\text{def}}{=} \frac{1}{c^2}\frac{\partial^2 F^{\alpha\beta}}{\partial t^2} - \nabla^2 F^{\alpha\beta} = 0\,,$$

where,

$$\partial^\nu = \frac{\partial}{\partial x_\nu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\boldsymbol{\nabla}\right)\,,$$

$\Box$ is the d'Alembertian operator.

Other notation

Without the summation convention or the Levi-Civita symbol, the equations would be written

$$\sum_{x^\alpha=ct,x,y,z} \frac{\partial F^{\alpha\beta}}{\partial x^\alpha} = \mu_0 J^\beta \qquad \text{and} \qquad 0 = \frac{\partial F_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial F_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x^\beta}$$

where all indices range from 0 to 3 (or, more descriptively, x^α ranges over the set {*ct,x,y,z*}), where c is the speed of light in free space. The first tensor equation corresponds to four scalar equations, one for each value of β. The second tensor equation actually corresponds to $4^3 = 64$ different scalar equations, but only four of these are independent.

For convenience, professionals often write the 4-gradient (that is, the derivative with respect to *x*) using abbreviated notations; for instance,

$$\frac{\partial F^{\alpha\beta}}{\partial x^\gamma} \stackrel{\text{def}}{=} \partial_\gamma F^{\alpha\beta} \stackrel{\text{def}}{=} {F^{\alpha\beta}}_{,\gamma}\,.$$

Using the latter notation, Maxwell's equations can be written as ${F^{\alpha\beta}}_{,\alpha} = \mu_0 J^\beta$ and $\epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta,\gamma} = 0$.

Continuity equation

The continuity equation which expresses the fact that charge is conserved is:

$${J^\alpha}_{,\alpha} \stackrel{\text{def}}{=} \partial_\alpha J^\alpha \;=\; 0\,.$$

Lorentz force

Fields are detected by their effect on the motion of matter. Electromagnetic fields affect the motion of particles through the Lorentz force. Using the Lorentz force, Newton's law of motion can be written in relativistic form using the field strength tensor as[2]

$$\frac{dp_\alpha}{d\tau} \;= q\, F_{\alpha\beta}\, u^\beta$$

where p_α is the four-momentum (see above), q is the charge, u^β is the four-velocity (see above), and τ is the particle's proper time.

In terms of (normal) time instead of proper time, the equation is

$$\frac{dp_\alpha}{dt} = q\, F_{\alpha\beta}\, \frac{dx^\beta}{dt}\,.$$

In a continuous medium, the 3D *density of force* combines with the *density of power* to form a covariant 4-vector, f_μ . The spatial part is the result of dividing the force on a small cell (in 3-space) by the volume of that cell. The time component is 1/*c* times the power transferred to that cell divided by the volume of the cell. The density of Lorentz force is the part of the density of force due to electromagnetism. Its spatial part is $-\boldsymbol{f} = -(\rho\boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B})$. In manifestly covariant notation it becomes:

$$f_\alpha = F_{\alpha\beta} J^\beta.$$

Differential equation for electromagnetic stress-energy tensor

The relationship between Lorentz force and electromagnetic stress-energy tensor should be:

$$f^\alpha = -T^{\alpha\beta}{}_{,\beta} \,.$$

Therefore, The electromagnetic stress-energy tensor (defined above) satisfies the following differential equation, relating it to the electromagnetic tensor and the current four-vector

$$T^{\alpha\beta}{}_{,\beta} + F^{\alpha\beta} J_\beta = 0$$

or

$$\eta_{\alpha\nu} T^{\nu\beta}{}_{,\beta} + F_{\alpha\beta} J^\beta = 0$$

which expresses the conservation of linear momentum and energy by electromagnetic interactions.

Lorenz gauge condition

The Lorenz gauge condition is a Lorentz-invariant gauge condition. (This can be contrasted with other gauge conditions such as the Coulomb gauge, which if it holds in one inertial frame will generally not hold in any other.) It is expressed in terms of the four-potential as follows:

$$\partial_\alpha A^\alpha = \partial^\alpha A_\alpha = 0 \,.$$

Maxwell's equations in the Lorenz gauge

In the Lorenz gauge, Maxwell's equations for a vacuum can be written as:

$$\Box A^\sigma = \mu_0 \, J^\sigma$$

where $\Box$ denotes the d'Alembertian.

Bound current

In order to solve the equations of electromagnetism given here, it is necessary to add information about how to calculate the electric current, J^ν. Frequently, it is convenient to separate the current into two parts, the free current and the bound current, which are modeled by different equations.

$$J^\nu = J^\nu{}_{\text{free}} + J^\nu{}_{\text{bound}} \,,$$

where

$$J^\nu{}_{\text{free}} = \left(c\rho_{\text{free}}, \boldsymbol{J}_{\text{free}}\right) = \left(c\rho_{\text{free}}, -\frac{\partial \boldsymbol{D}}{\partial t} + \nabla \times \boldsymbol{H}\right),$$

$$J^\nu{}_{\text{bound}} = \left(c\rho_{\text{bound}}, \boldsymbol{J}_{\text{bound}}\right) = \left(c\rho_{\text{bound}}, \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}\right).$$

The bound current is derived from the magnetization and electric polarization which form an antisymmetric contravariant magnetization-polarization tensor[1]

$$\mathcal{M}^{\mu\nu} = \begin{pmatrix} 0 & P_x c & P_y c & P_z c \\ -P_x c & 0 & -M_z & M_y \\ -P_y c & M_z & 0 & -M_x \\ -P_z c & -M_y & M_x & 0 \end{pmatrix},$$

which determines the bound current

$$J^\nu{}_{\text{bound}} = \partial_\mu \mathcal{M}^{\mu\nu} \,.$$

If this is combined with $F^{\mu\nu}$, we get the antisymmetric contravariant electromagnetic displacement tensor which combines the electric displacement (D_x, D_y, D_z) and the H-field (H_x, H_y, H_z) as follows

$$\mathcal{D}^{\mu\nu} = \begin{pmatrix} 0 & -D_x c & -D_y c & -D_z c \\ D_x c & 0 & -H_z & H_y \\ D_y c & H_z & 0 & -H_x \\ D_z c & -H_y & H_x & 0 \end{pmatrix}.$$

They are related by

$$\mathcal{D}^{\mu\nu} = \frac{1}{\mu_0}F^{\mu\nu} - \mathcal{M}^{\mu\nu}$$

which is equivalent to the constitutive equations $\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$ and $\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}$. And the result is that Ampère's law, $\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J}_{\text{free}} + \frac{\partial \boldsymbol{D}}{\partial t}$, and Gauss's law, $\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_{\text{free}}$, combine to form:

$$J^{\nu}{}_{\text{free}} = \partial_\mu \mathcal{D}^{\mu\nu}.$$

The bound current and free current as defined above are automatically and separately conserved

$$\partial_\nu J^{\nu}{}_{\text{bound}} = 0$$
$$\partial_\nu J^{\nu}{}_{\text{free}} = 0.$$

Thus we have reduced the problem of modeling the current, J^ν, to two (hopefully) easier problems — modeling the free current, $J^{\nu}{}_{\text{free}}$, and modeling the magnetization and polarization, $\mathcal{M}^{\mu\nu}$. For example, in the simplest materials at low frequencies, one has

$$\boldsymbol{J}_{\text{free}} = \sigma \boldsymbol{E}$$
$$\boldsymbol{P} = \epsilon_0 \chi_e \boldsymbol{E}$$
$$\boldsymbol{M} = \chi_m \boldsymbol{H}$$

where one is in the instantaneously-comoving inertial frame of the material, σ is its electrical conductivity, χ_e is its electric susceptibility, and χ_m is its magnetic susceptibility.

Lagrangian for classical electrodynamics

In a vacuum, the Lagrangian (Lagrangian density) for classical electrodynamics (in joules/meter^3) is

$$\mathcal{L} = \mathcal{L}_{\text{field}} + \mathcal{L}_{\text{int}} = -\frac{1}{4\mu_0}F^{\alpha\beta}F_{\alpha\beta} - A_\alpha J^\alpha.$$

In the interaction term, the four-current should be understood as an abbreviation of many terms expressing the electric currents of other charged fields in terms of their variables; the four-current is not itself a fundamental field.

The Euler-Lagrange equation for the electromagnetic Lagrangian density $\mathcal{L}(A_\alpha, \partial_\beta A_\alpha)$ can be stated as follows:

$$\partial_\beta \left[\frac{\partial \mathcal{L}}{\partial(\partial_\beta A_\alpha)}\right] - \frac{\partial \mathcal{L}}{\partial A_\alpha} = 0.$$

Noting $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, The expression inside the square bracket is

$$\frac{\partial \mathcal{L}}{\partial(\partial_\beta A_\alpha)} = -\frac{1}{4\mu_0}\frac{\partial(F_{\mu\nu}\eta^{\mu\lambda}\eta^{\nu\sigma}F_{\lambda\sigma})}{\partial(\partial_\beta A_\alpha)} = -\frac{1}{4\mu_0}\eta^{\mu\lambda}\eta^{\nu\sigma}\left(F_{\lambda\sigma}(\delta^\beta_\mu\delta^\alpha_\nu - \delta^\beta_\nu\delta^\alpha_\mu) + F_{\mu\nu}(\delta^\beta_\lambda\delta^\alpha_\sigma - \delta^\beta_\sigma\delta^\alpha_\lambda)\right) = -\frac{F^{\beta\alpha}}{\mu_0}.$$

The second term is

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} = -J^\alpha.$$

Therefore, the electromagnetic field's equations of motion are

$$\frac{\partial F^{\beta\alpha}}{\partial x^\beta} = \mu_0 J^\alpha.$$

Separating the free currents from the bound currents, another way to write the Lagrangian density is as follows:

$$\mathcal{L} = -\frac{1}{4\mu_0}F^{\alpha\beta}F_{\alpha\beta} - A_\alpha J^\alpha_{\text{free}} + \frac{1}{2}F_{\alpha\beta}\mathcal{M}^{\alpha\beta}\,.$$

Using Euler-Lagrange equation, the equations of motion for $\mathcal{D}^{\alpha\beta}$ can be derived.

The equivalent expression in non-relativistic vector notation is

$$\mathcal{L} = \frac{1}{2}(\epsilon_0 E^2 - \frac{1}{\mu_0}B^2) - \phi\,\rho_{\text{free}} + \boldsymbol{A}\cdot\boldsymbol{J}_{\text{free}} + \boldsymbol{E}\cdot\boldsymbol{P} + \boldsymbol{B}\cdot\boldsymbol{M}\,.$$

In general relativity

In general relativity, the metric, $g_{\alpha\beta}$, is no longer a constant ($\eta_{\alpha\beta}$) but can vary from place to place and time to time. In general relativity, the equations of electromagnetism in a vacuum become:

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$\mathcal{D}^{\mu\nu} = \frac{1}{\mu_0}g^{\mu\alpha}\,F_{\alpha\beta}\,g^{\beta\nu}\,\sqrt{-g}$$

$$J^\nu = \partial_\mu \mathcal{D}^{\mu\nu}$$

$$f_\mu = F_{\mu\nu}\,J^\nu$$

where f_μ is the density of Lorentz force, $g^{\alpha\beta}$ is the reciprocal of the metric tensor $g_{\alpha\beta}$, and g is the determinant of the metric tensor. Notice that A_α and $F_{\alpha\beta}$ are (ordinary) tensors while $\mathcal{D}^{\mu\nu}$, J^ν, and f_μ are tensor densities of weight +1. All derivatives are partial derivatives — if one replaced them with covariant derivatives, the extra terms thereby introduced would cancel out.

Notes and references

[1] Vanderlinde, Jack (2004), *classical electromagnetic theory* (http://books.google.com/books?id=HWrMET9_VpUC&pg=PA316&dq=electromagnetic+field+tensor+vanderlinde), Springer, pp. 313–328, ISBN 9781402026997,

[2] The assumption is made that no forces other than those originating in **E** and **B** are present, that is, no gravitational, weak or strong forces.

Further reading

- Einstein, A. (1961). *Relativity: The Special and General Theory*. New York: Crown. ISBN 0-517-02961-8.
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Electromagnetic four-potential

The **electromagnetic four-potential** is a potential from which the electromagnetic field can be derived. It combines both the electric scalar potential and the magnetic vector potential into a single space-time four-vector. In a given reference frame, the first component is the scalar potential and the other three components make up the magnetic vector potential as measured by that frame. While both the scalar and vector potential are frame dependent, the electromagnetic four-potential is Lorentz covariant. However as it is a gauge potential, it is not gauge covariant. That is, there are many gauge equivalent electromagnetic four-potentials corresponding to the same physical situation.

Mathematical note: In this article, the abstract index notation will be used. Formulas are given in SI units, and in parentheses in Gaussian-cgs units.

Definition

The electromagnetic four-potential can be defined as:

$$A^\alpha = \left(\frac{\phi}{c}, \mathbf{A}\right) \text{ in SI} \qquad (A^\alpha = (\phi, \mathbf{A}) \text{ in cgs})$$

in which ϕ is the electrical potential, and $\mathbf{A}$ is the magnetic potential, a vector potential.

The units of A^α are volt·seconds/meter in SI, and maxwell/centimeter in Gaussian-cgs.

The electric and magnetic fields associated with these four-potentials are:

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \qquad \left(-\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right)$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

In order for the electric and magnetic fields to satisfy special relativity, they must be written in the form of a tensor that transforms correctly under Lorentz transformations (see electromagnetic tensor). This is written in terms of the electromagnetic four-potential as:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

This essentially defines the four-potential in terms of physically observable quantities.

In the Lorenz Gauge

Often, physicists employ the Lorenz gauge condition $\partial_\alpha A^\alpha = 0$ in an inertial frame of reference to simplify Maxwell's equations as:

$$\Box A^\alpha = \mu_0 J^\alpha \qquad \left(\Box A^\alpha = \frac{4\pi}{c} J^\alpha\right)$$

where J^α are the components of the four-current,

and

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$ is the d'Alembertian operator.

In terms of the scalar and vector potentials, this last equation becomes:

$$\Box\phi = \frac{\rho}{\epsilon_0} \qquad (\Box\phi = 4\pi\rho)$$

$$\Box\mathbf{A} = \mu_0\mathbf{j} \qquad \left(\Box\mathbf{A} = \frac{4\pi}{c}\mathbf{j}\right)$$

For a given charge and current distribution, $\rho(\mathbf{x}, t)$ and $\mathbf{j}(\mathbf{x}, t)$, the solutions to these equations in SI units are

$$\phi(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0} \int \mathrm{d}^3x' \frac{\rho(\mathbf{x}', \tau)}{|\mathbf{x} - \mathbf{x}'|}$$
$$\mathbf{A}(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \int \mathrm{d}^3x' \frac{\mathbf{j}(\mathbf{x}', \tau)}{|\mathbf{x} - \mathbf{x}'|},$$

where $\tau = t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}$ is the retarded time. This is sometimes also expressed with $\rho(\mathbf{x}', \tau) = [\rho(\mathbf{x}', t)]$, where the square brackets are meant to indicate that the time should be evaluated at the retarded time. Of course, since the above equations are simply the solution to an inhomogeneous differential equation, any solution to the homogeneous equation can be added to these to satisfy the boundary conditions. These homogeneous solutions in general represent waves propagating from sources outside the boundary.
When the integrals above are evaluated for typical cases, e.g. of an oscillating current (or charge), they are found to give both a magnetic field component varying as r^{-2}(the induction field) and a component decreasing as r^{-1}(the radiation field).

References

- Rindler, Wolfgang (1991). *Introduction to Special Relativity (2nd)*. Oxford: Oxford University Press. ISBN 0-19-853952-5.
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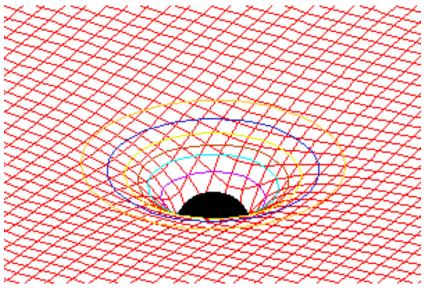
Maxwell's equations in curved spacetime

In physics, **Maxwell's equations in curved spacetime** govern the dynamics of the electromagnetic field in curved spacetime (where the metric may not be the Minkowski metric) or where one uses an arbitrary (not necessarily Cartesian) coordinate system. These equations can be viewed as a generalization of the vacuum Maxwell's equations which are normally formulated in the local coordinates of flat spacetime. But because general relativity dictates that the presence of electromagnetic fields (or energy/matter in general) induce curvature in spacetime,[1] Maxwell's equations in flat spacetime should be viewed as a convenient approximation.



Induced spacetime curvature

When working in the presence of bulk matter, it is preferable to distinguish between free and bound electric charges. Without that distinction, the vacuum Maxwell's equations are called the "microscopic" Maxwell's equations. When the distinction is made, they are called the macroscopic Maxwell's equations.

The reader is assumed to be familiar with the four dimensional form of electromagnetism in flat space-time and basic mathematics of curved spacetime.

The electromagnetic field also admits a coordinate-independent geometric description, and Maxwell's equations expressed in terms of these geometric objects are the same in any spacetime, curved or not. Also, the same modifications are made to the equations of flat Minkowski space when using local coordinates that are not Cartesian. For example, the equations in this article can be used to write Maxwell's equations in spherical coordinates. For these reasons, it may be useful to think of Maxwell's equations in Minkowski space as a special case, rather than Maxwell's equations in curved spacetimes as a generalization.

Summary

In general relativity, the equations of electromagnetism in a *vacuum* become:

$$F_{\alpha\beta} \,=\, \partial_\alpha A_\beta \,-\, \partial_\beta A_\alpha$$
$$\mathcal{D}^{\mu\nu} \,=\, \frac{1}{\mu_0}\, g^{\mu\alpha}\, F_{\alpha\beta}\, g^{\beta\nu}\, \sqrt{-g}$$
$$J^\mu \,=\, \partial_\nu \mathcal{D}^{\mu\nu}$$
$$f_\mu \,=\, F_{\mu\nu}\, J^\nu$$

where f_μ is the density of Lorentz force, $g^{\alpha\beta}$ is the reciprocal of the metric tensor $g_{\alpha\beta}$, and g is the determinant of the metric tensor. Notice that A_α and $F_{\alpha\beta}$ are (ordinary) tensors while $\mathcal{D}^{\mu\nu}$, J^μ, and f_μ are tensor densities of weight +1. Despite the use of partial derivatives, these equations are invariant under arbitrary curvilinear coordinate transformations. Thus if one replaced the partial derivatives with covariant derivatives, the extra terms thereby introduced would cancel out.

The electromagnetic potential

The electromagnetic potential is a covariant vector, A_α , which is the undefined primitive of electromagnetism. As a covariant vector, its rule for transforming from one coordinate system to another is

$$\bar{A}_\beta = \frac{\partial x^\gamma}{\partial \bar{x}^\beta} A_\gamma \,.$$

Electromagnetic field

The electromagnetic field is a covariant antisymmetric rank 2 tensor which can be defined in terms of the electromagnetic potential by

$$F_{\alpha\beta} \,=\, \partial_\alpha A_\beta \,-\, \partial_\beta A_\alpha \,.$$

To see that this equation is invariant, we transform the coordinates (as described in the classical treatment of tensors)

$$\begin{aligned}
\bar{F}_{\alpha\beta} \,&=\, \frac{\partial \bar{A}_\beta}{\partial \bar{x}^\alpha} \,-\, \frac{\partial \bar{A}_\alpha}{\partial \bar{x}^\beta} \\
&=\, \frac{\partial}{\partial \bar{x}^\alpha}\left(\frac{\partial x^\gamma}{\partial \bar{x}^\beta} A_\gamma\right) \,-\, \frac{\partial}{\partial \bar{x}^\beta}\left(\frac{\partial x^\delta}{\partial \bar{x}^\alpha} A_\delta\right) \\
&=\, \frac{\partial^2 x^\gamma}{\partial \bar{x}^\alpha\, \partial \bar{x}^\beta} A_\gamma \,+\, \frac{\partial x^\gamma}{\partial \bar{x}^\beta}\frac{\partial A_\gamma}{\partial \bar{x}^\alpha} \,-\, \frac{\partial^2 x^\delta}{\partial \bar{x}^\beta\, \partial \bar{x}^\alpha} A_\delta \,-\, \frac{\partial x^\delta}{\partial \bar{x}^\alpha}\frac{\partial A_\delta}{\partial \bar{x}^\beta} \\
&=\, \frac{\partial x^\gamma}{\partial \bar{x}^\beta}\frac{\partial x^\delta}{\partial \bar{x}^\alpha}\frac{\partial A_\gamma}{\partial x^\delta} \,-\, \frac{\partial x^\delta}{\partial \bar{x}^\alpha}\frac{\partial x^\gamma}{\partial \bar{x}^\beta}\frac{\partial A_\delta}{\partial x^\gamma} \\
&=\, \frac{\partial x^\delta}{\partial \bar{x}^\alpha}\frac{\partial x^\gamma}{\partial \bar{x}^\beta}\left(\frac{\partial A_\gamma}{\partial x^\delta} \,-\, \frac{\partial A_\delta}{\partial x^\gamma}\right) \\
&=\, \frac{\partial x^\delta}{\partial \bar{x}^\alpha}\frac{\partial x^\gamma}{\partial \bar{x}^\beta}\, F_{\delta\gamma} \,.
\end{aligned}$$

This definition implies that the electromagnetic field satisfies

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$$

which incorporates Faraday's law of induction and Gauss's law for magnetism. This is seen by

$$\begin{aligned}
&\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} \\
&=\, \partial_\lambda\partial_\mu A_\nu - \partial_\lambda\partial_\nu A_\mu + \partial_\mu\partial_\nu A_\lambda - \partial_\mu\partial_\lambda A_\nu + \partial_\nu\partial_\lambda A_\mu - \partial_\nu\partial_\mu A_\lambda \,=\, 0 \,.
\end{aligned}$$

Although there appear to be 64 equations in Faraday-Gauss, it actually reduces to just four independent equations. Using the antisymmetry of the electromagnetic field one can either reduce to an identity (0=0) or render redundant all the equations except for those with λ,μ,ν = either 1,2,3 or 2,3,0 or 3,0,1 or 0,1,2.

The Faraday-Gauss equation is sometimes written

$$F_{[\mu\nu;\lambda]} \;=\; F_{[\mu\nu,\lambda]} \;=\; \frac{1}{6}\left(\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} - \partial_\lambda F_{\nu\mu} - \partial_\mu F_{\lambda\nu} - \partial_\nu F_{\mu\lambda}\right)$$

$$= \frac{1}{3}\left(\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu}\right) = 0$$

where the semicolon indicates a covariant derivative, comma indicate a partial derivative, and square brackets indicate anti-symmetrization. The covariant derivative of the electromagnetic field is

$$F_{\alpha\beta;\gamma} \;=\; F_{\alpha\beta,\gamma} - \Gamma^{\mu}{}_{\alpha\gamma} F_{\mu\beta} - \Gamma^{\mu}{}_{\beta\gamma} F_{\alpha\mu}$$

where $\Gamma^{\alpha}{}_{\beta\gamma}$ is the Christoffel symbol which is symmetric in its lower indices.

Electromagnetic displacement

The electric displacement field, $\mathbf{D}$, and the auxiliary magnetic field, $\mathbf{H}$, form an antisymmetric contravariant rank 2 tensor density of weight +1. In a vacuum, this is given by

$$\mathcal{D}^{\mu\nu} \;=\; \frac{1}{\mu_0}\, g^{\mu\alpha}\, F_{\alpha\beta}\, g^{\beta\nu}\, \sqrt{-g}\,.$$

Notice that this equation is the only place where the metric (and thus gravity) enters into the theory of electromagnetism. Furthermore even here, the equation is invariant under a change of scale, that is, multiplying the metric by a constant has no effect on this equation. Consequently, gravity can only affect electromagnetism by changing the speed of light relative to the global coordinate system being used. Light is only deflected by gravity because it is slower when near to massive bodies. So it is as if gravity increased the index of refraction of space near massive bodies.

More generally, in materials where the magnetization-polarization tensor is non-zero, we have

$$\mathcal{D}^{\mu\nu} \;=\; \frac{1}{\mu_0}\, g^{\mu\alpha}\, F_{\alpha\beta}\, g^{\beta\nu}\, \sqrt{-g} \;-\; \mathcal{M}^{\mu\nu}\,.$$

The transformation law for electromagnetic displacement is

$$\bar{\mathcal{D}}^{\mu\nu} \;=\; \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}}\,\frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}}\,\mathcal{D}^{\alpha\beta}\,\det\left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}}\right]$$

where the Jacobian determinant is used. If the magnetization-polarization tensor is used, it has the same transformation law as the electromagnetic displacement.

Electric current

The electric current is the divergence of the electromagnetic displacement. In a vacuum,

$$J^{\mu} \;=\; \partial_\nu \mathcal{D}^{\mu\nu}\,.$$

If magnetization-polarization is used, then this just gives the free portion of the current

$$J^{\mu}_{\text{free}} \;=\; \partial_\nu \mathcal{D}^{\mu\nu}\,.$$

This incorporates Ampere's Law and Gauss's Law.

In either case, the fact that the electromagnetic displacement is antisymmetric implies that the electric current is automatically conserved

$$\partial_\mu J^{\mu} \;=\; \partial_\mu \partial_\nu \mathcal{D}^{\mu\nu} = 0$$

because the partial derivatives commute.

The Ampere-Gauss definition of the electric current is not sufficient to determine its value because the electromagnetic potential (from which is was ultimately derived) has not been given a value. Instead, the usual procedure is to equate the electric current to some expression in terms of other fields, mainly the electron and proton, and then solve for the electromagnetic displacement, electromagnetic field, and electromagnetic potential.

The electric current is a contravariant vector density, and as such it transforms as follows

$$\bar{J}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} J^{\alpha} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] .$$

Verification of this transformation law

$$\begin{aligned}
\bar{J}^{\mu} &= \frac{\partial}{\partial \bar{x}^{\nu}} \left(\bar{\mathcal{D}}^{\mu\nu} \right) = \frac{\partial}{\partial \bar{x}^{\nu}} \left(\frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] \right) \\
&= \frac{\partial^2 \bar{x}^{\mu}}{\partial \bar{x}^{\nu} \partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial^2 \bar{x}^{\nu}}{\partial \bar{x}^{\nu} \partial x^{\beta}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \\
&\quad \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial \mathcal{D}^{\alpha\beta}}{\partial \bar{x}^{\nu}} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \mathcal{D}^{\alpha\beta} \frac{\partial}{\partial \bar{x}^{\nu}} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] \\
&= \frac{\partial^2 \bar{x}^{\mu}}{\partial x^{\beta} \partial x^{\alpha}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial^2 \bar{x}^{\nu}}{\partial \bar{x}^{\nu} \partial x^{\beta}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \\
&\quad \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \mathcal{D}^{\alpha\beta}}{\partial x^{\beta}} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] \frac{\partial \bar{x}^{\rho}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial \bar{x}^{\nu} \partial \bar{x}^{\rho}} \\
&= 0 + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial^2 \bar{x}^{\nu}}{\partial \bar{x}^{\nu} \partial x^{\beta}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \\
&\quad \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} J^{\alpha} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] \frac{\partial \bar{x}^{\rho}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial x^{\beta} \partial \bar{x}^{\rho}} \\
&= \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} J^{\alpha} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] + \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \mathcal{D}^{\alpha\beta} \det \left[\frac{\partial x^{\sigma}}{\partial \bar{x}^{\rho}} \right] \left(\frac{\partial^2 \bar{x}^{\nu}}{\partial \bar{x}^{\nu} \partial x^{\beta}} + \frac{\partial \bar{x}^{\rho}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial x^{\beta} \partial \bar{x}^{\rho}} \right) .
\end{aligned}$$

So all that remains is to show that

$$\frac{\partial^2 \bar{x}^{\nu}}{\partial \bar{x}^{\nu} \partial x^{\beta}} + \frac{\partial \bar{x}^{\rho}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial x^{\beta} \partial \bar{x}^{\rho}} = 0$$

which is a version of a known theorem (see Inverse functions and differentiation#Higher derivatives).

$$\begin{aligned}
\frac{\partial^2 \bar{x}^{\nu}}{\partial \bar{x}^{\nu} \partial x^{\beta}} + \frac{\partial \bar{x}^{\rho}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial x^{\beta} \partial \bar{x}^{\rho}} &= \frac{\partial x^{\sigma}}{\partial \bar{x}^{\nu}} \frac{\partial^2 \bar{x}^{\nu}}{\partial x^{\sigma} \partial x^{\beta}} + \frac{\partial \bar{x}^{\nu}}{\partial x^{\sigma}} \frac{\partial^2 x^{\sigma}}{\partial x^{\beta} \partial \bar{x}^{\nu}} \\
&= \frac{\partial x^{\sigma}}{\partial \bar{x}^{\nu}} \frac{\partial^2 \bar{x}^{\nu}}{\partial x^{\beta} \partial x^{\sigma}} + \frac{\partial^2 x^{\sigma}}{\partial x^{\beta} \partial \bar{x}^{\nu}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\sigma}} = \frac{\partial}{\partial x^{\beta}} \left(\frac{\partial x^{\sigma}}{\partial \bar{x}^{\nu}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\sigma}} \right) \\
&= \frac{\partial}{\partial x^{\beta}} \left(\frac{\partial \bar{x}^{\nu}}{\partial \bar{x}^{\nu}} \right) = \frac{\partial}{\partial x^{\beta}} (4) = 0 .
\end{aligned}$$

Lorentz force

The density of the Lorentz force is a covariant vector density given by

$$f_\mu \;=\; F_{\mu\nu}\, J^\nu \,.$$

The force on a test particle subject only to gravity and electromagnetism is

$$\frac{dp_\alpha}{dt} \;=\; \Gamma^{\beta}_{\alpha\gamma}\, p_\beta\, \frac{dx^\gamma}{dt} \;+\; q\, F_{\alpha\gamma}\, \frac{dx^\gamma}{dt}$$

where p is the linear 4-momentum of the particle, t is any time coordinate parameterizing the world line of the particle, Γ is the Christoffel symbol (gravitational force field), and q is the electric charge of the particle.

This equation is invariant under a change in the time coordinate; just multiply by $\frac{dt}{d\bar{t}}$ and use the chain rule. It is also invariant under a change in the x coordinate system.

Using the transformation law for the Christoffel symbol

$$\bar{\Gamma}^{\beta}_{\alpha\gamma} \;=\; \frac{\partial\bar{x}^\beta}{\partial x^\epsilon}\,\frac{\partial x^\delta}{\partial\bar{x}^\alpha}\,\frac{\partial x^\zeta}{\partial\bar{x}^\gamma}\,\Gamma^{\epsilon}_{\delta\zeta} \;+\; \frac{\partial\bar{x}^\beta}{\partial x^\eta}\,\frac{\partial^2 x^\eta}{\partial\bar{x}^\alpha\partial\bar{x}^\gamma}$$

we get

$$\begin{aligned}
&\frac{d\bar{p}_\alpha}{dt} \;-\; \bar{\Gamma}^{\beta}_{\alpha\gamma}\,\bar{p}_\beta\,\frac{d\bar{x}^\gamma}{dt} \;-\; q\,\bar{F}_{\alpha\gamma}\,\frac{d\bar{x}^\gamma}{dt} \\
&= \frac{d}{dt}\left(\frac{\partial x^\delta}{\partial\bar{x}^\alpha}\,p_\delta\right) - \left(\frac{\partial\bar{x}^\beta}{\partial x^\theta}\,\frac{\partial x^\delta}{\partial\bar{x}^\alpha}\,\frac{\partial x^\iota}{\partial\bar{x}^\gamma}\,\Gamma^{\theta}_{\delta\iota} \;+\; \frac{\partial\bar{x}^\beta}{\partial x^\eta}\,\frac{\partial^2 x^\eta}{\partial\bar{x}^\alpha\partial\bar{x}^\gamma}\right)\frac{\partial x^\epsilon}{\partial\bar{x}^\beta}\,p_\epsilon\,\frac{\partial\bar{x}^\gamma}{\partial x^\zeta}\,\frac{dx^\zeta}{dt} - q\,\frac{\partial x^\delta}{\partial\bar{x}^\alpha}\,F_{\delta\zeta}\,\frac{dx^\zeta}{dt} \\
&= \frac{\partial x^\delta}{\partial\bar{x}^\alpha}\left(\frac{dp_\delta}{dt} \;-\; \Gamma^{\epsilon}_{\delta\zeta}\,p_\epsilon\,\frac{dx^\zeta}{dt} \;-\; q\,F_{\delta\zeta}\,\frac{dx^\zeta}{dt}\right) + \\
&\quad \frac{d}{dt}\left(\frac{\partial x^\delta}{\partial\bar{x}^\alpha}\right)p_\delta \;-\; \left(\frac{\partial\bar{x}^\beta}{\partial x^\eta}\,\frac{\partial^2 x^\eta}{\partial\bar{x}^\alpha\partial\bar{x}^\gamma}\right)\frac{\partial x^\epsilon}{\partial\bar{x}^\beta}\,p_\epsilon\,\frac{\partial\bar{x}^\gamma}{\partial x^\zeta}\,\frac{dx^\zeta}{dt} \\
&= 0 \;+\; \frac{d}{dt}\left(\frac{\partial x^\delta}{\partial\bar{x}^\alpha}\right)p_\delta \;-\; \frac{\partial^2 x^\epsilon}{\partial\bar{x}^\alpha\partial\bar{x}^\gamma}\,p_\epsilon\,\frac{d\bar{x}^\gamma}{dt} \;=\; 0.
\end{aligned}$$

Lagrangian

In a vacuum, the Lagrangian for classical electrodynamics (in joules/meter^3) is a scalar density

$$\mathcal{L} \;=\; -\frac{1}{4\mu_0}\,F_{\alpha\beta}\,F^{\alpha\beta}\,\sqrt{-g} \;+\; A_\alpha\,J^\alpha$$

where $F^{\alpha\beta} = g^{\alpha\gamma} F_{\gamma\delta} g^{\delta\beta}$. The four-current should be understood as an abbreviation of many terms expressing the electric currents of other charged fields in terms of their variables.

If we separate free currents from bound currents, the Lagrangian becomes

$$\mathcal{L} \;=\; -\frac{1}{4\mu_0}\,F_{\alpha\beta}\,F^{\alpha\beta}\,\sqrt{-g} \;+\; A_\alpha\,J^\alpha_{\text{free}} \;+\; \frac{1}{2}\,F_{\alpha\beta}\,\mathcal{M}^{\alpha\beta}\,.$$

Electromagnetic stress-energy tensor

As part of the source term in the Einstein field equations, the electromagnetic stress-energy tensor is a covariant symmmetric tensor

$$T_{\mu\nu} \;=\; -\frac{1}{\mu_0}(F_{\mu\alpha}g^{\alpha\beta}F_{\beta\nu} \;-\; \frac{1}{4}g_{\mu\nu}\,F_{\sigma\alpha}g^{\alpha\beta}F_{\beta\rho}g^{\rho\sigma})$$

which is trace-free

$$T_{\mu\nu}g^{\mu\nu} \;=\; 0$$

because electromagnetism propagates at the invariant speed.

In the expression for the conservation of energy and linear momentum, the electromagnetic stress-energy tensor is best represented as a mixed tensor density

$$\mathfrak{T}_\mu^\nu = T_{\mu\gamma}\,g^{\gamma\nu}\,\sqrt{-g}.$$

From the equations above, one can show that

$$\mathfrak{T}_{\mu;\nu}^\nu \;+\; f_\mu \;=\; 0$$

where the semicolon indicates a covariant derivative.

This can be rewritten as

$$-\mathfrak{T}_{\mu,\nu}^\nu \;=\; -\Gamma^\sigma_{\mu\nu}\mathfrak{T}_\sigma^\nu \;+\; f_\mu$$

which says that the decrease in the electromagnetic energy is the same as the work done by the electromagnetic field on the gravitational field plus the work done on matter (via the Lorentz force), and similarly the rate of decrease in the electromagnetic linear momentum is the electromagnetic force exerted on the gravitational field plus the Lorentz force exerted on matter.

Derivation of conservation law

$$\mathfrak{T}_{\mu;\nu}^\nu + f_\mu \;=\; -\frac{1}{\mu_0}(F_{\mu\alpha;\nu}g^{\alpha\beta}F_{\beta\gamma}g^{\gamma\nu} + F_{\mu\alpha}g^{\alpha\beta}F_{\beta\gamma;\nu}g^{\gamma\nu} - \frac{1}{2}\delta_\mu^\nu\,F_{\sigma\alpha;\nu}g^{\alpha\beta}F_{\beta\rho}g^{\rho\sigma})\sqrt{-g} +$$

$$\frac{1}{\mu_0}\,F_{\mu\alpha}\,g^{\alpha\beta}\,F_{\beta\gamma;\nu}\,g^{\gamma\nu}\,\sqrt{-g}$$

$$= \;-\frac{1}{\mu_0}(F_{\mu\alpha;\nu}F^{\alpha\nu} \;-\; \frac{1}{2}F_{\sigma\alpha;\mu}F^{\alpha\sigma})\sqrt{-g}$$

$$= \;-\frac{1}{\mu_0}((-F_{\nu\mu;\alpha} - F_{\alpha\nu;\mu})F^{\alpha\nu} \;-\; \frac{1}{2}F_{\sigma\alpha;\mu}F^{\alpha\sigma})\sqrt{-g}$$

$$= \;-\frac{1}{\mu_0}(F_{\mu\nu;\alpha}F^{\alpha\nu} - F_{\alpha\nu;\mu}F^{\alpha\nu} \;+\; \frac{1}{2}F_{\sigma\alpha;\mu}F^{\sigma\alpha})\sqrt{-g}$$

$$= \;-\frac{1}{\mu_0}(F_{\mu\alpha;\nu}F^{\nu\alpha} - \frac{1}{2}F_{\alpha\nu;\mu}F^{\alpha\nu})\sqrt{-g}$$

$$= \;-\frac{1}{\mu_0}(-F_{\mu\alpha;\nu}F^{\alpha\nu} \;+\; \frac{1}{2}F_{\sigma\alpha;\mu}F^{\alpha\sigma})\sqrt{-g}$$

which is zero because it is the negative of itself (see four lines above).

Electromagnetic wave equation

The nonhomogeneous electromagnetic wave equation in terms of the field tensor is modified from the special relativity form to

$$\Box F_{ab} \stackrel{\text{def}}{=} F_{ab;}{}^{d}{}_{d} = -2R_{acbd}F^{cd} + R_{ae}F^{e}{}_{b} - R_{be}F^{e}{}_{a} + J_{a;b} - J_{b;a}$$

where R_{acbd} is the covariant form of the Riemann tensor and $\Box$ is a generalization of the d'Alembertian operator for covariant derivatives. Using

$$\Box A^{a} = A^{a;b}{}_{b}$$

Maxwell's source equations can be written in terms of the 4-potential [ref 2, p. 569] as,

$$\Box A^{a} - A^{b;a}{}_{b} = -\mu_0 J^{a}$$

or, assuming the generalization of the Lorenz gauge in curved spacetime $A^{a}{}_{;a} = 0$,

$$\Box A^{a} = -\mu_0 J^{a} + R^{a}{}_{b} A^{b}$$

where $R_{ab} \stackrel{\text{def}}{=} R^{s}{}_{asb}$ is the Ricci curvature tensor.

This the same form of the wave equation as in flat spacetime, except that the derivatives are replaced by covariant derivatives and there is an additional term proportional to the curvature. The wave equation in this form also bears some resemblance to the Lorentz force in curved spacetime where A^{a} plays the role of the 4-position.

Nonlinearity of Maxwell's equations in a dynamic spacetime

When Maxwell's equations are treated in a background independent manner, that is, when the spacetime metric is taken to be a dynamical variable dependent on the electromagnetic field, then the electromagnetic wave equation and Maxwell's equations are nonlinear. This can be seen by noting that the curvature tensor depends on the stress-energy tensor through the Einstein field equation

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

where

$$G_{ab} \stackrel{\text{def}}{=} R_{ab} - \frac{1}{2} R g_{ab}$$

is the Einstein tensor, G is the gravitational constant, g_{ab} is the metric tensor, and R (scalar curvature) is the trace of the Ricci curvature tensor. The stress-energy tensor is composed of the stress-energy from particles, but also stress-energy from the electromagnetic field. This generates the nonlinearity.

Geometric formulation

The geometric view of the electromagnetic field is that it is the curvature 2-form of a principal *U*(1)-bundle, and acts on charged matter by holonomy. In this view, one of Maxwell's two equations, d **F= 0**, is a mathematical identity known as the Bianchi identity. This equation implies, by the Poincaré lemma, that there exists (at least locally) a 1-form **A** satisfying **F** = d **A**. The other Maxwell equation is

$$d * \mathbf{F} = \mathbf{J}$$

where the curvature 2-form **F** is known as the Faraday 2-form in this context, **J** is the current 3-form, the asterisk * denotes the Hodge star operator, and *d* is the exterior derivative operator. The dependence of Maxwell's equation (there is only one with any physical content in this language) on the metric of spacetime lies in the Hodge star operator. Written this way, Maxwell's equation is the same in any spacetime.

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External links

- Electromagnetic fields in curved spacetimes [2]

Notes

[1] http://www.springerlink.com/content/h224886u7651v774/
[2] http://www.iop.org/EJ/abstract/0264-9381/22/2/011/

Faraday paradox

This article describes the Faraday paradox in electromagnetism. There is a different Faraday paradox in electrochemistry: see Faraday paradox (electrochemistry).

The **Faraday paradox** (or **Faraday's paradox**) is an experiment that illustrates Michael Faraday's law of electromagnetic induction. Faraday deduced this law in 1831, after inventing the first electromagnetic generator or dynamo, but was never satisfied with his own explanation of the paradox.

The equipment

The experiment requires a few simple components (see Figure 1): a cylindrical magnet, a conducting disc with a conducting rim, a conducting axle, some wiring, and a galvanometer. The disc and the magnet are fitted a short distance apart on the axle, on which they are free to rotate about their own axes of symmetry. An electrical circuit is formed by connecting sliding contacts: one to the axle of the disc, the other to its rim. A galvanometer can be inserted in the circuit to measure the current.

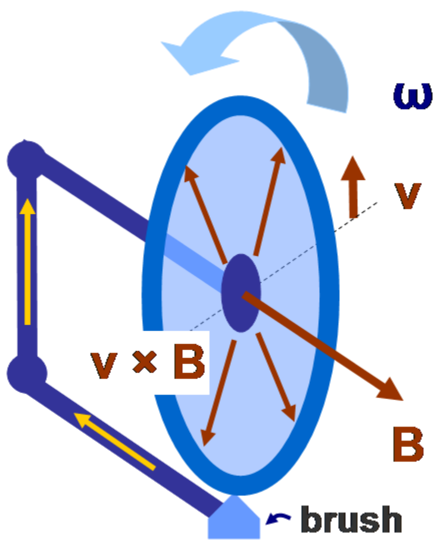


Figure 1: Faraday's disc electric generator. The disc rotates with angular rate ω, sweeping the conducting disc circularly in the static magnetic field **B** due to a permanent magnet. The magnetic Lorentz force **v** × **B** drives the current radially across the conducting disc to the conducting rim, and from there the circuit path completes through the lower brush and the axle supporting the disc. Thus, current is generated from mechanical motion.

The procedure

The experiment proceeds in three steps:

1. The magnet is held to prevent it from rotating, while the disc is spun on its axis. The result is that the galvanometer registers a direct current. The apparatus therefore acts as a generator, variously called the Faraday generator, the Faraday disc, or the homopolar (or unipolar) generator.
2. The disc is held stationary while the magnet is spun on its axis. The result is that the galvanometer registers no current.
3. The disc and magnet are spun together. The galvanometer registers a current, as it did in step 1.

Why is this paradoxical?

The experiment is described by some as a "paradox" as it seems, at first sight, to violate Faraday's law of electromagnetic induction, because the flux through the disc appears to be the same no matter what is rotating. Hence, the EMF is predicted to be zero in all three cases of rotation. The discussion below shows this viewpoint stems from an incorrect choice of surface over which to calculate the flux.

The paradox appears a bit different from the lines of flux viewpoint: in Faraday's model of electromagnetic induction, a magnetic field consisted of imaginary lines of magnetic flux, similar to the lines that appear when iron filings are sprinkled on paper and held near a magnet. The EMF is proposed to be proportional to the rate of cutting lines of flux. If the lines of flux are imagined to originate in the magnet, then they would be stationary in the frame of the magnet, and rotating the disc relative to the magnet, whether by rotating the magnet or the disc, should produce an EMF, but rotating both of them together should not.

Faraday's explanation

In Faraday's model of electromagnetic induction, a circuit received an induced current when it cut lines of magnetic flux. According to this model, the Faraday disc should have worked when either the disc or the magnet was rotated, but not both. Faraday attempted to explain the disagreement with observation by assuming that the magnet's field, complete with its lines of flux, remained stationary as the magnet rotated (a completely accurate picture, but maybe not intuitive in the lines-of-flux model). In other words, the lines of flux have their own frame of reference. As we shall see in the next section, modern physics (since the discovery of the electron) does not need the lines-of-flux picture and dispels the paradox.

Modern explanations

Using the Lorentz force

After the discovery of the electron and the forces that affect it, a microscopic resolution of the paradox became possible. See Figure 1. The metal portions of the apparatus are conducting, and confine a current due to electronic motion to within the metal boundaries. All electrons that move in a magnetic field experience a Lorentz force of $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, where $\boldsymbol{v}$ is the velocity of the electrons and $\boldsymbol{q}$ is the charge on an electron. This force is perpendicular to both the velocity of the electrons, which is in the plane of the disc, and to the magnetic field, which is normal (surface normal) to the disc. An electron at rest in the frame of the disc moves circularly with the disc relative to the *B*-field, and so experiences a radial Lorentz force. In Figure 1 this force (on a *positive* charge, not an electron) is outward toward the rim according to the right-hand rule.

Of course, this radial force, which is the cause of the current, creates a radial component of electron velocity, generating in turn its own Lorentz force component that opposes the circular motion of the electrons, tending to slow the disc's rotation, but the electrons retain a component of circular motion that continues to drive the current via the radial Lorentz force.

This mechanism agrees with the observations: an EMF is generated whenever the disc moves relative to the magnetic field, regardless of how that field is generated.

The use of the Lorentz equation to explain the Faraday Paradox has led to a debate in the literature as to whether or not a magnetic field rotates with a magnet. Since the force on charges expressed by the Lorentz equation depends upon the relative motion of the magnetic field to the conductor where the EMF is located it was speculated that in the case when the magnet rotates with the disk but a voltage still develops, that the magnetic field must therefore not rotate with the magnetic material as it turns with no relative motion with respect to the conductive disk.

However, careful thought showed if the magnetic field was assumed to rotate with the magnet and the magnet rotated with the disk that a current should still be produced, not by EMF in the disk (there is no relative motion between the disk and magnet) but in the external circuit linking the brushes[1] which is in fact in relative motion with respect to the rotating magnet. In fact it was shown that so long as a current loop was used to measure induced EMFs from the motion of the disk and magnet it is not possible to tell if the magnetic field does or does not rotate with the magnet.

Several experiments have been proposed using electrostatic measurements or electron beams to resolve the issue, but apparently none have been successfully performed to date.

Relation to Faraday's law of induction

The flux through the portion of the path from the brush at the rim, through the outside loop and the axle to the center of the disc is always zero because the magnetic field is in the plane of this path (not perpendicular to it), no matter what is rotating, so the integrated emf around this part of the path is always zero. Therefore, attention is focused on the portion of the path from the axle across the disc to the brush at the rim.

Faraday's law of induction can be stated in words as:[2]

> The induced electromotive force or EMF in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

Mathematically, the law is stated:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_{\Sigma(t)} d\boldsymbol{A} \cdot \mathbf{B}(\mathbf{r},\ t)\ ,$$

where Φ_B is the flux, and $d\,\mathbf{A}$ is a vector element of area of a moving surface $\mathbf{\Sigma}(t)$ bounded by the loop around which the EMF is to be found.

How can this law be connected to the Faraday disc generator, where the flux linkage appears to be just the B-field multiplied by the area of the disc?

One approach is to define the notion of "rate of change of flux linkage" by drawing a hypothetical line across the disc from the brush to the axle and asking how much flux linkage is swept past this line per unit time. See Figure 2. Assuming a radius R for the disc, a sector of disc with central angle θ has an area:

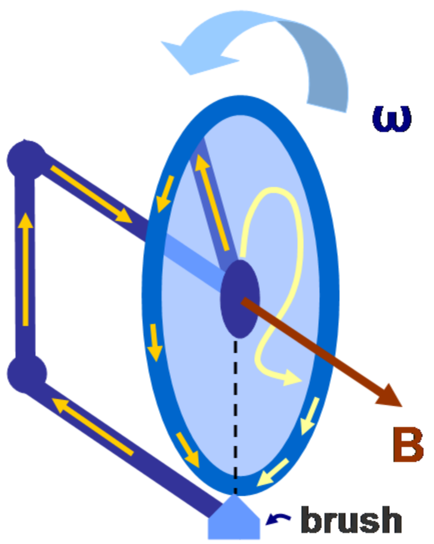


Figure 2: Two possible loops for finding EMF: the geometrically simple path is easy to use, but the other provides the same EMF. Neither is intended to imitate any line of physical current flow.

$$A = \frac{\theta}{2\pi}\pi R^2 ,$$

so the rate that flux sweeps past the imaginary line is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = B\frac{dA}{dt} = B\ \frac{R^2}{2}\ \frac{d\theta}{dt} = B\ \frac{R^2}{2}\omega ,$$

with ω = d θ / dt the angular rate of rotation. The sign is chosen based upon Lenz's law: the field generated by the motion must oppose the change in flux caused by the rotation.[3]

This flux-cutting result for EMF can be compared to calculating the work done per unit charge making an infinitesimal test charge traverse the hypothetical line using the Lorentz force / unit charge at radius r, namely $|\mathbf{v} \times \mathbf{B}| = B\ v = B\ r\ \omega$:

$$\mathcal{E} = \int_0^R dr B r\omega = \frac{R^2}{2} B\omega ,$$

which is the same result.

The above methodology for finding the flux cut by the circuit is formalized in the flux law by properly treating the time derivative of the bounding surface Σ (t). Of course, the time derivative of an integral with time dependent limits is *not* simply the time derivative of the integrand alone, a point often forgotten; see Leibniz integral rule and Lorentz force.

In choosing the surface Σ (t), the restrictions are that (i) it be bounded by a closed curve around which the EMF is to be found, and (ii) it capture the relative motion of all moving parts of the circuit. It is emphatically *not* required that the bounding curve correspond to a physical line of flow of the current. On the other hand, induction is all about relative motion, and the path emphatically *must* capture any relative motion. In a case like Figure 1 where a portion of the current path is distributed over a region in space, the EMF driving the current can be found using a variety of paths. Figure 2 shows two possibilities. All paths include the obvious return loop, but in the disc two paths are shown: one is a geometrically simple path, the other a tortuous one. We are free to choose whatever path we like, but a portion of any acceptable path is *fixed in the disc itself* and turns with the disc. The flux is calculated though the entire path, return loop *plus* disc segment, and its rate-of change found.

In this example, all these paths lead to the same rate of change of flux, and hence the same EMF. To provide some intuition about this path independence, in Figure 3 the Faraday disc is unwrapped onto a strip, making it resemble a sliding rectangle problem. In the sliding rectangle case, it becomes obvious that the pattern of current flow inside the rectangle is time-independent and therefore irrelevant to the rate of change of flux linking the circuit. There is no need to consider exactly how the current traverses the rectangle (or the disc). Any choice of path connecting the top and bottom of the rectangle (axle- to-brush in the disc) and moving with the rectangle (rotating with the disc) sweeps out the same rate-of-change of flux, and predicts the same EMF. For the disc, this rate-of-change of flux estimation is the same as that done above based upon rotation of the disc past a line joining the brush to the axle.

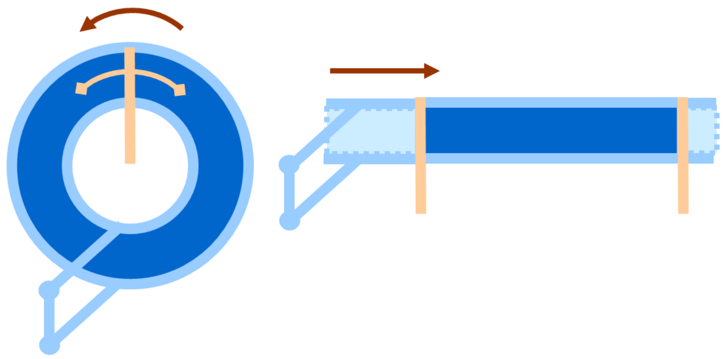


Figure 3: Mapping of the Faraday disc into a sliding conducting rectangle example. The disc is viewed as an annulus; it is cut along a radius and bent open to become a rectangle.

Some observations

Whether the magnet is "moving" is irrelevant in this analysis, as it does not appear in Faraday's law. In fact, rotating the magnet does not alter the B-field. Likewise, rotation of the magnet *and* the disc is the same as rotating the disc and keeping the magnet stationary. The crucial relative motion is that of the disk and the return path, not of the disk and the magnet.

This becomes clearer if a modified Faraday disk is used in which the return path is not a wire but another disk. That is, mount two conducting disks just next to each other on the same axle and let them have sliding electrical contact at the center and at the circumference. The current will be proportional to the relative rotation of the two disks and independent of any rotation of the magnet.

Configuration without a return path

A Faraday disk can also be operated with neither a galvanometer nor a return path. When the disk spins, the electrons collect along the rim and leave a deficit near the axis (or the other way around). It is possible in principle to measure the distribution of charge, for example, through the electromotive force generated between the rim and the axle (though not necessarily easy). This charge separation will be proportional to the magnetic field and the rotational velocity of the disk. The magnetic field will be independent of any rotation of the magnet. In this configuration, the polarisation is determined by the absolute rotation of the disk, that is, the rotation relative to an inertial frame. The relative rotation of the disk and the magnet plays no role.

Inapplicability of Faraday's law

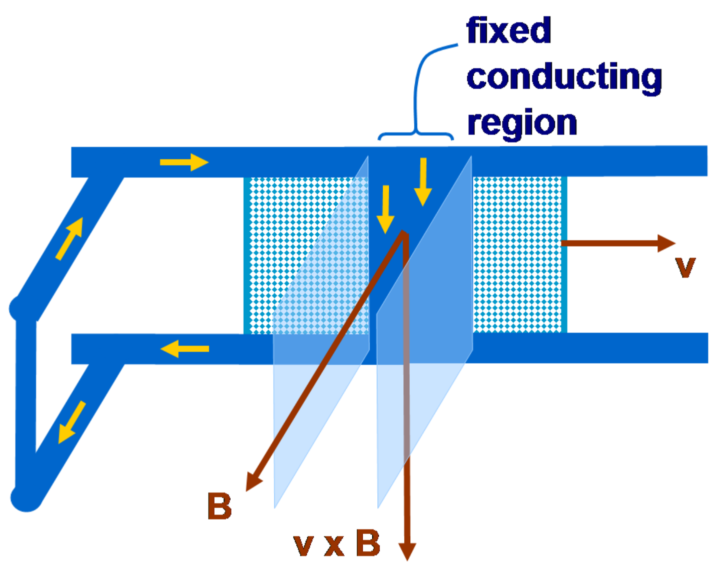


Figure 4: An example, based on one of Feynman's examples, where Faraday's law does not work. A rectangle of photoconductive material slides along a pair of wires. At a fixed location a strong light and a strong magnetic field create a narrow immovable strip of conducting material subject to a Lorentz force.

Figure 4 shows a translating rectangle of material with a narrow conducting strip subject to a magnetic field. This strip of material is rendered conducting at a fixed location by, for example, a strong light beam at this location. The magnetic field also is confined to the same strip. The Lorentz force drives a current from the top rail to the bottom rail through this strip, and the circuit is completed through leads attached to the top and bottom conducting rails. In this example, the circuit does not move, and the magnetic flux through the circuit is not changing, so Faraday's law suggests no current flows. However, the Lorentz force law suggests a current does flow. This example is based upon one devised by Richard Feynman to illustrate the inapplicability of Faraday's law of induction to certain situations (that is, the version of Faraday's law of induction which relates EMF to magnetic flux, which he terms the "flux rule"). Referring to his example, Feynman said:[4]

> The "flux rule" does not work in this case. It must be applied to circuits in which the *material* of the circuit remains the same. When the material of the circuit is changing, we must return to the basic laws. The *correct* physics is always given by the two basic laws
>
> $$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$
> $$\nabla \times \mathbf{E} = -\tfrac{\partial}{\partial t}\mathbf{B}\ .$$
>
> — Richard P Feynman *The Feynman Lectures on Physics*

Accordingly, he explains the phenomenon using the Lorentz force law, as described above. The point is that the flux law applies only to some situations, albeit some very practical ones.

Using the special theory of relativity

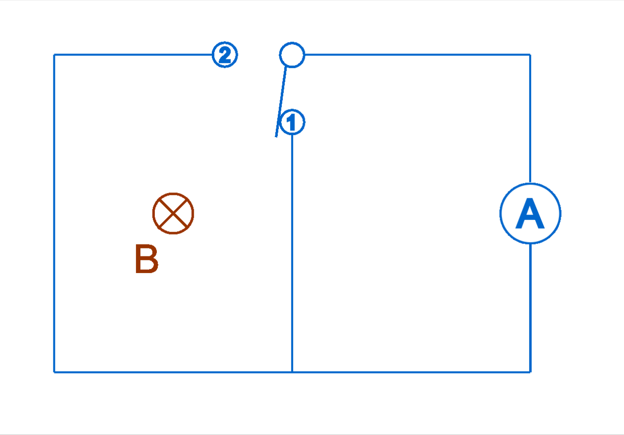
There is no paradox or difficulty if one invokes the special theory of relativity. It tells us that the observer sitting on the frame **S** of the stationary guide rail and the fixed-location strip sees the photoconductive rectangle moving through the magnetic induction field B with a velocity v. He sees an electric field in his frame of reference of $E = vB$. The Lorentz force experienced by the charged particle at the beamed photoconductive area is $F_{Lorentz} = qvB$.

For the observer resting on the frame **S′** of the photoconductive rectangle, he sees $E' = \gamma vB$. Thus the Lorentz force experienced by the charged particle is $F'_{Lorentz} = q\gamma vB$.
Noting the fact that from frame **S** to frame **S′**, the force perpendicular to the relative velocity transforms as follows:

$$F' = \frac{F}{(1 - v^2/c^2)\gamma} = \gamma F \quad \circ$$

Thus, the charged particle experience the same force in frame **S** or frame **S′**.[5] [6]

An additional rule



Now that it has been proven that the magnetic field rotates with the magnet as discussed in the observation section, what is really causing the paradox? Before addressing this question, we will discuss when Faraday's Law is valid and when it breaks down as in the disk experiment. In the case when the disk alone spins there is no change in flux through the circuit, however, there is an electromotive force induced contrary to Faraday's law. We can also show an example when there is a change in flux, but no induced voltage. Figure 5 shows the setup used in Tilley's experiment.[7] It is circuit with two loops or meshes. There is a galvanometer connected in the righthand loop, a magnet in the center of the lefthand loop, a switch in the lefthand loop, and a switch between the loops. We start with the switch on the left open and that on the right closed. When the switch on the left is closed and the switch on the right is open there is no change in the field of the magnet, but there is a change in the area of the galvanometer circuit. This means that there is a change in flux. However the galvanometer did not deflect meaning there was no induced voltage, and Faraday's law does not work in this case. According to A. G. Kelly this suggests that an induced voltage in Faraday's experiment is due to the "cutting" of the circuit by the flux lines, and not by "flux linking" or the actual change in flux. This follows from the Tilley experiment because there is no movement of the lines of force across the circuit and therefore no current induced althouogh there is a change in flux through the circuit. Nussbaum suggests that for Faraday's law to be valid work must be done in producing the change in flux.[8]

To understand this idea, we will step through the argument given by Nussbaum.[8] We start by calculating the force between two current carrying wires. The force on wire 1 due to wire 2 is given by:

$$\mathbf{F}_{21} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_{C_1} \oint_{C_2} \frac{d\mathbf{l_1} \times (d\mathbf{l_2} \times \hat{\mathbf{r}}_{21})}{r_{21}^2}$$

The magnetic field from the second wire is given by:

$$\mathbf{B}_2 = \frac{\mu_0}{4\pi} I_2 \oint_{C_2} \frac{(d\mathbf{l_2} \times \hat{\mathbf{r}}_{21})}{r_{21}^2}$$

So we can rewrite the force on wire 1 as:

$$\mathbf{F}_{21} = I_1 \oint_{C_1} d\mathbf{l_1} \times \mathbf{B_2}$$

Now consider a segment $d\mathbf{l}$ of a conductor displaced $d\mathbf{r}$ in a constant magnetic field. The work done is found from:

$$d\mathbf{W} = d\mathbf{F} \cdot d\mathbf{r}$$

If we plug in what we previously found for $d\mathbf{F}$ we get:

$$d\mathbf{W} = (Id\mathbf{l} \times \mathbf{B}) \cdot d\mathbf{r}$$

The area covered by the displacement of the conductor is:

$$d\mathbf{S} = d\mathbf{r} \times d\mathbf{l}$$

Therefore:

$$d\mathbf{W} = I\mathbf{B} \cdot d\mathbf{s} = Id\Phi$$

The differential work can also be given in terms of charge $d\mathbf{q}$ and potential difference V:

$$d\mathbf{W} = Vd\mathbf{q} = VId\mathbf{t}$$

By setting the two equations for differential work equal to each other we arrive at Faraday's Law.

$$d\Phi = V d\mathbf{t}$$

Furthermore, we now see that this is only true if $d\mathbf{W}$ is nonvanishing. Meaning, Faraday's Law is only valid if work is performed in bringing about the change in flux.

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[1] A. G. Kelly, Monographs 5 & 6 of the Institution of Engineers of Ireland, 1998, ISBN 1-898012-37-3 and ISBN 1-898012-42-3]

[2] See, for example, M N O Sadiku (2007). *Elements of Electromagnetics* (http://books.google.com/?id=w2ITHQAACAAJ&dq=isbn:0-19-530048-3) (Fourth ed.). NY/Oxford UK: Oxford University Press. pp. §9.2 pp. 386 ff. ISBN 0-19-530048-3. .

[3] For example, the circuit with the radial segment in Figure 2 according to the right-hand rule *adds* to the applied B-field, tending to increase the flux linkage. That suggests that the flux through this path is decreasing due to the rotation, so $d\theta / dt$ is *negative*.

[4] Richard Phillips Feynman, Leighton R B & Sands M L (2006). *The Feynman Lectures on Physics* (http://books.google.com/?id=zUt7AAAACAAJ&dq=intitle:Feynman+intitle:Lectures+intitle:on+intitle:Physics). San Francisco: Pearson/Addison-Wesley. Vol. II, pp. 17–2, 17–3. ISBN 0805390499. .

[5] Griffiths, David J. (1998), *Introduction to Electrodynamics (3rd ed.)*, Prentice Hall, pp. 516–532, ISBN 0-13-805326-X

[6] Hughes, William; Young, Frederick (1966), *The electromagnetodynamics of fluids*, Wiley, pp. 31

[7] Tilley, D. E., Am. J. Phys. 36, 458 (1968)

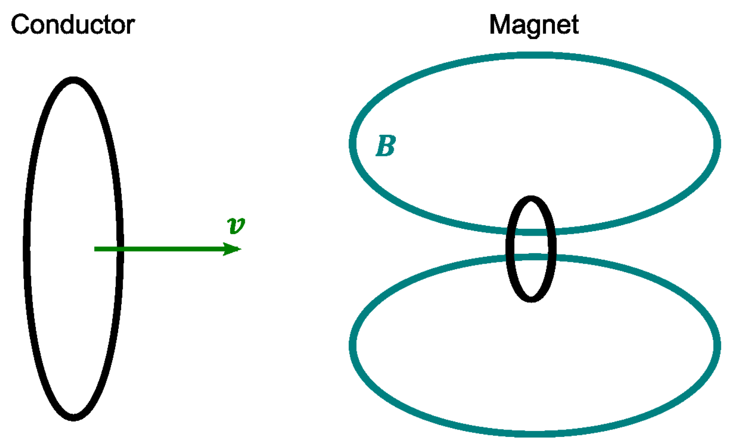
[8] Nussbaum, A., "Faraday's Law Paradox's", http://www.iop.org/EJ/article/0031-9120/7/4/006/pev7i4p231.pdf?request-id=49fbce3f-dbc4-4d6c-98e9-8258814e6c30

Further reading

- Michael Faraday,Experimental Researches in Electricity, Vol I, First Series, 1831 in Great Books of the Western World, Vol 45, R. M. Hutchins, ed., Encyclopædia Britannica, Inc., The University of Chicago, 1952. (http://manybooks.net/titles/faradaym1498614986-8.html)
- "Electromagnetic induction: physics and flashbacks" (PDF) (http://www.brera.unimi.it/sisfa/atti/2001/giuliani.pdf) by Giuseppe Giuliani - details of the Lorentz force in Faraday's disc
- "Homopolar Electric Dynamo" (http://www.spots.ab.ca/~belfroy/Homopolars/homopoltext.html) - contains derivation of equation for EMF of a Faraday disc
- Don Lancaster's "Tech Musings" column, Feb 1998 (http://www.tinaja.com/glib/muse121.pdf) - on practical inefficiencies of Faraday disc
- "Faraday's Final Riddle; Does the Field Rotate with a Magnet?" (PDF) (http://exvacuo.free.fr/div/Sciences/Experiences/Em/Homopolar IEI.pdf) - contrarian theory, but contains useful references to Faraday's experiments
- P. J. Scanlon, R. N. Henriksen, and J. R. Allen, "Approaches to electromagnetic induction," Am. J. Phys. 37, 698–708 (1969). - describes how to apply Faraday's law to Faraday's disc
- Jorge Guala-Valverde, Pedro Mazzoni, Ricardo Achilles "The homopolar motor: A true relativistic engine," Am. J. Phys. 70 (10), 1052–1055 (Oct. 2002). - argues that only the Lorentz force can explain Faraday's disc and describes some experimental evidence for this
- Frank Munley, Challenges to Faraday's flux rule, Am. J. Phys. 72, 1478 (2004). - an updated discussion of concepts in the Scanlon reference above.
- Richard Feynman, Robert Leighton, Matthew Sands, "The Feynman Lectures on Physics Volume II", Chapter 17 - In addition to the Faraday "paradox" (where linked flux does not change but an emf is induced), he describes the "rocking plates" experiment where linked flux changes but no emf is induced. He shows that the correct physics is always given by the combination of the Lorentz force with the Maxwell-Faraday equation (see quotation box) and poses these two "paradoxes" of his own.
- The rotation of magnetic field (http://web.archive.org/web/20091027003359/http://geocities.com/terella1/) by Vanja Janezic - describes a simple experiment that anyone can do. Because it only involves two bodies, its result is less ambiguous than the three-body Faraday, Kelly and Guala-Valverde experiments.

- W. F. Hughes and F. J. Young, The Electromagnetodynamics of Fluids, John Wiley & Sons (1965) LCCC #66-17631. Chapters 1. Principles of Special Relativity and 2. The Electrodynamics of Moving Media. From these chapters it is possible to work all induced emf problems and explain all the associated paradoxes found in the literature.

Moving magnet and conductor problem



Conductor moving in a magnetic field.

The **moving magnet and conductor problem** is a famous thought experiment, originating in the 19th century, concerning the intersection of classical electromagnetism and special relativity. In it, the current in a conductor moving with constant velocity, *v*, with respect to a magnet is calculated in the frame of reference of the magnet and in the frame of reference of the conductor. The observable quantity in the experiment, the current, is the same in either case, in accordance with the basic *principle of relativity*, which states: "Only *relative* motion is observable; there is no absolute standard of rest".[1] However, according to Maxwell's equations, the charges in the conductor experience a **magnetic force** in the frame of the magnet and an **electric force** in the frame of the conductor. The same phenomenon would seem to have two different descriptions depending on the frame of reference of the observer.

This problem, along with Michelson-Morley experiment, formed the basis of Einstein's theory of relativity.

Introduction

Einstein's 1905 paper that introduced the world to relativity opens with a description of the magnet/conductor problem.[2]

> It is known that Maxwell's electrodynamics--as usually understood at the present time--when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighborhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighborhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise--assuming equality of relative motion in the two cases discussed--to electric currents of the same path and intensity as those produced by the electric forces in the former case.
>
> — ***On the electrodynamics of moving bodies***, **A. Einstein**, 1905

An overriding requirement on the descriptions in different frameworks is that they be consistent. Consistency is an issue because Newtonian mechanics predicts one transformation (so-called Galilean invariance) for the *forces* that drive the charges and cause the current, while electrodynamics as expressed by Maxwell's equations predicts that the *fields* that give rise to these forces transform differently (according to Lorentz invariance). Observations of the aberration of light, culminating in the Michelson Morley experiment, established the validity of Lorentz invariance, and the development of special relativity resolved the resulting disagreement with Newtonian mechanics. Special relativity revised the transformation of forces in moving reference frames to be consistent with Lorentz invariance. The details of these transformations are discussed below.

In addition to consistency, it would be nice to consolidate the descriptions so they appear to be frame-independent. A clue to a framework-independent description is the observation that magnetic fields in one reference frame become electric fields in another frame. Likewise, the solenoidal portion of electric fields (the portion that is not originated by electric charges) becomes a magnetic field in another frame: that is, the solenoidal electric fields and magnetic fields are aspects of the same thing.[3] That means the paradox of different descriptions may be only semantic. A description that uses scalar and vector potentials φ and $\boldsymbol{A}$ instead of $\boldsymbol{B}$ and $\boldsymbol{E}$ avoids the semantical trap. A Lorentz-invariant four vector $A^{\alpha} = (\varphi / c_0, \boldsymbol{A})$ replaces **E** and **B**[4] and provides a frame-independent description (albeit less visceral than the **E**– **B**–description).[5] An alternative unification of descriptions is to think of the physical entity as the electromagnetic field tensor, as described later on. This tensor contains both **E** and **B** fields as components, and has the same form in all frames of reference.

Background

Electromagnetic fields are not directly observable. The existence of classical electromagnetic fields can be inferred from the motion of charged particles, whose trajectories are observable. Electromagnetic fields do explain the observed motions of classical charged particles.

A strong requirement in physics is that all observers of the motion of a particle agree on the trajectory of the particle. For instance, if one observer notes that a particle collides with the center of a bullseye, then all observers must reach the same conclusion. This requirement places constraints on the nature of electromagnetic fields and on their transformation from one reference frame to another. It also places constraints on the manner in which fields affect the acceleration and, hence, the trajectories of charged particles.

Perhaps the simplest example, and one that Einstein referenced in his 1905 paper introducing special relativity, is the problem of a conductor moving in the field of a magnet. In the frame of the magnet, a conductor experiences a *magnetic* force. In the frame of a conductor moving relative to the magnet, the conductor experiences a force due to an *electric* field. The magnetic field in the magnet frame and the electric field in the conductor frame must generate consistent results in the conductor. At the time of Einstein in 1905, the field equations as represented by Maxwell's equations were properly consistent. Newton's law of motion, however, had to be modified to provide consistent particle trajectories.[6]

Transformation of fields, assuming Galilean transformations

Assuming that the magnet frame and the conductor frame are related by a Galilean transformation, it is straightforward to compute the fields and forces in both frames. This will demonstrate that the induced current is indeed the same in both frames. As a byproduct, this argument will *also* yield a general formula for the electric and magnetic fields in one frame in terms of the fields in another frame.[7]

In reality, the frames are *not* related by a Galilean transformation, but by a Lorentz transformation. Nevertheless, it will be a Galilean transformation *to a very good approximation*, at velocities much less than the speed of light.

Unprimed quantities correspond to the rest frame of the magnet, while primed quantities correspond to the rest frame of the conductor. Let **v** be the velocity of the conductor, as seen from the magnet frame.

Magnet frame

In the rest frame of the magnet, the magnetic field is some fixed field **B**(**r**), determined by the structure and shape of the magnet. The electric field is zero.

In general, the force exerted upon a particle of charge q in the conductor by the electric field and magnetic field is given by (SI units):

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

where q is the charge on the particle, $\mathbf{v}$ is the particle velocity and **F** is the Lorentz force. Here, however, the electric field is zero, so the force on the particle is

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$

Conductor frame

In the conductor frame, the magnetic field **B**' will be related to the magnetic field **B** in the magnet frame according to:[8]

$$\mathbf{B}'(\mathbf{x}', t) = \mathbf{B}(\mathbf{x}' + \mathbf{v}t).$$

In this frame, there *is* an electric field, generated by the Maxwell-Faraday equation:

$$\nabla \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t} \ .$$

Using the above expression for **B**',

$$\nabla \times \mathbf{E}' = -(\mathbf{v} \cdot \nabla)\mathbf{B} = -\nabla \times (\mathbf{B} \times \mathbf{v}) - \mathbf{v}(\nabla \cdot \mathbf{B}) = -\nabla \times (\mathbf{B} \times \mathbf{v})$$

(using the chain rule and Gauss's law for magnetism). This has the solution:

$$\mathbf{E}' = -\mathbf{B} \times \mathbf{v} = \mathbf{v} \times \mathbf{B} \ .$$

A charge q in the conductor will be at rest in the conductor frame. Therefore, the magnetic force term of the Lorentz force has no effect, and the force on the charge is given by

$$\mathbf{F}' = q\mathbf{E}' = q\mathbf{v} \times \mathbf{B} \ .$$

This demonstrates that *the force is the same in both frames* (as would be expected), and therefore any observable consequences of this force, such as the induced current, would also be the same in both frames. This is despite the fact that the force is seen to be an electric force in the conductor frame, but a magnetic force in the magnet's frame.

Galilean transformation formula for fields

A similar sort of argument can be made if the magnet's frame also contains electric fields. (The Ampere-Maxwell equation also comes into play, explaining how, in the conductor's frame, this moving electric field will contribute to the magnetic field.) The end result is that, in general,

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

$$\mathbf{B}' = \mathbf{B} - \frac{1}{{c_0}^2} \ \mathbf{v} \times \mathbf{E} \ ,$$

with c_0 the speed of light in free space.

By plugging these transformation rules into the full Maxwell's equations, it can be seen that if Maxwell's equations are true in one frame, then they are *almost* true in the other, but contain incorrect terms pro by the Lorentz transformation, and the field transformation equations also must be changed, according to the expressions given below.

Transformation of fields as predicted by Maxwell's equations

In a frame moving at velocity **v**, the **E**-field in the moving frame when there is no **E**-field in the stationary magnet frame Maxwell's equations transform as:[9]

$$\mathbf{E}' = \gamma \mathbf{v} \times \mathbf{B}$$

where

$$\gamma = \frac{1}{\sqrt{1-(v/c_0)^2}}$$

is called the Lorentz factor and c_0 is the speed of light in free space. This result is a consequence of requiring that observers in all inertial frames arrive at the same form for Maxwell's equations. In particular, all observers must see the same speed of light c_0. That requirement leads to the Lorentz transformation for space and time. Assuming a Lorentz transformation, invariance of Maxwell's equations then leads to the above transformation of the fields for this example.

Consequently, the force on the charge is

$$\mathbf{F}' = q\mathbf{E}' = q\gamma \mathbf{v} \times \mathbf{B} \ .$$

This expression differs from the expression obtained from the nonrelativistic Newton's law of motion by a factor of γ. Special relativity modifies space and time in a manner such that the forces and fields transform consistently.

Modification of dynamics for consistency with Maxwell's equations

The Lorentz force has the same *form* in both frames, though the fields differ, namely:

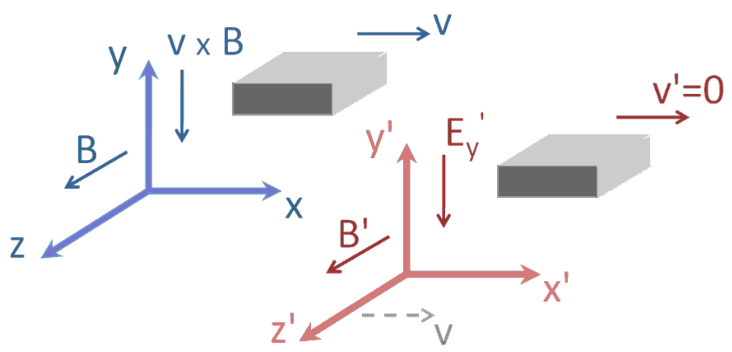


Figure 1: Conducting bar seen from two inertial frames; in one frame the bar moves with velocity **v**; in the *primed* frame the bar is stationary because the primed frame moves at the same velocity as the bar. The **B**-field varies with position in the *x*-direction

$$\mathbf{F} = q\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right] \ .$$

See Figure 1. To simplify, let the magnetic field point in the *z*-direction and vary with location *x*, and let the conductor translate in the positive *x*-direction with velocity *v*. Consequently, in the magnet frame where the conductor is moving, the Lorentz force points in the negative *y*-direction, perpendicular to both the velocity, and the *B*-field. The force on a charge, here due only to the *B*-field, is

$$F_y = -q\ v\ B\ ,$$

while in the conductor frame where the magnet is moving, the force is also in the negative *y*-direction, and now due only to the **E**-field with a value:

$$F_y{}' = q\ E' = -q\ \gamma v\ B.$$

The two forces differ by the Lorentz factor γ. This difference is expected in a relativistic theory, however, due to the change in space-time between frames, as discussed next.

Relativity takes the Lorentz transformation of space-time suggested by invariance of Maxwell's equations and imposes it upon dynamics as well (a revision of Newton's laws of motion). In this example, the Lorentz transformation affects the *x*-direction only (the relative motion of the two frames is along the *x*-direction). The relations connecting time and space are (*primes* denote the moving conductor frame) :[10]

$$x' = \gamma(x - vt) \ x = \gamma(x' + vt') \ ,$$
$$t' = \gamma(t - \frac{vx}{c_0^2}) \ t = \gamma(t' + \frac{vx'}{c_0^2}) \ .$$

These transformations lead to a change in the *y*-component of a force:

$$F_y{}' = \gamma F_y \ .$$

That is, within Lorentz invariance, force is ***not*** the same in all frames of reference, unlike Galilean invariance. But, from the earlier analysis based upon the Lorentz force law:

$$\gamma F_y \ = -q \ \gamma v \ B \ , \ F_y{}' \ = -q \ \gamma v \ B \ ,$$

which agrees completely. So the force on the charge is ***not*** the same in both frames, but it transforms as expected according to relativity.

Newton's law of motion in modern notation

The modern approach to obtaining the relativistic version of Newton's law of motion can be obtained by writing Maxwell's equations in covariant form and identifying a covariant form that is a generalization of Newton's law of motion.

Newton's law of motion can be written in modern covariant notation in terms of the field strength tensor as (cgs units):

$$mc\frac{du^\alpha}{d\tau} = \ F^{\alpha\beta} q u_\beta$$

where m is the particle mass, q is the charge, and

$$u_\beta = \eta_{\beta\alpha} u^\alpha = \eta_{\beta\alpha}\frac{dx^\alpha}{d\tau}$$

is the 4-velocity of the particle. Here, τ is c times the proper time of the particle and η is the Minkowski metric tensor.

The field strength tensor is written in terms of fields as:

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & cB_z & -cB_y \\ -E_y & -cB_z & 0 & cB_x \\ -E_z & cB_y & -cB_x & 0 \end{pmatrix} .$$

Alternatively, using the four vector:

$$A^\alpha = (\phi/c, \ A_x, \ A_y, \ A_z) \ ,$$

related to the electric and magnetic fields by:

$$\mathbf{E} = -\nabla\phi - \partial_t \mathbf{A} \ \mathbf{B} = \nabla \times \mathbf{A} \ ,$$

the field tensor becomes:[11]

$$F^{\alpha\beta} = \frac{\partial A^\beta}{\partial x_\alpha} - \frac{\partial A^\alpha}{\partial x_\beta} \ ,$$

where:

$$x_\alpha = (-ct, \ x, \ y, \ z) \ .$$

The fields are transformed to a frame moving with constant relative velocity by:

$$\acute{F}^{\mu\nu} = \Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta}F^{\alpha\beta},$$

where $\Lambda^{\mu}{}_{\alpha}$ is a Lorentz transformation.

In the magnet/conductor problem this gives

$$\mathbf{E}' = \gamma\frac{\mathbf{v}}{c} \times \mathbf{B},$$

which agrees with the traditional transformation when one takes into account the difference between SI and cgs units. Thus, the relativistic modification to Newton's law of motion using the traditional Lorentz force yields predictions for the motion of particles that are consistent in all frames of reference with Maxwell's equations.

References and notes

[1] The *Laws of Physics* are the same in all inertial frames.

[2] http://www.fourmilab.ch/etexts/einstein/specrel/www/

[3] There are *two* constituents of electric field: a solenoidal field (or *incompressible field*) and a conservative field (or *irrotational field*). The first is transformable to a magnetic field by changing the frame of reference, the second originates in electric charge, and transforms always into an electric field, albeit of different magnitude.

[4] The symbol c_0 represents the speed of light in free space.

[5] However, φ and $\boldsymbol{A}$ are not completely disentangled, so the two types of E-field are not separated completely. See Jackson *From Lorenz to Coulomb and other explicit gauge transformations* (http://arxiv.org/abs/physics/0204034) The author stresses that *Lorenz* is *not* a typo..

[6] Roger Penrose (Martin Gardner: foreword) (1999). *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics* (http://books.google.com/books?id=oI0grArWHUMC&pg=PA248&dq=reference+"laws+of+physics"). Oxford University Press. p. 248. ISBN 0192861980. .

[7] See Jackson, *Classical Electrodynamics*, Section 5.15.

[8] This expression can be thought of as an assumption based on our experience with magnets, that their fields are independent of their velocity. At relativistic velocities, or in the presence of an electric field in the magnet frame, this equation would not be correct.

[9] Tai L. Chow (2006). *Electromagnetic theory* (http://books.google.com/books?id=dpnpMhw1zo8C&pg=PA153&dq=isbn=0763738271&sig=PgEEBA6TQEZ5fD_AhJQ8dd7MGHo#PPA368,M1). Sudbury MA: Jones and Bartlett. Chapter 10.21; p. 402–403 ff. ISBN 0-7637-3827-1. .

[10] Tai L. Chow (2006). *Electromagnetic theory* (http://books.google.com/books?id=dpnpMhw1zo8C&pg=PA153&dq=isbn=0763738271#PPA368,M1). Sudbury MA: Jones and Bartlett. Chapter 10.5; p. 368 ff. ISBN 0-7637-3827-1. .

[11] DJ Griffiths (1999). *Introduction to electrodynamics*. Saddle River NJ: Pearson/Addison-Wesley. p. 541. ISBN 0-13-805326-X.

External links

- Magnets and conductors in special relativity (http://www.physics.ucla.edu/demoweb/demomanual/modern_physics/special_relativity/special_relativity.html)

Further reading

[1] Einstein, A. (1961). *Relativity: The Special and General Theory*. New York: Crown. ISBN 0-517-02961-8.

[2] Misner, Charles; Thorne, Kip S. & Wheeler, John Archibald (1973). *Gravitation*. San Francisco: W. H. Freeman. ISBN 0-7167-0344-0.

[3] Landau, L. D. and Lifshitz, E. M. (1975). *Classical Theory of Fields (Fourth Revised English Edition)*. Oxford: Pergamon. ISBN 0-08-018176-7.

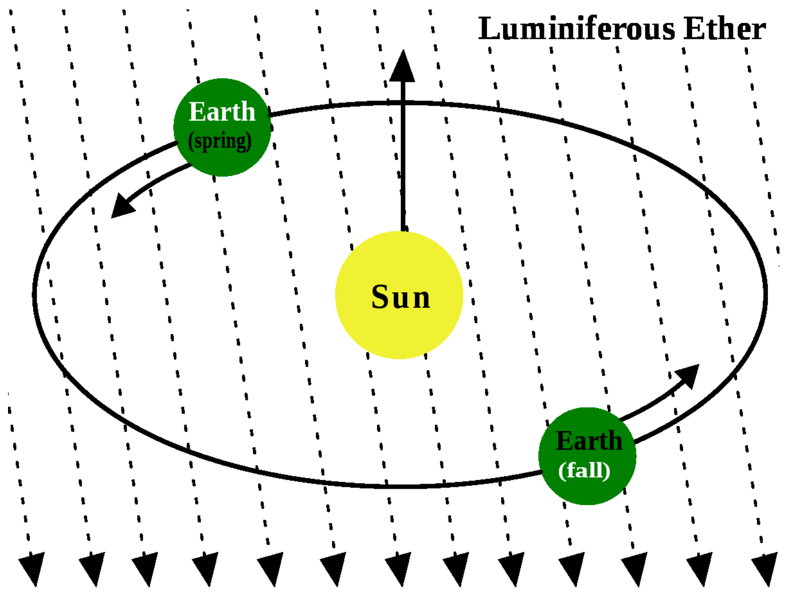
[4] Jackson, John D. (1998). *Classical Electrodynamics (3rd ed.)*. Wiley. ISBN 0-471-30932-X.

[5] C Møller (1976). *The Theory of Relativity* (http://worldcat.org/oclc/220221617&referer=brief_results) (Second Edition ed.). Oxford UK: Oxford University Press. ISBN 019560539X.

Luminiferous aether

In the late 19th century, **luminiferous aether** or **ether**, meaning light-bearing aether, was the term used to describe a medium for the propagation of light.[1]

Due to the negative outcome of aether-drift experiments like the Michelson-Morley experiment, the aether as a mechanical medium having a state of motion, is not used anymore in modern physics, and is replaced by the theory of relativity and quantum theory.



The luminiferous aether: it was hypothesised that the Earth moves through a "medium" of aether that carries light

The history of light and aether

To Robert Boyle in the 17th century, a little before Isaac Newton, the aether was a probable hypothesis and consisted of subtle particles, one sort of which explained the absence of vacuum and the mechanical interactions between bodies, and the other sort of which explained phenomenon such as magnetism (and possibly gravity) that were inexplicable based on the purely mechanical interactions of macroscopic bodies:

> ...though in the ether of the ancients there was nothing taken notice of but a diffused and very subtle substance; yet we are at present content to allow that there is always in the air a swarm of steams moving in a determinate course between the north pole and the south.[2]

Isaac Newton contended that light was made up of numerous small particles. This could explain such features as light's ability to travel in straight lines and reflect off surfaces. This theory was known to have its problems: although it explained reflection well, its explanation of refraction and diffraction was less satisfactory. In order to explain refraction, Newton's *Opticks* (1704) postulated an "Aethereal Medium" transmitting vibrations *faster* than light, by which light, when overtaken, is put into "Fits of easy Reflexion and easy Transmission", which caused refraction and diffraction. Newton believed that these vibrations were related to heat radiation:

> Is not the Heat of the warm Room convey'd through the vacuum by the Vibrations of a much subtiler Medium than Air, which after the Air was drawn out remained in the Vacuum? And is not this Medium the same with that Medium by which Light is refracted and reflected, and by whose Vibrations Light communicates Heat to Bodies, and is put into Fits of easy Reflexion and easy Transmission?[3]

The modern understanding is that heat radiation *is*, like light, electromagnetic radiation. However, Newton considered them to be two different phenomena. He believed heat vibrations to be excited "when a Ray of Light falls upon the Surface of any pellucid Body". He wrote, "I do not know what this Aether is", but that if it consists of particles then they must be "exceedingly smaller than those of Air, or even than those of Light: The exceeding smallness of its Particles may contribute to the greatness of the force by which those Particles may recede from one another, and thereby make that Medium exceedingly more rare and elastic than Air, and by consequence exceedingly less able to resist the motions of Projectiles, and exceedingly more able to press upon gross Bodies, by endeavoring to expand itself."

Christiaan Huygens, prior to Newton, had hypothesized that light was a wave propagating through an aether, but Newton rejected this idea. The main reason for his rejection stemmed from the fact that both men could apparently only envision light to be a longitudinal wave, like sound and other mechanical waves in fluids. However, longitudinal waves by necessity have only one form for a given propagation direction, rather than two polarizations as in a transverse wave, and thus they were unable to explain the phenomenon of birefringence, where two polarizations of light are refracted differently by a crystal. Instead, Newton preferred to imagine non-spherical particles, or "corpuscles", of light with different "sides" that give rise to birefringence. A further reason why Newton rejected light as waves in a medium was because such a medium would have to extend everywhere in space, and would thereby "disturb and retard the Motions of those great Bodies" (the planets and comets) and thus "as it [light's medium] is of no use, and hinders the Operation of Nature, and makes her languish, so there is no evidence for its Existence, and therefore it ought to be rejected."

In 1720 James Bradley carried out a series of experiments attempting to measure stellar parallax. Although he failed to detect any parallax, thereby placing a lower limit on the distance to stars, he discovered another effect, stellar aberration, an effect which depends not on position (as in parallax), but on speed. He noticed that the apparent position of the star changed as the Earth moved around its orbit. Bradley explained this effect in the context of Newton's corpuscular theory of light, by showing that the aberration angle was given by simple vector addition of the Earth's orbital velocity and the velocity of the corpuscles of light, just as vertically falling raindrops strike a moving object at an angle. Knowing the Earth's velocity and the aberration angle, this enabled him to estimate the speed of light. To explain stellar aberration in the context of an aether-based theory of light was regarded as more problematic, because it requires that the aether be stationary even as the Earth moves through it—precisely the problem that led Newton to reject a wave model in the first place.

However, a century later, Young and Fresnel revived the wave theory of light when they pointed out that light could be a transverse wave rather than a longitudinal wave—the polarization of a transverse wave (like Newton's "sides" of light) could explain birefringence, and in the wake of a series of experiments on diffraction the particle model of Newton was finally abandoned. Physicists still assumed, however, that like mechanical waves, light waves required a medium for propagation, and thus required Huygens's idea of an aether "gas" permeating all space.

However, a transverse wave apparently required the propagating medium to behave as a solid, as opposed to a gas or fluid. The idea of a solid that did not interact with other matter seemed a bit odd , and Augustin-Louis Cauchy suggested that perhaps there was some sort of "dragging", or "entrainment", but this made the aberration measurements difficult to understand. He also suggested that the *absence* of longitudinal waves suggested that the aether had negative compressibility. George Green pointed out that such a fluid would be unstable. George Gabriel Stokes became a champion of the entrainment interpretation, developing a model in which the aether might be (by analogy with pine pitch) rigid at very high frequencies and fluid at lower speeds. Thus the Earth could move through it fairly freely, but it would be rigid enough to support light.

Later, Maxwell's equations showed that light is an electromagnetic wave. The apparent need for a propagation medium for such Hertzian waves can be seen by the fact that they consist of perpendicular electric (E) and magnetic (B or H) waves. The E waves consist of undulating dipolar electric fields, and all such dipoles appeared to require separated and opposite electric charges. Electric charge is an inextricable property of matter, so it appeared that some form of matter was required to provide the alternating current that would seem to have to exist at any point along the propagation path of the wave. Propagation of waves in a true vacuum would imply the existence of electric fields without associated electric charge, or of electric charge without associated matter. Albeit compatible with Maxwell's equations, electromagnetic induction of electric fields could not be demonstrated in vacuum, because all methods of detecting electric fields required electrically charged matter.

In addition, Maxwell's equations required that all electromagnetic waves in vacuum propagate at a fixed speed, c. As this can only occur in one reference frame in Newtonian physics (see Galilean-Newtonian relativity), the aether was hypothesized as the absolute and unique frame of reference in which Maxwell's equations hold. That is, the aether must be "still" universally, otherwise *c* would vary along with any variations that might occur in its supportive medium. Maxwell himself proposed several mechanical models of aether based on wheels and gears, and George FitzGerald even constructed a working model of one of them. These models had to agree with the fact that the electromagnetic waves are transverse but never longitudinal.

Nevertheless, by this point the mechanical qualities of the aether had become more and more magical: it had to be a fluid in order to fill space, but one that was millions of times more rigid than steel in order to support the high frequencies of light waves. It also had to be massless and without viscosity, otherwise it would visibly affect the orbits of planets. Additionally it appeared it had to be completely transparent, non-dispersive, incompressible, and continuous at a very small scale. Maxwell wrote in *Encyclopædia Britannica*:[4]

> Aethers were invented for the planets to swim in, to constitute electric atmospheres and magnetic effluvia, to convey sensations from one part of our bodies to another, and so on, until all space had been filled three or four times over with aethers.... The only aether which has survived is that which was invented by Huygens to explain the propagation of light.

Contemporary scientists were aware of the problems, but aether theory was so entrenched in physical law by this point that it was simply assumed to exist. In 1908 Oliver Lodge gave a speech in behalf of Lord Rayleigh [5] to the Royal Institution on this topic, in which he outlined its physical properties, and then attempted to offer reasons why they were not impossible. Nevertheless he was also aware of the criticisms, and quoted Lord Salisbury as saying that "aether is little more than a nominative case of the verb *to undulate*". Others criticized it as an "English invention", although Rayleigh jokingly corrected them to state it was actually an invention of the Royal Institution.

By the early 20th Century, aether theory was in trouble. A series of increasingly complex experiments had been carried out in the late 19th century to try to detect the motion of the Earth through the aether, and had failed to do so. A range of proposed aether-dragging theories could explain the null result but these were more complex, and tended to use arbitrary-looking coefficients and physical assumptions. Lorentz and Fitzgerald offered within the framework of Lorentz ether theory a more elegant solution to how the motion of an absolute aether could be undetectable (length contraction), but if their equations were correct, the new special theory of relativity (1905) could generate the same mathematics without referring to an aether at all. Aether fell to Occam's Razor.[6] [7] [8] [9]

Maxwell's sea of molecular vortices and the indirect experimental determination of the speed of light

In 1856 Wilhelm Eduard Weber and Rudolf Kohlrausch performed an experiment to measure the numerical value of the ratio of the electromagnetic unit of charge to the electrostatic unit of charge. The result came out to be equal to the product of the speed of light and the square root of two. The following year, Gustav Kirchhoff wrote a paper in which he showed that the speed of a signal along an electric wire was equal to the speed of light. These are the first recorded historical links between the speed of light and electromagnetic phenomena. Meanwhile, James Clerk Maxwell was working on Faraday's lines of force. In his 1861 paper *On Physical Lines of Force* he modelled these magnetic lines of force using a sea of molecular vortices that he considered to be partly made of aether and partly made of ordinary matter. He derived expressions for the dielectric constant and the magnetic permeability in terms of the transverse elasticity and the density of this elastic medium. He then equated the ratio of the dielectric constant to the magnetic permeability with a suitably adapted version of Weber and Kohlrausch's result of 1856, and he substituted this result into Newton's equation for the speed of sound. On obtaining a value that was close to the speed of light as measured by Fizeau, Maxwell concluded that light consists in undulations of the same medium that is the cause of electric and magnetic phenomena.[6] [7] [8] [9] Maxwell had however expressed some uncertainties surrounding the precise nature of his molecular vortices and so he began to embark on a purely dynamical approach to the problem. He wrote another famous paper in 1864 under the title of A Dynamical Theory of the Electromagnetic Field in which the details of the luminiferous medium were less explicit.[10] Although Maxwell did not explicitly mention the sea of molecular vortices, his derivation of Ampère's circuital law was carried over from the 1861 paper and he used a dynamical approach involving rotational motion within the electromagnetic field which he likened to the action of flywheels. Using this approach to justify the electromotive force equation (the precursor of the Lorentz force equation), he derived a wave equation from a set of eight equations which appeared in the paper and which included the electromative force equation and Ampère's circuital law.[10] Maxwell once again used the experimental results of Wilhelm Eduard Weber and Rudolf Kohlrausch to show that this wave equation represented an electromagnetic wave that propagates at the speed of light, hence supporting the view that light is a form of electromagnetic radiation.

Relative motion between the Earth and aether

Aether drag

The two most important models, which were aimed to describe the relative motion of the Earth and aether, were Augustin-Jean Fresnel's (1818) model of the (nearly) stationary aether including a partial aether drag determined by Fresnel's dragging coefficient,[11] and George Gabriel Stokes' (1844)[12] model of complete aether drag. The latter theory was not considered as correct, since it was not compatible to the aberration of light, and the auxiliary hypotheses developed to explain this problem, was not convincing. Also, subsequent experiments as the Sagnac effect (1913) also showed that is model is untenable. However, the most important experiment supporting Fresnel's theory, was Fizeau's 1851 experimental confirmation of Fresnel's 1818 prediction that a medium with refractive index *n* moving with a velocity *v* would increase the speed of light traveling through the medium in the same direction as *v* from *c*/*n* to:[13] [14]

$$\frac{c}{n} + \left(1 - \frac{1}{n^2}\right) v.$$

That is, movement adds only a fraction of the medium's velocity to the light (predicted by Fresnel in order to make Snell's law work in all frames of reference, consistent with stellar aberration). This was initially interpreted to mean that the medium drags the aether along, with a *portion* of the medium's velocity, but that understanding became very problematic after Wilhelm Veltmann demonstrated that the index *n* in Fresnel's formula depended upon the wavelength of light, so that the aether could not be moving at a wavelength-independent speed. This implied that there must be a separate aether for each of the infinitely many frequencies.

Negative aether-drift experiments

The key difficulty with Fresnel's aether hypothesis arose from the juxtaposition of the two well-established theories of Newtonian dynamics and Maxwell's electromagnetism. Under a Galilean transformation the equations of Newtonian dynamics are invariant, whereas those of electromagnetism are not. Basically this means that while physics should remain the same in non-accelerated experiments, light would not follow the same rules because it is traveling in the universal "aether frame". Some effect caused by this difference should be detectable.

A simple example concerns the model on which aether was originally built: sound. The speed of propagation for mechanical waves, the speed of sound, is defined by the mechanical properties of the medium. For instance, if one is in an airliner, you can still carry on a conversation with the person beside you because the sound of your words are traveling along with the air inside the aircraft. This effect is basic to all Newtonian dynamics, which says that everything from sound to the trajectory of a thrown baseball should all remain the same in the aircraft as sitting still on the Earth. This is the basis of the Galilean transformation, and the concept of frame of reference.

But the same was not true for light, since Maxwell's mathematics demanded a single universal speed for the propagation of light, based, not on local conditions, but on two measured properties, the permittivity and permeability of free space, that were assumed to be the same throughout the universe. If these numbers did change, there should be noticeable effects in the sky; stars in different directions would have different colors, for instance .

Thus at any point there should be one special coordinate system, "at rest relative to the aether". Maxwell noted in the late 1870s that detecting motion relative to this aether should be easy enough—light traveling along with the motion of the Earth would have a different speed than light traveling backward, as they would both be moving against the unmoving aether. Even if the aether had an overall universal flow, changes in position during the day/night cycle, or over the span of seasons, should allow the drift to be detected.

First order experiments

Although the aether is almost stationary according to Fresnel, his theory predicts a positive outcome of aether drift experiments only to *second* order in v/c, while Fresnel's dragging coefficient would cause a negative outcome of all optical experiments capable of measuring effects to *first* order in v/c. This was confirmed by the following first-order experiments, which all gave negative results (the following list is based on the description of Wilhelm Wien (1898), with changes and additional experiments according to the descriptions of Edmund Taylor Whittaker (1910) and Jakob Laub (1910):[15] [6] [16]

- The experiment of François Arago (1810), to confirm whether refraction, and thus the aberration of light, is influenced by Earth's motion. Similar experiments were conducted by George Biddell Airy (1871) by means of a telescope filled with water, and Éleuthère Mascart (1872).[17] [18] [19]
- The experiment of Fizeau (1860), to find whether the rotation of the polarization plane through glass columns is changed by Earth's motion. He obtained a positive result, but Lorentz could show that the results have been contradictory. DeWitt Bristol Brace (1905) and Straßer (1907) repeated the experiment with improved accuracy, and obtained negative results.[20] [21] [22]
- The experiment of Martin Hoek (1868). This experiment is a more precise variation of the famous Fizeau experiment (1851). Two light rays were sent in opposite directions – one of them traverses a path filled with resting water, the other one follows a path through air. In agreement with Fresnel's dragging coefficient, he obtained a negative result.[23]
- The experiment of Wilhelm Klinkerfues (1870) investigated, whether an influence of Earth's motion on the absorption line of sodium exists. He obtained a positive result, but this was shown to be an experimental error, because a repetition of the experiment by Haga (1901) gave a negative result.[24] [25]
- The experiment of Ketteler (1872), in which two rays of an interferometer were sent in opposite directions through two mutually inclined tubes filled with water. No change of the interference fringes occurred. Later, Mascart (1872) showed that the interference fringes of polarized light in calcite remained uninfluenced as well.[26] [27]
- The experiment of Éleuthère Mascart (1872) to find a change of rotation of the polarization plane in quartz. No change of rotation was found when the light rays had the direction of Earth's motion, and then the opposite direction. Lord Rayleigh conducted similar experiments with improved accuracy, and obtained a negative result as well.[19] [27] [28]

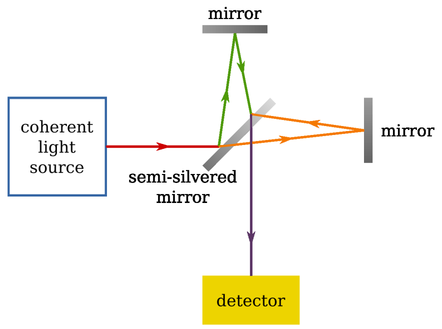
Besides those optical experiments, also electrodynamic first-order experiments were conducted, which should lead to positive results according to Fresnel. However, Hendrik Antoon Lorentz (1895) modified Fresnel's theory and showed that those experiments can be explained by a stationary aether as well:[29]

- The experiment of Wilhelm Röntgen (1888), to find whether a charged condenser produces magnetic forces due to Earth's motion.[30]
- The experiment of Theodor des Coudres (1889), to find whether the inductive effect of two wire rolls upon a third one, is influenced by the direction of Earth's motion. Lorentz showed, that this effect is canceled to first order by the electrostatic charge (produced by Earth's motion) upon the conductors.[31]
- The experiment of Königsberger (1905). The plates of a condenser are located in the field of a strong electromagnet. Due to Earth's motion, the plates shall be charged. No such effect was observed.[32]

- The experiment of Frederick Thomas Trouton (1902). A condenser was brought parallel to Earth's motion and it was assumed, that momentum is produced when the condenser is charged. The negative result can be explained by Lorentz's theory, according to which the electromagnetic momentum compensates the momentum due to Earth's motion. Lorentz could also show, that the sensibility of the apparatus was much too low to observe such an effect.[33]

Second order experiments

While the *first*-order experiments could be explained by a modified stationary aether, more precise *second*-order experiments should bring positive results, however, no such results could be found.



The Michelson–Morley experiment compared the time for light to reflect from mirrors in two orthogonal directions. It is commonly held to disprove light propagation through a luminiferous aether.

The famous Michelson-Morley experiment compared the source light with itself after being sent in different directions, looking for changes in phase in a manner that could be measured with extremely high accuracy.[34] [35] The publication of their result in 1887, the null result, was the first clear demonstration that something was seriously wrong with the aether concept of that time (after Michelson's first experiment in 1881 that wasn't fully conclusive). In this case the MM experiment yielded a shift of the fringing pattern of about 0.01 of a fringe, corresponding to a small velocity. However, it was incompatible with the expected aether wind effect due to the Earth's (seasonally varying) velocity which would have required a shift of 0.4 of a fringe, and the error was small enough that the value may have indeed been zero. Therefore, the null hypothesis, the hypothesis that there was no aether wind, could not be rejected. More modern experiments have since reduced the possible value to a number very close to zero, about 10^{-17}.

A series of experiments using similar but increasingly sophisticated apparatuses all returned the null result as well. Conceptually different experiments that also attempted to detect the motion of the aether were the Trouton-Noble experiment (1903)[36] and the Experiments of Rayleigh and Brace (1902, 1904)[37] [38] to detect double refraction, which like Michelson-Morley (MM) obtained a null result.

These "aether-wind" experiments led to the abandonment of the aether concept by some scientists like Emil Cohn or Alfred Bucherer, and to a flurry of efforts to "save" aether by assigning it ever more complex properties by others. Of particular interest was the possibility of "aether entrainment" or "aether drag", which would lower the magnitude of the measurement, perhaps enough to explain MMX results. However, as noted earlier, aether dragging already had problems of its own, notably aberration. In addition, the interference experiments of Lodge (1893, 1897) and Ludwig Zehnder (1895), aimed to show whether the aether is dragged by various, rotating masses, showed no aether drag.[39] [40] [41] A more precise measurement was made in the Hammar experiment (1935), which ran a complete MM experiment with one of the "legs" placed between two massive lead blocks.[42] If the aether was dragged by mass then this experiment would have been able to detect the drag caused by the lead, but again the null result was found. The theory was again modified, this time to suggest that the entrainment only worked for very large masses or those masses with large magnetic fields. This too was shown to be incorrect by the Michelson–Gale–Pearson experiment, which detected the Sagnac effect due to Earth's rotation (s. Aether drag hypothesis)

Another, completely different, attempt to save "absolute" aether was made in the Lorentz-Fitzgerald contraction hypothesis, which posited that *everything* was affected by travel through the aether. In this theory the reason the Michelson-Morley experiment "failed" was that the apparatus contracted in length in the direction of travel. That is, the light was being affected in the "natural" manner by its travel though the aether as predicted, but so was the apparatus itself, canceling out any difference when measured. Fitzgerald had inferred this hypothesis from a paper by Oliver Heaviside. Without referral to an aether, this physical interpretation of relativistic effects was shared by

Kennedy and Thorndike in 1932 as they concluded that the interferometer's arm contracts and also the frequency of its light source "very nearly" varies in the way required by relativity.[43] [44]

Similarly the Sagnac effect, observed by G. Sagnac in 1913 was immediately seen to be fully consistent with special relativity.[45] [46] In fact, the Michelson-Gale-Pearson experiment in 1925 was proposed specifically as a test to confirm the relativity theory, although it was also recognized that such tests, which merely measure absolute rotation, are also consistent with non-relativistic theories.[47]

During the 1920s, the experiments pioneered by Michelson were repeated by Dayton Miller, who publicly proclaimed positive results on several occasions, although not large enough to be consistent with any known aether theory. In any case, other researchers were unable to duplicate Miller's claimed results, and in subsequent years the experimental accuracy of such measurements has been raised by many orders of magnitude, and no trace of any violations of Lorentz invariance has been seen. (A later re-analysis of Miller's results concluded that he had underestimated the variations due to temperature.)

Since the Miller experiment and its unclear results there have been many more experiments to detect the aether. Many of the experimenters have claimed positive results. These results have not gained much attention from mainstream science, since they are in contradiction to a large quantity of high-precision measurements, all of them confirming special relativity.[48]

Lorentz aether theory

Between 1892 and 1904, Hendrik Lorentz created an electron/aether theory, in which he introduced a strict separation between matter (electrons) and aether. In his model the aether is completely motionless, and it won't be set in motion in the neighborhood of ponderable matter. Contrary to other electron models before, the electromagnetic field of the aether appears as a mediator between the electrons, and changes in this field can propagate not faster than the speed of light. A fundamental concept of Lorentz's theory in 1895 was the "theorem of corresponding states" for terms of order v/c. This theorem states that a moving observer (relative to the aether) in his "fictitious" field makes the same observations as a resting observers in his "real" field. Lorentz noticed that it was necessary to change the space-time variables when changing frames and introduced concepts like physical length contraction (1892)[49] to explain the Michelson-Morley experiment, and the mathematical concept of local time (1895) to explain the aberration of light and the Fizeau experiment. That resulted in the formulation of the so called Lorentz transformation by Joseph Larmor (1897, 1900)[50] [51] and Lorentz (1899, 1904),[29] [52] whereby it was noted by Larmor that the complete formulation of local time is accompanied by some sort of time dilation of moving electrons in the aether.[53] [8] [54]

The work of Lorentz was mathematically perfected by Henri Poincaré who formulated on many occasions the Principle of Relativity and tried to harmonize it with electrodynamics. He declared simultaneity only a convenient convention which depends on the speed of light, whereby the constancy of the speed of light would be a useful postulate for making the laws of nature as simple as possible. In 1900 and 1904[55] [56] he interpreted Lorentz's local time as the result of clock synchronization by light signals. And finally in June and July 1905[57] [58] he declared the relativity principle a general law of nature, including gravitation. He corrected some mistakes of Lorentz and proved the Lorentz covariance of the electromagnetic equations. However, he used the notion of an aether as a perfectly undetectable medium and distinguished between apparent and real time, so most historians of science argue that he failed to invent special relativity.[53] [59] [8]

End of aether?

Special relativity

Aether theory was dealt another blow when the Galilean transformation and Newtonian dynamics were both modified by Albert Einstein's special theory of relativity, giving the mathematics of Lorentzian electrodynamics a new, "non-aether" context.[60] Unlike most major shifts in scientific thought, special relativity was adopted by the scientific community remarkably quickly, consistent with Einstein's later comment that the laws of physics described by the Special Theory were "ripe for discovery" in 1905.[61] Max Planck's early advocacy of the special theory, along with the elegant formulation given to it by Minkowski, contributed much to the rapid acceptance of special relativity among working scientists.

Einstein based his theory on Lorentz's earlier work. Instead of suggesting that the mechanical properties of objects changed with their constant-velocity motion through an undetectable aether, Einstein proposed to deduce the characteristics that any successful theory must possess in order to be consistent with the most basic and firmly established principles, independent of the existence of a hypothetical aether. He found that the Lorentz transformation must transcend its connection with Maxwell's equations, and must represent the fundamental relations between the space and time coordinates of inertial frames of reference. In this way he demonstrated that the laws of physics remained invariant as they had with the Galilean transformation, but that light was now invariant as well.

With the development of the special relativity, the need to account for a single universal frame of reference had disappeared — and acceptance of the 19th century theory of a luminiferous aether disappeared with it. For Einstein, the Lorentz transformation implied a conceptual change: that the concept of position in space or time was not absolute, but could differ depending on the observer's location and velocity.

Moreover, in another paper published the same month in 1905, Einstein made several observations on a then-thorny problem, the photoelectric effect. In this work he demonstrated that light can be considered as particles that have a "wave-like nature". Particles obviously do not need a medium to travel, and thus, neither did light. This was the first step that would lead to the full development of quantum mechanics, in which the wave-like nature *and* the particle-like nature of light are both considered to be descriptions of the same thing. A summary of Einstein's thinking about the aether hypothesis, relativity and light quanta may be found in his 1909 (originally German) lecture "The Development of Our Views on the Composition and Essence of Radiation".[62]

Lorentz on his side continued to use the aether concept. In his lectures of around 1911 he pointed out that what "the theory of relativity has to say ... can be carried out independently of what one thinks of the aether and the time". He commented that "whether there is an aether or not, electromagnetic fields certainly exist, and so also does the energy of the electrical oscillations" so that, "if we do not like the name of "aether", we must use another word as a peg to hang all these things upon." He concluded that "one cannot deny the bearer of these concepts a certain substantiality".[63] [53]

In later years there have been a few individuals who advocated a neo-Lorentzian approach to physics, which is Lorentzian in the sense of positing an absolute true state of rest that is undetectable and which plays no role in the predictions of the theory. (No violations of Lorentz covariance have ever been detected, despite strenuous efforts.) Hence these theories resemble the 19th century aether theories in name only. For example, the founder of quantum field theory, Paul Dirac, stated in 1951 in an article in Nature, titled "Is there an Aether?" that "we are rather forced to have an aether".[64] [65] However, Dirac never formulated a complete theory, and so his speculations found no acceptance by the scientific community.

Einstein's views on the aether

In 1916, after Einstein completed his foundational work on general relativity, Lorentz wrote a letter to him in which he speculated that within general relativity the aether was re-introduced. In his response Einstein wrote that one can actually speak about a "new aether", but one may not speak of motion in relation to that aether. This was further elaborated by Einstein in some semi-popular articles (1918, 1920, 1924, 1930).[66] [67] [68] [69] [70] [71] [72]

In 1918 Einstein publicly alluded to that new definition for the first time.[66] Then, in the early 1920s, in a lecture which he was invited to give at Lorentz's university in Leiden, Einstein sought to reconcile the theory of relativity with his mentor's cherished concept of the aether. In this lecture Einstein stressed that special relativity took away the last mechanical property of Lorentz's aether: immobility. However, he continued that special relativity does not necessarily rule out the aether, because the latter can be used to give physical reality to acceleration and rotation. This concept was fully elaborated within general relativity, in which physical properties (which are partially determined by matter) are attributed to space, but no substance or state of motion can be attributed to that "aether" (aether = curved space-time).[72] [67] [73]

In another paper of 1924, named "Concerning the Aether", Einstein argued that Newton's absolute space, in which acceleration is absolute, is the "Aether of Mechanics". And within the electromagnetic theory of Maxwell and Lorentz one can speak of the "Aether of Electrodynamics", in which the aether possesses an absolute state of motion. As regards special relativity, also in this theory acceleration is absolute as in Newton's mechanics. However, the difference from the electromagnetic aether of Maxwell and Lorentz lies in the fact, that "*because it was no longer possible to speak, in any absolute sense, of simultaneous states at different locations in the aether, the aether became, as it were, four dimensional, since there was no objective way of ordering its states by time alone.*". Now the "aether of special relativity" is still "absolute", because matter is affected by the properties of the aether, but the aether is not affected by the presence of matter. This asymmetry was solved within general relativity. Einstein explained that the "aether of general relativity" is not absolute, because matter is influenced by the aether, just as matter influences the structure of the aether.[68]

So the only similarity of this relativistic aether concept with the classical aether models lies in the presence of physical properties in space. Therefore, as historians such as John Stachel argue, Einstein's views on the "new aether" are not in conflict with his abandonment of the aether in 1905. For, as Einstein himself pointed out, no "substance" and no state of motion can be attributed to that new aether. In addition, Einstein's use of the word "aether" found little support in the scientific community, and played no role in the continuing development of modern physics.[70] [71] [72]

Aether concepts

- Aether theories
- Aether (classical element)
- Aether drag hypothesis

Notes

[1] The 19th century science book A Guide to the Scientific Knowledge of Things Familiar provides a brief summary of scientific thinking in this field at the time.

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Experiments

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External links

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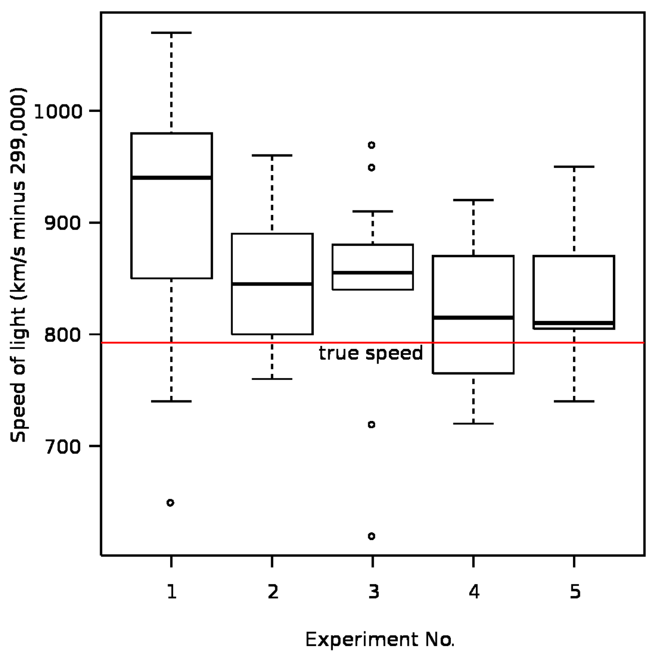
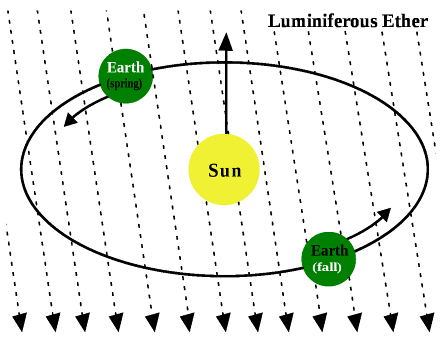
Michelson–Morley experiment

The **Michelson–Morley experiment** was performed in 1887 by Albert Michelson and Edward Morley at what is now Case Western Reserve University in Cleveland, Ohio. Its results are generally considered to be the first strong evidence against the theory of a luminiferous ether and in favor of special relativity. The most immediate effect at the time was to put an end to Lord Kelvin's Vortex theory, which said that atoms were vortices in the ether.[1] The experiment has also been referred to as "the moving-off point for the theoretical aspects of the Second Scientific Revolution".[2] See also Tests of special relativity.



Box plots based on data from the Michelson–Morley experiment

Measuring ether



A depiction of the concept of the "aether wind"

Physics theories of the late 19th century postulated that, just as water waves must have a medium to move across (water), and audible sound waves require a medium to move through (such as air or water), so also light waves require a medium, the "luminiferous aether". Because light can travel through a vacuum, it was assumed that the vacuum must contain the medium of light. Because the speed of light is so great, designing an experiment to detect the presence and properties of this aether took considerable ingenuity.

Earth travels a tremendous distance in its orbit around the Sun, at a speed of around 30 km/s or over 108,000 km per hour. The Sun itself is travelling about the Galactic Center at even greater speeds, and there are other motions at higher levels of the structure of the universe. Since the Earth is in motion, it was expected that the flow of aether across the Earth should produce a detectable "aether wind". Although it would be possible, in theory, for the Earth's motion to match that of the aether at one moment in time, it was not possible for the Earth to remain at rest with respect to the aether at all times, because of the variation in both the direction and the speed of the motion.

At any given point on the Earth's surface, the magnitude and direction of the wind would vary with time of day and season. By analysing the return speed of light in different directions at various different times, it was thought to be possible to measure the motion of the Earth relative to the aether.

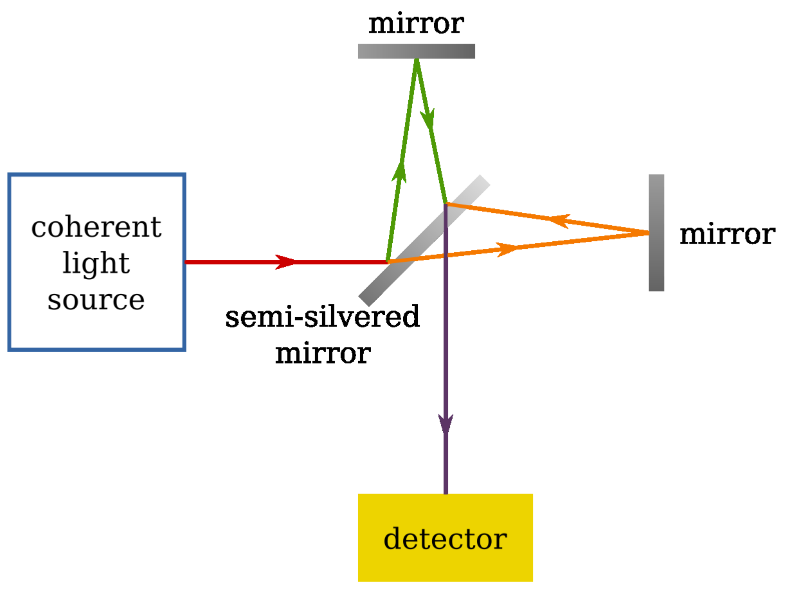
The expected difference in the measured speed of light was quite small, given that the velocity of the Earth in its orbit around the Sun was about one hundredth of one percent of the speed of light. A number of physicists had attempted to make this measurement during the mid-19th century, but the accuracy demanded was simply too great for existing experimental setups. For instance, the Fizeau–Foucault apparatus could measure the speed of light to perhaps 5% accuracy, not nearly enough to make any sort of aether wind measurement.

The 1881 and 1887 experiments

Michelson had a solution to the problem of how to construct a device sufficiently accurate to detect aether flow. The device he designed, later known as an interferometer, sent a single source of white light through a half-silvered mirror that was used to split it into two beams travelling at right angles to one another. After leaving the splitter, the beams travelled out to the ends of long arms where they were reflected back into the middle on small mirrors. They then recombined on the far side of the splitter in an eyepiece, producing a pattern of constructive and destructive interference based on the spent time to transit the arms. If the Earth is traveling through an ether medium, a beam reflecting back and forth parallel to the flow of ether would take longer than a beam reflecting perpendicular to the ether because the time gained from traveling downwind is less than that lost traveling upwind. The result would be a delay in one of the light beams that could be detected when the beams were recombined through interference. Any slight change in the spent time would then be observed as a shift in the positions of the interference fringes. If the aether were stationary relative to the Sun, then the Earth's motion would produce a fringe shift 4% the size of a single fringe.

While teaching at his alma mater in 1877, the U.S. Naval Academy in Annapolis, Michelson conducted his first known successful light speed experiments as a part of a classroom demonstration. He left active U.S. Naval service in 1881, while he was in Germany concluding studies there. In that year and while there, Michelson had used an experimental device to make several more measurements, in which he noticed that the expected shift of 0.04 was not seen, and a smaller shift of (at most) about 0.02 was.[3] However his apparatus was a prototype, and had experimental errors far too large to say anything about the aether wind. For a measurement of the aether wind, a much more accurate and tightly controlled experiment would have to be carried out. The prototype was, however, successful in demonstrating that the basic method was feasible.

Six years later he collaborated with Edward Morley, spending considerable time and money to create an improved version with more than enough accuracy to detect the drift.[4] At this time Michelson was professor of physics at the Case School of Applied Science, and Morley was professor of chemistry at Western Reserve University, which shared a campus with the Case School on the eastern edge of Cleveland. The experiment was performed in several periods of concentrated observations between April and July 1887, in Adelbert Dormitory of WRU (later renamed Pierce Hall, demolished in 1962).[5] [6]



Though using a contemporary laser, this Michelson interferometer is the same in principle as those used in the original experiment.

In their experiment, the light was repeatedly reflected back and forth along the arms of the interferometer, increasing the path length to 11 m. At this length, the drift would be about 0.4 fringes. To make that easily detectable, the apparatus was assembled in a closed room in the basement of the heavy stone dormitory, eliminating most thermal and vibrational effects. Vibrations were further reduced by building the apparatus on top of a large block of sandstone, about a foot thick and five feet square, which was then floated in an annular trough of mercury. They calculated that effects of about 1/100th of a fringe would be detectable.

The mercury pool allowed the device to be easily turned, so that given a single steady push it would slowly rotate through the entire range of possible angles to the "aether wind", while measurements were continuously observed by looking through the eyepiece. Even over a period of minutes some sort of effect would be noticed, since one of the arms would inevitably turn into the direction of the wind and the other away. Over longer periods day/night cycles or yearly cycles would also be easily measurable.

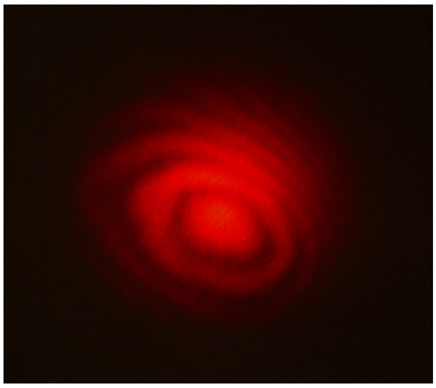
During each full rotation of the device, each arm would be parallel to the wind twice (facing into and away from the wind) and perpendicular to the wind twice. This effect would show readings in a sine wave formation with two peaks and two troughs. Additionally, if the wind were only from Earth's orbit around the Sun, the wind would fully change directions east/west during a 12-hour period. In this ideal conceptualization, the sine wave of day/night readings would be of opposing phase.

Because it was assumed that the motion of the Earth around the Sun would cause an additional component to the wind, the yearly cycles would be detectable as an alteration of the magnitude of the wind. An example of this effect is a helicopter flying forward. While hovering, a helicopter's blades would be measured as travelling around typically at 300 mph at the tips. However, if the helicopter is travelling forward at 150 mph, there are points where the tips of the blades are travelling through the air at 150 mph (downwind) and 450 mph (upwind). The same effect would cause the magnitude of an aether wind to decrease and increase on a yearly basis.

Most famous "failed" experiment

Subsequent experiments

After all this thought and preparation, the experiment became what might be called the most famous failed experiment to date.[7] Instead of providing insight into the properties of the aether, Michelson and Morley's article in the *American Journal of Science* reported the measurement to be as small as one-fortieth of the expected displacement but "since the displacement is proportional to the square of the velocity" they concluded that the measured velocity was "probably less than one-sixth" of the expected velocity of the Earth's motion in orbit and "certainly less than one-fourth." Although this small "velocity" was measured, it was considered far too small to be used as evidence of speed relative to the aether, and it was later said to be within the range of an experimental error that would allow the speed to actually be zero.



Interference pattern produced with a Michelson Interferometer using a red laser.

Although Michelson and Morley went on to different experiments after their first publication in 1887, both remained active in the field. Other versions of the experiment were carried out with increasing sophistication. Roy J. Kennedy and K. K. Illingworth both modified the mirrors to include a half-wave "step", eliminating the possibility of some sort of standing wave pattern within the apparatus. Illingworth could detect changes on the order of 1/300th of a fringe, Kennedy up to 1/1500th. Miller later built a non-magnetic device to eliminate magnetostriction, while Michelson built one of non-expanding invar to eliminate any remaining thermal effects. Others from around the world increased accuracy, eliminated possible side effects, or both.

Morley was not convinced of his own results, and went on to conduct additional experiments with Dayton Miller. Miller worked on increasingly large experiments, culminating in one with a 32 m (effective) arm length at an installation at the Mount Wilson observatory. To avoid the possibility of the aether wind being blocked by solid walls, he used a special shed with thin walls, mainly of canvas. He consistently measured a small positive effect that varied with each rotation of the device, the sidereal day and on a yearly basis. His measurements amounted to approximately 10 km/s instead of the nearly 30 km/s expected from the Earth's orbital motion alone. He remained convinced this was due to partial entrainment, though he did not attempt a detailed explanation.

Though Kennedy later also carried out an experiment at Mount Wilson, finding 1/10 the drift measured by Miller, and no seasonal effects, Miller's findings were considered important at the time, and were discussed by Michelson, Lorentz and others at a meeting reported in 1928 (ref below). There was general agreement that more experimentation was needed to check Miller's results. Lorentz recognised that the results, whatever their cause, did not quite tally with either his or Einstein's versions of special relativity. Einstein was not present at the meeting and felt the results could be dismissed as experimental error (see Shankland ref below). To date, no-one has been able to replicate Miller's results, and modern experimental accuracies are considered to have ruled them out.[8]

Also note, that the expected values are related to the relative speed between Earth and Sun of 30 km/s. With respect to the speed of the solar system around the galactic center of ca. 220 km/s, or the speed of the solar system relative to the CMB rest frame of ca. 368 km/s, the zero results of those experiments are even more obvious.

Name	**Location**	**Year**	**Arm length (meters)**	**Fringe shift expected**	**Fringe shift measured**	**Ratio**	**Upper Limit on V_{aether}**	**Experimental Resolution**	**Null result**
Michelson[3]	Potsdam	1881	1.2	0.04	≤ 0.02	2	~ 20 km/s	0,02	≈ yes
Michelson and Morley[4]	Cleveland	1887	11.0	0.4	< 0.02 or ≤ 0,01	40	~ 4–8 km/s	0,01	≈ yes
Morley and Miller[9] [10]	Cleveland	1902–1904	32.2	1.13	≤ 0.015	80	~ 3,5 km/s	0,015	yes
Miller[11]	Mt. Wilson	1921	32.0	1.12	≤ 0.08	15	~ 8–10 km/s	unclear	unclear
Miller[11]	Cleveland	1923–1924	32.0	1.12	≤ 0.03	40	~ 5 km/s	0.03	yes
Miller (sunlight)[11]	Cleveland	1924	32.0	1.12	≤ 0.014	80	~ 3 km/s	0.014	yes
Tomaschek (star light)[12]	Heidelberg	1924	8.6	0.3	≤ 0.02	15	~ 7 km/s	0.02	yes
Miller[11] [13]	Mt. Wilson	1925–1926	32.0	1.12	≤ 0.088	13	~ 8–10 km/s	unclear	unclear
Kennedy[14]	Pasadena/Mt. Wilson	1926	2.0	0.07	≤ 0.002	35	~ 5 km/s	0.002	yes
Illingworth[15]	Pasadena	1927	2.0	0.07	≤ 0.0004	175	~ 2 km/s	0.0004	yes
Piccard & Stahel[16]	with a Balloon	1926	2.8	0.13	≤ 0.006	20	~ 7 km/s	0.006	yes
Piccard & Stahel[17]	Brussels	1927	2.8	0.13	≤ 0.0002	185	~ 2,5 km/s	0.0007	yes
Piccard & Stahel[18]	Rigi	1927	2.8	0.13	≤ 0.0003	185	~ 2,5 km/s	0.0007	yes
Michelson et al.[19]	Mt. Wilson	1929	25.9	0.9	≤ 0.01	90	~ 3 km/s	0.01	yes
Joos[20]	Jena	1930	21.0	0.75	≤ 0.002	375	~ 1,5 km/s	0.002	yes

Recent experiments

Further information: Modern searches for Lorentz violation

In recent times experiments similar to the Michelson–Morley experiment have become commonplace. Lasers and masers amplify light by repeatedly bouncing it back and forth inside a carefully tuned cavity, thereby inducing high-energy atoms in the cavity to give off more light. The result is an effective path length of kilometers. Better yet, the light emitted in one cavity can be used to start the same cascade in another set at right angles, thereby creating an interferometer of extreme accuracy.[21] The first such experiment was led by Charles H. Townes, one of the co-creators of the first maser. Their 1958 experiment put an upper limit on drift, including any possible experimental errors, of only 30 m/s. In 1974 a repeat with accurate lasers in the triangular Trimmer experiment reduced this to 0.025 m/s, and included tests of entrainment by placing one leg in glass. [22]

The most precise experiments of this kind (using laser, maser, cryogenic optical resonators, etc.) were made in recent years. In some of those experiments, the devices were rotated or remained stationary, and some were combined with the Kennedy–Thorndike experiment. Tests on Lorentz invariance achieving a comparable precision, are the Hughes–Drever experiments.

Author	Year	Maximum anisotropy of c
Brillet & Hall[23]	1979	$\lesssim 10^{-15}$
Wolf et al.[24]	2003	
Müller et al.[25]	2003	
Wolf et al.[26]	2004	
Wolf et al.[27]	2004	
Antonini et al.[28]	2005	$\lesssim 10^{-16}$
Stanwix et al.[29]	2005	
Herrmann et al.[30]	2005	
Stanwix et al.[31]	2006	
Müller et al.[32]	2007	
Eisele et al.[33]	2009	$\lesssim 10^{-17}$
Herrmann et al.[34]	2009	

Fallout

Further information: History of special relativity

Einstein and special relativity

The constancy of the speed of light was postulated by Albert Einstein in 1905,[35] motivated by Maxwell's theory of electromagnetism and the lack of evidence for the luminiferous ether but not, contrary to widespread belief, by the null result of the Michelson–Morley experiment.[36] However the null result of the Michelson–Morley experiment helped the notion of the constancy of the speed of light gain widespread and rapid acceptance.

Aether dragging

Initially, the experiment of 1881 was meant to distinguish between the theory of Augustin-Jean Fresnel (1818), who proposed an almost stationary aether, and in which the aether is only partially dragged with a certain coefficient by matter; and the theory of George Gabriel Stokes (1845), who stated that the aether was fully dragged in the vicinity of the earth. Michelson initially believed the negative outcome confirmed the theory of Stokes. However, Hendrik Lorentz showed in 1886, that Stokes's explanation of aberration is contradictory.[37] [38]

Also the assumption that the aether is not carried in the vicinity, but only *within* matter, was very problematic as shown by the Hammar experiment (1935). Hammar placed one arm of the interferometer between two huge lead blocks. If aether were dragged by mass, the blocks would, it was theorized, have been enough to cause a visible effect. Once again, no effect was seen, so any such theory is considered as disproved.

Emission theory

Walter Ritz's emitter theory (or ballistic theory), was also consistent with the results of the experiment, not requiring aether. The theory postulates that light has always the same velocity in respect to the source.[39] However it also led to several "obvious" optical effects that were not seen in astronomical photographs, notably in observations of binary stars in which the light from the two stars could be measured in an interferometer. If this was correct, the light from the stars should cause fringe shifting due to the velocity of the stars being added to the speed of the light, but again, no such effect could be seen.

The Sagnac experiment placed a modified apparatus on a constantly rotating turntable; the main modification was that the light trajectory encloses an area. In doing so any ballistic theories such as Ritz's could be tested directly, as the light going one way around the device would have a different length to travel than light going the other way (the eyepiece and mirrors would be moving toward/away from the light). In Ritz's theory there would be no shift, because the net velocity between the light source and detector was zero (they were both mounted on the turntable). However in this case an effect *was* seen, thereby eliminating any simple ballistic theory. This fringe-shift effect is used today in laser gyroscopes.

Length contraction

The explanation was found in the FitzGerald–Lorentz contraction, also simply called length contraction. According to this physical law all objects physically contract along the line of motion (originally thought to be relative to the aether), so while the light may indeed transit slower on that arm, it also ends up travelling a shorter distance that exactly cancels out the drift. In 1932 the Kennedy–Thorndike experiment modified the Michelson–Morley experiment by making the path lengths of the split beam unequal, with one arm being very short. In this version a change of the velocity of the earth would still result in a fringe shift except if also the predicted time dilation is correct. Once again, no effect was seen, which they presented as evidence for both length contraction and time dilation, both key effects of relativity.

Einstein derived the FitzGerald–Lorentz contraction from the relativity postulate; thus his description of special relativity was also consistent with the apparently null results of most experiments (though not, as was recognized at the 1928 meeting, with Miller's observed seasonal effects). Today special relativity is generally considered the "solution" to the Michelson–Morley null result. However, this was not universally recognized at the time. As late as 1920, Einstein himself still spoke of a different concept of ether that was not a "ponderable medium" but something of significance nonetheless.[40]

The Trouton–Noble experiment is regarded as the electrostatic equivalent of the Michelson–Morley optical experiment, though whether or not it can ever be done with the necessary sensitivity is debatable. On the other hand, the 1908 Trouton–Rankine experiment, which can be regarded as the electrical equivalent to the Kennedy–Thorndike experiment, achieved very high sensitivity.

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External links

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